Jordan Higher Reverse Derivations on Prime Γ-Semirings

Hussein Ali Oreibi [a] Dr. Auday Hikmat Mahmoud [b]

Mustansiriyah University, College of Education, Department of Mathematics

a) hu99uh@uomustansiriyah.edu.iq
b) audaymath@gmail.com

Abstract

The aim of this paper is to investigate Jordan higher reverse derivations on prime Γ -semirings. We introduce a higher revers derivation and a Jordan higher derivation in Γ -semirings. For a 2-torsion free prime Γ -semiring M such that $x\alpha y\beta z = x\beta y\alpha z$ for all x,y, z∈M and α,β ∈Γ we prove that every Jordan higher reverse derivation of M is a higher reverse derivation of M.

Keywords: higher reverse derivation, Jordan higher reverse derivation, prime Γ -semiring

المشتقات العكسية العلياJordan على شبه الحلقات الاولية من النمط –Γ

حسين علي عريبي د. عدي حكمت محمود

الجامعة المستنصرية، كلية التربية، قسم الرياضيات

الملخص

في هذا العمل قدمنا مفاهيم المشتقات العكسية العليا ومشتقات جوردان المعكوسة العليا على شبة الحلقة S من النمط Γ. عرفنا المشتقات الثلاثية المعكوسة العليا على الحلقات شبه الأولية من النمط كاما. برهنا أن كل مشتقة جوردان معكوسة عليا معرفة على شبه الحلقة الأولية S من النمط كاما تكون مشتقة معكوسة عليا.

الكلمات المفتاحية: اشتقاق عكسي أعلى، اشتقاق عكسي أعلى في Jordan، شبة زمرة اولية من النمط _ Γ.

1. Introduction

Γ-semirings were first studied by M. K. Rao [11] as a generalization of Γ-ring as well as of semiring. It is noted that Γ-rings were considered by N. Nobusawa in 1964 in [9], there have been a few slightly different definitions for a Γ-ring. The concepts of Γ-semirings by M.Murali Krishna Rao [10] 1et M and Γ be two additive semigroups. If there exists a mapping M ×Γ × M → M (images to be denoted by $x\alpha y$ for x,y ∈ M and $\alpha \in \Gamma$) satisfying, for all x,y,z ∈ M and α, β ∈ Γ,
(i) $x\alpha(y\beta z) = (x\alpha y)\beta z$ (ii) $x\alpha(y + z) = x\alpha y + x\alpha z$ (iii) $(x + y)\alpha z = x\alpha z + y\alpha z$
(iv) $x(\alpha + \beta)y = x\alpha y + x\beta y$ then M is called a Γ −semiring. [2] Throughout this PaPer M denotes a Γ -semiring with center Z(M)[1], recall that a Γ - semiring M is called prime if a Γ M Γ b = (0) implies $a = 0$ or $b = 0$[8], and it is called semiprime if $a\Gamma M\Gamma a = (0)$ implies $a = o$[6], a prime Γ − semiring is obviously semiprime and a Γ − semiring M is called 2 -torision free if 2a = 0 implies a = 0 for every a ∈ M[5], an additive mapping d from M into itself is called a derivation s if $d(a\alpha b) = d(a)\alpha b + a\alpha d(b)$, for all a, b ∈ M, $\alpha \in \Gamma$[7] and d is said to be Jordan derivation of a Γ -semiring M if $d(a\alpha a) = d(a)\alpha a + a\alpha d(a)$, for all a ∈ M, $\alpha \in \Gamma$[4] Bresar and Vukman [3] have introduced the notion of a reverse derivation as an additive mapping d from a semiring S into itself satisfying $d(xy) = d(y)x + yd(x)$ for all x, y ∈ S M. Sammn[13] presented the study between the derivation and reverse derivation in semiprime ring SAlso it is shown that non-commutative prime rings don't admit a non-trivial skew commuting derivation. We defined in [12] the concepts of higher reverse derivation of Γ -semiring M

we introduce a higher reverse derivations and a Jordan higher reverse derivation s in Γ –semirings.we definition a Jordan triple higher reverse derivations on Γ –semirings we prove every Jordan higher reverse derivation of a prime *Γ*-semiring is higher reverse derivation.

2. Jordan higher Reverse Derivations on Γ-semirings

Definition (2.1):

Let M be a Γ-semirings and D $=(d_i)_{i\in N}$ be a family of additive mappings of M, such that $d_0 = Id_M$ then D is called a higher reverse derivation s on M if for every a, ƀ∈ M, λ ∈ Γand n ∈N

$$d_n(a\lambda ƀ) = \sum_{i+j=n} d_i(ƀ)\,\lambda\, d_j(a) \;\ldots.(i)$$

D is called a Jordan higher reverse derivations on M if for every a ∈ M, λ ∈Γand n ∈N.

$$d_n(a\lambda a) = \sum_{i+j=n} d_i(a)\lambda\, d_j(a) \;\ldots\ldots\ldots(ii)$$

D is, called a Jordan triple higher reverse derivations on M if for every a, ƀ∈ M, λ, β∈Γand n ∈N

$$d_n(a\lambda ƀ\,\beta a) = d_n(a)\beta a\lambda ƀ + \sum_{\substack{i+j+r=n\\ i<n}} d_i(a)\beta\, d_j(ƀ)\lambda\, d_r(a) \;\ldots..(iii)$$

Any higher reverse derivation on a Γ-semirings M is obviously a Jordan higher reverse derivation on M, but this is not always the case, as the following example demonstrates:

Example (2.2):

Let M be a Γ-semirings and a ∈ M such that xΓa Γx = 0 for all x ∈ M and let a Γa = 0, 1et $D=(d_i)_{i\in N}$ be a family of additive mappings of M into itself define by for each n ∈N:

$d_n(\boldsymbol{x}) = n\boldsymbol{x}\lambda a + a\lambda\boldsymbol{x}$ For all $\boldsymbol{x}$ ∈ M, λ∈Γ

We note that *D* is Jordan higher reverse derivation on M but not higher reverse derivation on M.

Lemma (2.3):

Let M be a Γ-semiring and D = $(d_i)_{i\in N}$ be a higher reverse derivation on M then for all a, ƀ, ƈ ∈ M and $\lambda, \beta \in \Gamma$ the following statements are hold:

$$(i)\; d_n(a\lambda ƀ + ƀ\lambda a) = \sum_{i+j=n} d_i(ƀ)\lambda d_j(a) + d_i(a)\lambda d_j(ƀ)$$

In special case if ƀ∈ Z (M)

$$(ii)\; d_n(a\lambda ƀ\,\beta a + a\beta ƀ\,\lambda a) = d_n(a)\beta a\lambda ƀ + \sum_{\substack{i+j+r=n\\ i<n}} d_i(a)\beta\, d_j(ƀ)\lambda\, d_r(a) + d_n(a)\lambda a\beta ƀ + \sum_{\substack{i+j+r=n\\ i<n}} d_i(a)\beta\, d_j(ƀ)\lambda\, d_r(a)$$

$$(iii) d_n(a\lambda ƀ\,\lambda a) = d_n(a)\lambda a\lambda ƀ + \sum_{\substack{i+j+r=n\\ i<n}} d_i(a)\beta\, d_j(ƀ)\lambda\, d_r(a)$$

$$(iv)\; d_n(a\lambda ƀ\lambda ƈ + ƈ\lambda ƀ\lambda a) = d_n(ƈ)\lambda a\lambda ƀ + \sum_{\substack{i+j+r=n\\ i<n}} d_i(ƈ)\lambda d_j(ƀ)\lambda\, d_r(a) + d_n(a)\lambda ƈ\lambda ƀ + \sum_{\substack{i+j+r=n\\ i<n}} d_i(a)\lambda\, d_j(ƀ)\lambda\, d_r(ƈ)$$

Proof:

(i) Replace $(a+ƀ)$ for a in definition (2.1)(ii) we have:

$$d_n\big((a+ƀ)\lambda(a+ƀ)\big)=\sum_{i+j=n} d_i(a+ƀ)\lambda d_j(a+ƀ)$$
$$=\sum_{i+j=n}\big(d_i(a)+d_i(ƀ)\big)\lambda(d_j(a)+d_j(ƀ))$$
$$=\sum_{i+j=n} d_i(a)\lambda d_j(a)+d_i(ƀ)\lambda d_j(a)+d_i(a)\lambda d_j(ƀ)+d_i(ƀ)\lambda d_j(ƀ) \quad \ldots\ldots(1)$$

On the second party:

$$d_n((a+ƀ)\lambda(a+ƀ))=d_n(a\lambda a+a\lambda ƀ+ƀ\lambda a+ƀ\lambda ƀ)$$
$$=d_n(a\lambda a+ƀ\lambda ƀ)+d_n(a\lambda ƀ+ƀ\lambda a)$$
$$=\sum_{i+j=n} d_i(a)\lambda d_j(a)+d_i(ƀ)\lambda d_j(ƀ)+d_n(a\lambda ƀ+ƀ\lambda a) \quad \ldots\ldots(2)$$

Comparing (1) and (2) we get:

$$d_n(a\lambda ƀ+ƀ\lambda a)=\sum_{i+j=n} d_i(ƀ)\lambda d_j(a)+d_i(a)\lambda d_j(ƀ)$$

(ii) Replace $a\beta ƀ+ƀ\beta a$ for $ƀ$ in (i) we get:

$$d_n(a\lambda(a\beta ƀ+ƀ\beta a)+(a\beta ƀ+ƀ\beta a)\lambda a)$$
$$=d_n(a\lambda(a\beta ƀ)+a\lambda(ƀ\beta a)+(a\beta ƀ)\lambda a+(ƀ\beta a)\lambda a)$$
$$=d_n((a\lambda a)\beta ƀ+(a\lambda ƀ)\beta a+(a\beta ƀ)\lambda a+(ƀ\beta a)\lambda a)$$
$$=\sum_{i+j=n} d_i(ƀ)\beta d_j(a\lambda a)+d_i(a)\beta d_j(a\lambda ƀ)+d_i(a)\lambda d_j(a\beta ƀ)+d_i(a)\lambda d_j(ƀ\beta a)$$
$$=\sum_{i+j+r=n} d_i(ƀ)\beta d_j(a)\lambda d_r(a)+d_i(a)\beta d_j(ƀ)\lambda d_r(a)+d_i(a)\lambda d_j(ƀ)\beta d_r(a)$$
$$+d_i(a)\lambda d_j(a)\beta d_r(ƀ)$$
$$=d_n(ƀ)\beta a\lambda a+\sum_{\substack{i<n\\ i+j+r=n}} d_i(ƀ)\beta d_j(a)\lambda d_r(a)+d_n(a)\beta a\lambda ƀ+\sum_{\substack{i<n\\ i+j+r=n}} d_i(a)\beta d_j(ƀ)\lambda d_r(a)$$
$$+d_n(a)\lambda a\beta ƀ+\sum_{\substack{i<n\\ i+j+r=n}} d_i(a)\lambda d_j(ƀ)\beta d_r(a)$$
$$+d_n(a)\lambda ƀ\beta a+\sum_{\substack{i<n\\ i+j+r=n}} d_i(a)\lambda d_j(a)\beta d_r(ƀ) \quad \ldots\ldots(1)$$

On the Second party:

$$d_n(a\lambda(a\beta ƀ+ƀ\beta a)+(a\beta ƀ+ƀ\beta a)\lambda a)$$
$$=d_n(a\lambda a\beta ƀ+a\lambda ƀ\beta a+a\beta ƀ\lambda a+ƀ\beta a\lambda a)$$
$$=d_n(a\lambda a\beta ƀ+ƀ\beta a\lambda a)+d_n(a\lambda ƀ\beta a+a\beta ƀ\lambda a)$$
$$=d_n(ƀ)\beta a\lambda a+\sum_{\substack{i<n\\ i+j+r=n}} d_i(ƀ)\beta d_j(a)\lambda d_r(a)$$
$$+d_n(a)\lambda ƀ\beta a+\sum_{\substack{i<n\\ i+j+r=n}} d_i(a)\lambda d_j(a)\beta d_r(ƀ)+d_n(a\lambda ƀ\beta a+a\beta ƀ\lambda a) \quad \ldots.(2)$$

Comparing (1) and (2) we have the expected outcome.

(iii) Changing out λ for β in definition (2.1)(iii) we get:

$$d_n(a\lambda b\,\lambda a) = d_n(a)\lambda a\lambda b + \sum_{i+j+r=n}^{i<n} d_i(a)\lambda d_j(b)\lambda d_r(a)$$

(iv)Replacing $a + c$ for a in (iii) we have:

$$d_n\big((a + c)\lambda b\,\lambda(a + c)\big) = d_n(a + c)\lambda(a + c)\lambda b + \sum_{i+j+r=n}^{i<n} d_i(a + c)\lambda d_j(b)\lambda d_r(a + c)$$

$$= d_n(a)\lambda a\lambda b + \sum_{i+j+r=n}^{i<n} d_i(a)\lambda d_j(b)\lambda d_r(a)$$

$$+ d_n(c)\lambda a\lambda b + \sum_{i+j+r=n}^{i<n} d_i(c)\lambda d_j(b)\lambda d_r(a)$$

$$+ d_n(a)\lambda c\lambda b + \sum_{i+j+r=n}^{i<n} d_i(a)\lambda d_j(b)\lambda d_r(c)$$

$$+ d_n(c)\lambda c\lambda b + \sum_{i+j+r=n}^{i<n} d_i(c)\lambda d_j(b)\lambda d_r(c) \qquad \ldots\ldots(1)$$

On the second party:

$$d_n((a + c)\lambda b\,\lambda(a + c))$$
$$= d_n(a\lambda b\,\lambda a + a\lambda b\,\lambda c + c\lambda b\,\lambda a + c\lambda b\,\lambda c)$$
$$= d_n(a\lambda b\,\lambda a + c\lambda b\,\lambda c) + d_n(a\lambda b\,\lambda c + c\lambda b\,\lambda a)$$
$$= d_n(a)\lambda\, a\lambda\, b + \sum_{i+j+r=n}^{i<n} d_i(a)\lambda\, d_j(b)\lambda\, d_r(a)$$
$$+ d_n(c)\lambda\, c\lambda\, b + \sum_{i+j+r=n}^{i<n} d_i(c)\lambda\, d_j(b)\lambda\, d_r(c) + d_n(a\lambda b\,\lambda c + c\lambda b\,\lambda a) \ldots\ldots(2)$$

Comparing (1) and (2) we have the required resu1t.

Definition (2.4):

Let D= $(d_i)_{i\in N}$ be a Jordan higher reverse derivation on a Γ-semirings M with additive inverse and identity element for every n ∈N, a, b ∈ M and λ∈Γ we define:

$$\psi_n(a, b)_\lambda = d_n(a\lambda b) - \sum_{i+j=n} d_i(b)\lambda d_j(a)$$

Lemma (2.5):

Let D= $(d_i)_{i\in N}$ be a Jordan higher reverse derivation on a Γ-semirings M with additive inverse and identity element for all a, b, c ∈ M, λ, β∈Γand n ∈N then:

(i) $\psi_n(a, b)_\lambda = -\psi_n(b, a)_\lambda$

(ii) $\psi_n(a + b, c)_\lambda = \psi_n(a, c)_\lambda + \psi_n(b, c)_\lambda$

(iii) $\psi_n(a, b + c)_\lambda = \psi_n(a, b)_\lambda + \psi_n(a, c)_\lambda$

$(iv)$$\psi_n(a, b)_{\lambda+\beta} = \psi_n(a, b)_\lambda + \psi_n(a, b)_\beta$

proof:

(i) By 1emma (2.3)(i) and since d_n is additive mapping

$$d_n(a\lambda b + b\,\lambda a) = \sum_{i+j=n} d_i(b)\lambda d_j(a) + d_i(a)\lambda d_j(b)$$

$$d_n(a\lambda b) + d_n(b\,\lambda a) = \sum_{i+j=n} d_i(b)\lambda d_j(a) + \sum_{i+j=n} d_i(a)\lambda d_j(b)$$

$$d_n(a\lambda ƀ) - \sum_{i+j=n} d_i(ƀ)\lambda d_j(a) = -d_n(ƀ\lambda a) + \sum_{i+j=n} d_i(a)\lambda d_j(ƀ)$$
$$d_n(a\lambda ƀ) - \sum_{i+j=n} d_i(ƀ)\lambda d_j(a) = -(d_n(ƀ\lambda a) - \sum_{i+j=n} d_i(a)\lambda d_j(ƀ))$$
$$\psi_n(a,ƀ)_\lambda = -\psi_n(ƀ,a)_\lambda$$
$$(ii)\ \psi_n(a+ƀ,c)_\lambda = d_n((a+ƀ)\lambda c) - \sum_{i+j=n} d_i(c)\lambda d_j(a+ƀ)$$
$$= d_n(a\lambda c + ƀ\lambda c) - (\sum_{i+j=n} d_i(c)\lambda d_j(a) + d_i(c)\lambda d_j(ƀ))$$
$$= d_n(a\lambda c) - \sum_{i+j=n} d_i(c)\lambda d_j(a) + d_n(ƀ\lambda c) - \sum_{i+j=n} d_i(c)\lambda d_j(ƀ)$$
$$= \psi_n(a,c)_\lambda + \psi_n(ƀ,c)_\lambda$$

$(iii) - (iv)$: As the same way of (ii).

Remark (2.6):

Note that D= $(d_i)_{i\in N}$ is higher reverse derivations on Γ-semirings M with additive inverse and identity if and only if $\psi_n(a,ƀ)_\lambda = 0$ for all $a, ƀ \in$ M, $\lambda \in\Gamma$ and n ∈N.

3. The Main Results

Lemma 3.1: [5]

Let's M is a 2-torsion free semi prime Γ-semiring with additive identity and inverse and supposing that $a,ƀ \in$ M, if $a\Gamma m\Gamma ƀ + ƀ\Gamma m\Gamma a = 0$ for any m∈M, then $a\Gamma m\Gamma ƀ = ƀ\Gamma m\Gamma a = 0$.

Lemma (3.2):

Let d = $(d_i)_{i\in N}$ be a Jordan higher reverse derivations of a 2-torsion free Γ – semiring M with additive inverse and identity element. Let $n \in N$ and assume that $a, ƀ, m \in$ M; $\lambda, \beta \in \Gamma$ Then:

$$\psi_n(a,ƀ)_\lambda \beta m\beta [a,ƀ]_\lambda + [a,ƀ]_\lambda \beta m\beta\ \psi_n(a,ƀ)_\lambda = 0$$

Proof:

We consider U = $a\lambda ƀ\beta m\beta ƀ\lambda a + ƀ\lambda a\beta m\beta a\lambda ƀ$. first, we compute

d_n (U) = d_n $(a\lambda ƀ\beta m\beta ƀ\lambda a + ƀ\lambda a\beta m\beta a\lambda ƀ)$

= d_n $(a\lambda (ƀ\beta m\beta ƀ)\lambda a) + d_n (ƀ\lambda (a\beta m\beta a)\lambda ƀ)$

Since d_n is additive mapping then by lemma (2.3) *iii* we obtain on one hand

$$= \sum_{s+t=n} d_s(a\lambda (ƀ\beta m\beta ƀ)\lambda a) + d_t(ƀ\lambda(a\beta m\beta a)\lambda ƀ)$$
$$= \sum_{i+j+r+t=n} d_i(a)\lambda d_j(ƀ)\beta d_r(m)\beta d_s(ƀ)\lambda d_t(a)$$
$$+ \sum_{i+j+r+t=n} d_i(ƀ)\lambda d_j(a)\beta d_r(m)\beta d_s(a)\lambda d_t(ƀ)$$

On the other hand:

$d_n(U) = d_n\ (a\lambda ƀ\beta m\beta ƀ\lambda a + ƀ\lambda a\beta m\beta a\lambda ƀ)$

$= d_n((a\lambda ƀ)\beta m\beta(ƀ\lambda a) + (ƀ\lambda a)\beta m\beta(a\lambda ƀ))$

Using lemma (2.3 *iv*)

$$d_n(U) = \sum_{i+j+r=n} d_i(ƀ\lambda a)\beta d_j(m)\beta d_r(a\lambda ƀ) + \sum_{i+j+r=n} d_i(a\lambda ƀ)\beta d_j(m)\beta d_r(ƀ\lambda a)$$

Comparing the tow expressions for $d_n(U)$, we have:

$$\sum_{i+j+r+s+t=n} d_i(ƀ)\,\lambda\, d_j(a)\beta d_r(\mathrm{m})\beta d_s(a)\,\lambda\, d_t(ƀ) - \sum_{i+j+r=n} d_i(a\,\lambda\, ƀ)\beta d_j(\mathrm{m})\beta d_r(ƀ\,\lambda\, a)$$
$$+ \sum_{i+j+r+s+t=n} d_i(a)\,\lambda\, d_j(ƀ)\beta d_r(\mathrm{m})\beta d_s(ƀ)\,\lambda\, d_t(a)$$
$$- \sum_{i+j+r=n} d_i(ƀ\,\lambda\,\mathrm{a})\beta d_j(\mathrm{m})\beta d_r(a\,\lambda\, ƀ) \qquad \dots(1)$$

By the inductive assumption we can substitute:

$d_g(u\,\lambda\, v) for \sum_{i+j=g} d_i\,(v\,)\lambda\, d_j(u)\ where\ g < n\ dor\ u = a, ƀ$

and $v = ƀ, a$

Therefore:

$$\sum_{i+j+r+s+t=n} d_i(ƀ)\,\lambda\, d_j(a)\beta d_r(\mathrm{m})\beta d_s(a)\,\lambda\, d_t(ƀ) - \sum_{i+j+r=n} d_i(a\,\lambda\, ƀ)\beta d_j(\mathrm{m})\beta d_r(ƀ\,\lambda\, a)$$
$$= -\,(d_n(a\lambda ƀ) - \sum_{i+j=n} d_i(ƀ)\,\lambda\, d_j(a))\beta \mathrm{m}\beta ƀ\,\lambda\, a\ - a\lambda ƀ\beta \mathrm{m}\beta(d_n(ƀ\lambda a) - \sum_{s+t=n} d_s(a)\lambda\, d_t(ƀ)\,)$$

= - ($\psi_n(a, ƀ)_\lambda\ \beta m\beta\ ƀ\lambda a$ + $a\lambda ƀ\ \beta m\beta\ \psi_n(ƀ, a)_\lambda$) ……(2)

Similarly,

$$\sum_{i+j+r+t=n} d_i(a)\lambda\, d_j(ƀ)\beta d_r(\mathrm{m})\beta d_s(ƀ)\lambda\, d_t(a) - \sum_{i+j+r=n} d_i(ƀ\,\lambda\,\mathrm{a})\beta d_j(\mathrm{m})\beta d_r(a\lambda ƀ)$$

= - ($\psi_n(ƀ, a)_\lambda\ \beta m\beta\ a\lambda ƀ$+ $ƀ\lambda a\beta m\beta\ \psi_n(a, ƀ)_\lambda$ ………(3)

Hence, by using (2) and (3) we obtain:

-($\psi_n(a, ƀ)_\lambda\ \beta m\beta\ ƀ\lambda a\ +\ a\lambda ƀ\ \beta m\beta\ \psi_n(ƀ, a)_\lambda\ + \psi_n(ƀ, a)_\lambda\ \beta m\beta a\lambda ƀ$+
$ƀ\lambda a\beta m\beta\ \psi_n(a, ƀ)_\lambda\,) = 0$

By lemma(2.5*i*) we get:

$-\,(\psi_n(a, ƀ)_\lambda\ \beta m\beta\ ƀ\,\lambda\, a - \psi_n(a, ƀ)_\lambda\ \beta m\beta\ a\,\lambda\, ƀ$
$+\ ƀ\lambda\, a\ \beta m\beta\ \psi_n(a, ƀ)_\lambda - a\lambda\, ƀ\ \beta m\beta\ \psi_n(a, ƀ)_\lambda\,) = 0$

- ($\psi_n(a, ƀ)_\lambda\ \beta m\beta\ (ƀ\,\lambda\, a - a\,\lambda\, ƀ) + (ƀ\,\lambda\, a - a\,\lambda\, ƀ)\psi_n(a, ƀ)_\lambda\,) = 0$

$$\psi_n(a, ƀ)_\lambda \beta m\beta[a, ƀ]_\lambda +\ [a, ƀ]_\lambda \beta m\beta \psi_n(a, ƀ)_\lambda = 0$$

Lemma (3.3):

Let d = $(d_i)_{i\in N}$ be a Jordan higher reverse derivation s of a 2-torsion free prime Γ – semiring M with additive inverse and identity element. Let $n \in N$ and $a, ƀ, m\ \in$ M; $\lambda, \beta \in \Gamma$ Then:

$$\psi_n(a, ƀ)_\lambda \beta m\beta[a, ƀ]_\lambda = 0$$

Proof:

By lemma 3.2 we get:

$\psi_n(a, ƀ)_\lambda \beta m\beta[a, ƀ]_\lambda +\ [a, ƀ]_\lambda \beta m\beta \psi_n(a, ƀ)_\lambda = 0$

By Lemma (3.1):

$\psi_n(a, ƀ)_\lambda \beta m\beta[a, ƀ]_\lambda = 0$

Theorem (3.4):

Let d = $(d_i)_{i\in N}$ be aJordan higher reverse derivations of a 2-torsion free prime Γ – semiring M with additive inverse and identity element. Let $n \in N$ and $a, ƀ, m\ \in$ M; $\lambda, \beta \in \Gamma$ then:

$\psi_n(a, ƀ)_\lambda \beta m\beta[c\!\!', d]_\lambda = 0$

Proof:

Replacing a + $c\!\!'$ for a in lemma 3.3

$\psi_n(a + c\!\!', ƀ)_\lambda \beta m\beta[a + c\!\!', ƀ]_\lambda = 0$

$\psi_n(a, ƀ)_\lambda \beta m\beta[a, ƀ]_\lambda + \psi_n(a, ƀ)_\lambda \beta m\beta[c\!\!', ƀ]_\lambda + \psi_n(c\!\!', ƀ)_\lambda \beta m\beta[a, ƀ]_\lambda +$

$\psi_n(c,b)_\lambda \beta m \beta [c,b]_\lambda = 0$

By lemma 3.3 we get:

$\psi_n(a,b)_\lambda \beta m \beta [a,b]_\lambda = \psi_n(c,b)_\lambda \beta m \beta [c,b]_\lambda = 0$

Then we have:

$\psi_n(a,b)_\lambda \beta m \beta [c,b]_\lambda + \psi_n(c,b)_\lambda \beta m \beta [a,b]_\lambda = 0$

Therefore, we get:

$\psi_n(a,b)_\lambda \beta m \beta [c,b]_\lambda \beta m \beta \psi_n(a,b)_\lambda \beta m \beta [c,b]_\lambda$

$= -\psi_n(a,b)_\lambda \beta m \beta [c,b]_\lambda \beta m \beta \psi_n(c,b)_\lambda \beta m \beta [a,b]_\lambda = 0$

Hence,by primness of M:

$\psi_n(a,b)_\lambda \beta m \beta [c,b]_\lambda = 0$ ... (1)

Similarly, by replacing $b + d$ for b in lemma 3.3 we get:

$\psi_n(a,b)_\lambda \beta m \beta [a,d]_\lambda = 0$ ... (2)

Thus $\psi_n(a,b)_\lambda \beta m \beta [a+c, b+d]_\lambda = 0$

$\psi_n(a,b)_\lambda \beta m \beta [a,b]_\lambda + \psi_n(a,b)_\lambda \beta m \beta [a,d]_\lambda + \psi_n(a,b)_\lambda \beta m \beta [c,b]_\lambda$
$+\psi_n(a,b)_\lambda \beta m \beta [c,d]_\lambda = 0$

By (1),(2) and lemma (3.3) we get:

$\psi_n(a,b)_\lambda \beta m \beta [c,d]_\lambda = 0$

Theorem (3.5)

Let M be a 2-torsion free prime Γ-semiring. Then every Jordan higher reverse derivation of M is higher reverse derivation of M.

Proof:

Let $d = (d_i)_{i \in N}$ be a Jordan higher reverse derivation of 2-torsion free prime Γ–semiring M, by theorem 3.4 we get:

$\psi_n(a,b)_\lambda \beta m \beta [c,d]_\lambda = 0$

Since M is prime, we get either $\psi_n(a,b)_\lambda = 0$ or $[c,d]_\lambda = 0$ for all $a, b, c, d \in M$, $\lambda \in \Gamma$ and $n \in N$, if $[c,d]_\lambda \neq 0$ for all $c, d \in M$ and $\lambda \in \Gamma$.

Then $\psi_n(a,b)_\lambda = 0$ for all $a, b \in M$ and $\lambda \in \Gamma$ and $n \in N$ and by remark (2.6) d is higher reverse derivation of M.

But if $[c,d]_\lambda = 0$ for all $c, d \in M$ and $\lambda \in \Gamma$ then M commutative and therefore, we have from 1emma 2.3(i).

$$d_n(2a\lambda b) = 2 \sum_{i+j=n} d_i(b)\, \lambda\, d_j(a)$$

Since M is 2-torsion free, we find d is a higher reverse derivation

4. References

[1] M. Asci and S. Ceran, ”The commutativity in prime gamma rings with left derivation”, International Mathematical,Vol. 2, No.3, pp. 103-108, 2007.

[2] W. E. Barnes, “On The Γ-rings of Nobusawa ”, Pacific J. Math.,Vol. 18, pp.411-422, 1966.

[3] M. Bresar and J. Vukman, “On Some additive mappings in rings with involution“, Aequation Math., 38, pp.178-185, 1989.

[4] Y. Ceven and M. A. Ozlurk, “On Jordan Generalized Derivation in gamma rings“, hacettepe, J. of Mathematics and Statistics, Vol.33, pp. 11-14, 2004.

[5] S. Chakraborty and A.C. Paul, “On Jordan K-derivation of 2-torision free ΓN-ring”, Pun Jab University J-of Math., Vol. 40, pp.97-101, 2008.

[6] T. K. Dutta and S. K. Sadar, “Semmiprime Ideals and Irreducible Ideal Of Γ-semirings”, Novi Sad J. Math., Vol.30, No.1, pp.97-108, 2000.

[7] F. J. Jing, ”On Derivation Of Γ-rings”, QUFU Shi Fan Daxue Xuebeo Ziran Kexue Ban, Vol. 13, No. 4, PP. 159-161,1987.

[8] S.Kyuno, “On Prime gamma Rings”,pacific J. of Math., Vol.75, pp.185-190,1987

[9] T. K. Mukherjee, M. K. Sen, S. ghosh, Chain conditions on semirings, Int. J. Math. Math. Sci. 19(2), pp.321-326, 1996.

[10] M. Murali Krishna Rao, "Ideal Theory of gamma-semirings", Book, June 2019.

[11] K. Murata, Y. Kurata and, H. Marubayashi, “A generalization of prime ideals in rings“, Osaka J. Math. Vol.6, pp. 291-301, 1969.

[12] S. M. Salih and M. R. Salih, "On Jordan generalized higher Reverse Derivations in Γ-rings", Article in IOSR Journal of Mathematics, January 2014.

[13] M. Samman and N. Alyamani,”Derivation and Reverse Derivation in Semiprime Rings”, International Mathematical Forum, Vol. 2, No. 39, pp.1895-1902, 2007.