ON THE RESULTS OF CATEGORY IN TOPOLOGICAL GROUPOID

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Abstract: This search discuss the problem between topological group and topological groupoid . We shall take some consequences of groupoid space and group space and make a relation with the tensor product , As well we discuss the action between the action of topological groupoid and the action of topological group and we attach a relation with the tensor product . Some remarks , proposition and examples are our own except if else is referred to.

Keywords: Topological group, Topologicalgroupoid, Ehresmann groupoid, The action of topological groupoid .

Article History: We take a groupoid and include the topological space. We get that the topological groupoid. After that we inter the action we get after that the action of topological groupoid topological groupoid is a generalization for the group and we make a lot of relation between them

1. Introduction:

In this paper, we study the basic construction of the action of topological group and topological groupoid and divide into two sections. In section one , we recollection some meanings which are (Topological group , topological groupoid , tensor product , the morphism of topological groupoid , topological group and topo. Submersion map) . this category can be deemed a generalization of the weel-known category " Category of Topological Group".

In section two, we recollection the meaning of the actions of both topological group (Ehresmann groupoid) topological groupoid and used for erect a particular and substantial kinds of groupoids (action groupoid $H{*}\Gamma{*}H$). Then substantial morphism which are action morphisms S ans S^* will be formed.[11]

Both sections are submitted with explanation and propositions, most of these are novel (to the best of our knowledge) .

2. Properties of Topological group and Topological groupoid :

(2-1) Definition [2],[5],[7]:

A set Ω with two constructions which are concordant :

(1) Ω is a group (2) Ω is a topological space

i.e. the composition law $\delta : \Omega \times \Omega \to \Omega$ and (the inversion law) $\sigma : \Omega \to \Omega$ are both continuous this definition called *Topological Group.*

(2-2) Examples [5] , [9]:

(i) R is a topological group with respect to addition. Also, R \ {0} is a topological group with respect to multiplication.

(ii) Every group is a topological group when equipped with the discrete topology.

(iii) Every group is a topological group when equipped with the indiscrete topology.

(iv) The groups GL(n, K), K ∈ {R, ₵} of invertible real or complex matrices are topological groups.

(2-3) Definition [6]:

A morphism s : Ω→Ω' , of topological groups is a homomorphism of groups whom it continuous.

(2-4) Definition [1],[7]:

An isomorphism S: Ω→Ω'of topological groups is :

(1) An isomorphism of ((abstract groups)) .

(2) A homeomorphism of topological spaces .

(2-5) Definition [1],[4],[5]:

Let Ω be a topological groupoid. A subset (P) of Ω is called **topological subgroup** if (P) is a subgroup and (P) is subspace of Ω.

(2-6) Example [7]:

All open subgroup of topological group is a topological subgroup.

Let s: H→K be a continuous map. Then s is a (topological) submersion if for every $p_\circ \in H$ there is an open neighborhood U of $s(p_\circ)$ in K and continuous right inverse h:U→H to s s.t. $h(s(p_\circ)) = p_\circ$.

(2-7) Definition [6] , [10]:

Let H_1 and H_2 be groupoids spaces **the Tensor product** of $H_1\otimes H_2$ consist of linear combinations of elements of the form $h_1\otimes h_2$ where $h_1\in H_1$ and $h_2\in H_2$ with the following relations:

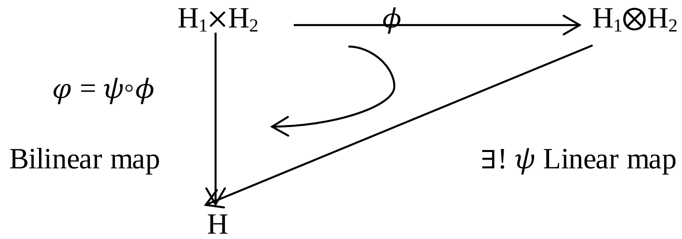
(1) $K(h_1\otimes h_2) = (kh_1)\otimes h_2 = h_1 \otimes (kh_2)$ for any scalars k.

(2) $h_1 \otimes h_2 + h_1' \otimes h_2 = (h_1 + h_1') \otimes h_2$.

(3) $(h_1 \otimes h_2) + (h_1 \otimes h_2') = h_1\otimes (h_2\otimes h_2')$ for all $h_1, h_1' \in H_1$ and h_2 , $h_2'\in H_2$.

(2-8) Definition [8]:

Let H_1 and H_2 be any groupoid spaces then **the tensor product** $H_1\otimes H_2$ exists. For there more if φ is any bilinear map of $H_1\times H_2$ into H then there exist a unique linear map, s.t. the following diagram commutes, this property of $H_1\otimes H_2$ is called universal property of tensor product, where H is groupoid space.



(2-9) Lemma:

Let s: H→K be a submersion map then s^c is a closed map.

Proof: Let s:H→K looks like a projection with respect to suitable charts:

$s' = \psi \circ f \circ \varphi^{-1} : R^h \to R^k$

with $h \geq k$ where $R^h \cong R^k \times R^{h-k}$. It is known in fact in topology that a projection is an open map.

Now, as both ψ and φ^{-1} are diffeomorphisms, the composed map s' is open iff s is open.

Now since s is open map then s^c is closed map.

(2-10) Definition [1],[5],[7]:

A **topological groupoid** is a groupoid (H,K) together with topologies on H and K s.t. the maps τ:H→K, λ:K→H, σ:H→H and ω:H∗H→H are continuous maps where H∗H has the subspace topology from H×H .

(2-11) Remark [3],[4]:

(1) the map ρ:H→K is continuous since $\rho = \eta \circ \gamma$

(2) In a topological group Ω the continuity of the map $\psi:\Omega\times\Omega\to\Omega$, $\psi(r , z) = rz^{-1}$ gives the continuity of the composition law and inversion law of Ω since $rz = r(z^{-1})^{-1}$ and $z^{-1} = e\, z^{-1}$.

(2-12) Proposition:

Any groupoid (f∘g , g∘f) is a topological groupoid then we have τ , ρ , λ and γ are continuous maps where δ is given in ex.(Let (H , K) be any groupoid. We denote by ∧H the subset of H× H ×H of triple of elements of H having the same source. This set forms a subgroupoid of Descartes groupoid H× H ×H which has H as base. Now let δ:∧H→H the map defined by δ(h_1 , h_2 , h_3)= $h_1.h_2.h_3^{-1}$. Then δ is a groupoid morphism over δ:H → K).

Proof: let (f∘g , g∘f) be topological groupoid then we have τ,ρ,λ and γ are continuous maps. The continuity of δ is given by the composition maps.

$$\wedge f\circ g \xrightarrow{(I\times\lambda)|\wedge f\circ g} f\circ g * f\circ g \xrightarrow{\gamma} f\circ g$$

$$(r_1 , r_2) \longrightarrow (r_1 , r_2^{-1}) \longrightarrow r_1 r_2^{-1}$$

Now let τ,ρ and δ are continuous maps the continuity of λ is given by the composition of continuous maps;

$$f\circ g \xrightarrow{\Delta} f\circ g \times f\circ g \xrightarrow{\tau\times I} g\circ f \times f\circ g \xrightarrow{\rho\times I} \wedge f\circ g \xrightarrow{\delta} f\circ g$$

$$r \longrightarrow (r , r) \longrightarrow (\tau(r) , r) \longrightarrow (\rho(\tau(r)) , r) \longrightarrow r^{-1}$$

where ∆ is the diagonal of f∘g . the continuity of γ is given by the composition of continuous maps;

$$f\circ g * f\circ g \xrightarrow{(I\times\lambda)|f\circ g * f\circ g} \wedge f\circ g \xrightarrow{\delta} f\circ g$$

$$((r_1,x_1) , (r_2,x_2)) \longrightarrow ((r_1,x_1) , (r_2,x_2)^{-1}) \longrightarrow (r_1,x_1)(r_2,x_2)$$

Then we get (f∘g , g∘f) is a topological groupoid.

(2-13) Remarks:

(1) Any topological group Ω is a topological groupoid.

(2) Any trivial groupoid H×Ω×K where Ω is a topological group and H and K are a topological spaces, are topological groupoid with the product topology.

(2-14) Definition[3],[4]:

A **topological subgroupoid** of a topological groupoid (H,K) is the subgroupoid (A,B) with the subspace topology from (H,K).

(2-15) Definition [1],[3],[4]:

A **morphism** of topological groupoid is a morphism of groupoids $(s , s_\circ) : (H , K) \rightarrow (H',K')$ s.t. s and $s_\circ$ are continuous maps.

(2-16) Definition [1],[4],[5],[7] :

An isomorphism of topological groupoids is a morphism of topological groupoids s.t.

$s: H \rightarrow H'$ is a homeomorphism.

(2-17) Example [5]:

The transitor of any topological groupoid (H ,K) is a morphism of topological groupoids. Ω is continuous since $\omega = (\gamma\times\Gamma) o \Delta$ where Δ is the diagonal map of H.

(2-18) Proposition:

Let $((H\times H),(K\times K))$ be a topological groupoid then :

(1) The source and the target maps are identification maps.

(2) The map of unities δ is an embedding.

(3) The inversion map γ is a homeomorphism.

Proof: (1) by definition of groupoid and the definition of topological groupoid we get $\alpha_\circ\delta = I_{KxK}$ and $\beta_\circ\delta = I_{KxK}$ hence, α and β are identification maps since they have δ as a continuous right inverse.

(2) by definition of groupoid and the definition of topological groupoid we get δ is continuous injective map and its continuous bijective map onto its image $\delta(K\times K)$. To prove δ is an open map let $V\times V$ be an open in $K\times K$. Then the set $\delta(V\times V)$ is ${}_{(VxV)}(H\times H)_{(VxV)} \cap \delta(K\times K)$ which is an open in $\delta(K\times K)$ since ${}_{(VxV)}(H\times H)_{(VxV)} = \varphi^{-1}(V\times V)$ is an open in $(G\times G)$ where $\delta(K\times K)$ has the subspace topology from $(H\times H)$ hence δ is a homeomorphism from $K\times K$ onto $\delta(K\times K)$ therefore δ is an embedding.

(3) by definition of groupoid and the definition of topological groupoid we get γ is a bijective continuous map and its inverse $\gamma^{-1}(d\times d) = \gamma\ ((d\times d)^{-1})$. For each $d\times d \in K\times K$ is also continuous hence γ is a homeomorphism.

(2-19) Proposition:

Let $(s , s_\circ) : (H,K) \rightarrow (H',K')$ is an isomorphism of topological groupoid then:

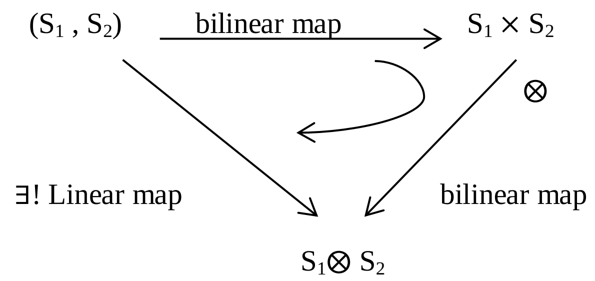
(1) The map $\otimes^{m}_{i=1}S_\circ : \otimes^{m}_{i=1} K_i \rightarrow \otimes^{m}_{i=1}K_i'$ is a homeomorphism.

(2) The map $\otimes^{m}_{i=1}S_x : \otimes^{m}_{i=1} H_ix \rightarrow \otimes^{m}_{i=1} H'_{S_\circ(x)}$ is a homeomorphism, for every $x \in K$.

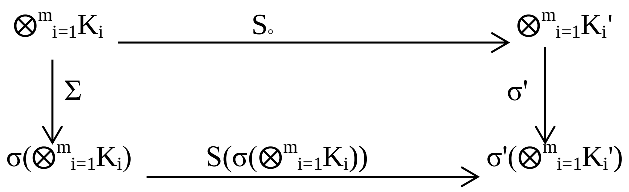
(3) The map $\otimes^{m}_{i=1}\ {}_xS_x: \otimes^{m}_{i=1}\ xK_ix \rightarrow \otimes^{m}_{i=1}\ {}_{S_\circ(x)}K_i'{}_{S_\circ(x)}$ is an isomorphism of topological groups, for each $x \in K$.

Proof: (1) let $S_1 \otimes S_2 \otimes S_3 \ldots\ldots\ldots.. \otimes S_n \in S$

We take $S_1 \otimes S_2$ and $S_1 \times S_2$ satisfy the universal property



Now by using the definition of morphism of topological groupoid and the following commutative diagram in T



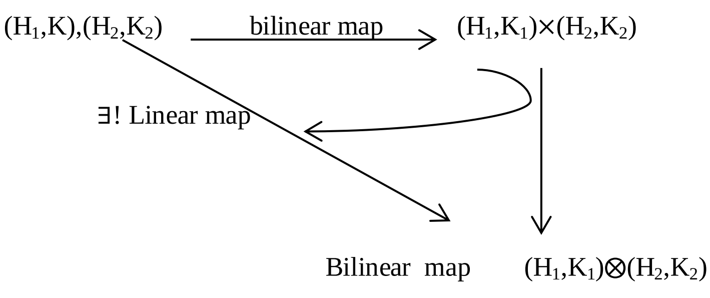
then $\otimes^{m}_{i=1}S_{\circ}$ is a homeomorphism.

(2) and (3) are clear.

(2-20) Proposition:

Let (H , K) be a topological groupoid and W be an open in K if p: $\otimes^{m}_{i=1}Wi \rightarrow \otimes^{m}_{i=1}Hi_b$ is continuous right inverse to $(\otimes^{m}_{i=1}Hi , \otimes^{m}_{i=1}Ki) : \otimes^{m}_{i=1}Hi_b \rightarrow \otimes^{m}_{i=1}Ki$ for some b∈Ki then the topological subgroupoid ${}_{W}Hi_{W}$ is isomorphic to the trivial subgroupoid $W\times_b Hi_b \times W$ in ***TG .***

Proof: firstly let $(H_1 , K_1)\otimes(H_2 , K_2)$…… …$\otimes(H_n , K_n) \in (H , K)$ we take $(H_1 , K_1) \otimes (H_2 , K_2)$



And complete as the composition of continuous maps.

3. The Action Of Topological Group and Topological Groupoid:

(3-1) Definition [5],[7]:

A topological group Ω acts continuously on a topological space $E_1\times E_2$ (from right) if it is given a right action $\theta :(E_1\times E_2)\times\Omega \rightarrow (E_1\times E_2)$ which is continuous map .

Here $(E_1\times E_2)\times\Omega$ has the product topology.

In similar way we can define a continuous left action.

(3-2) Definition [5],[6]:

We recall that the topological group Ω acts principally on a topological space T if the action of Ω on T is free and the map π:T→K=T/Ω is a submersion.

(3-3) Proposition:

Let φ: H×Ω×Ω→H be a law of continuous action of topological group Ω on a topological space H then :

(1) φ^c is a closed map.

(2) If g: Ω"→Ω' be a morphism of topological groups then g act continuously on H (on right side).

(3) If Ω' acts freely on H and f:Ω"→Ω' is injective then Ω" acts freely on H.

Proof: (3) If $\psi(z.r',r') = \psi(z,r'',r'') \Rightarrow \varphi(z,f(r',r')) = \varphi(z,f(r'',r'')) \Rightarrow f(r',r') = f(r'',r'') \Rightarrow r' = r''$ (since the action φ is free and f is injective). Hence Ω" acts freely on H.

(3-4) Proposition:

Let H×Ω×H → H be a law of continuous action of topological group Ω on a topological space H then :

(1) the pair (H×Ω×H , H) is a topological groupoid where H×Ω×H has the product topology.

(2) the fiber product $H \times_K H \times_K H$ of π: H×H→K = H×H/Ω by itself is a topological groupoid where $H \times_K H \times_K H$ has the subspace topology from H× H ×H.

Proof: (2) $H \times_K H \times_K H$ is the image of transitor of H×Ω×H which is subgroupoid of Descartes groupoid H×Ω×H by the definition of topological subgroupoid we have $H \times_K H \times_K H$ is a topological groupoid.

(3-5) proposition:

Let φ: H×Ω×H → H be a law of continuous free action of topological group Ω on a topological space H then the map $T:H \times_K H \times_K H \to \Omega$, T(k , k' , k) = n where k' = φ(k , n , k) is a morphism of topological groupoids over C_e : H → {e}.

Proof: the following diagrams are commutative in S

$$\begin{array}{ccc} H \times_K H \times_K H & \xrightarrow{\quad T \quad} & \Omega \\ \beta' \downarrow \downarrow \alpha' & & \beta \downarrow \downarrow \alpha \\ H \times \Omega & \xrightarrow{\quad C_e \quad} & \{e\} \end{array} \quad \text{and} \quad \begin{array}{ccc} (H \times_K H \times_K H).(H \times_K H \times_K H) & \xrightarrow{\quad \gamma' \quad} & H \times_K H \times_K H \\ \downarrow T\times T\times T \downarrow & & \downarrow T \\ \Omega\times\Omega\times\Omega & \xrightarrow{\quad \gamma \quad} & \Omega \end{array}$$

And T is continuous since if W be an open in Ω then $T^{-1}(W)=(H, H.W , H)\cap H \times_K H \times_K H$ be an open in $H \times_K H \times_K H$ where (H , H.W , H) be an open in H × H × H since $H.W = \bigcup_{t\in W} \varphi(H,t,H) = \bigcup_{t\in W} H.t.H$ is an open in H. Hence (T , C_e) is in **TG**.

(3-6) Remark :

The pair ($H\times\Omega\times H$, H) is called the action groupoid & T: $H \times_K H \times_K H \to \Omega$ is called the action morphism.

(3-7) Proposition:

Let $\delta : S \times \Omega \times S \to S$ be a law of continuous free action of topological group Ω on a topological space S then :

(1) $\otimes^{m}_{i=1}$Si (tensor product of $\mu : S\times S \to K = S\times S/\Omega$ by itself) isomorphic to $S\times\Omega\times S$ in topological groupoid.

(2) The orbit maps are embedding maps.

Proof: (2) Let $r \in S$ then the orbit map $\varphi_r : \Omega \to S$ is injective (the action is free). So we have a bijective continuous map $\varphi_r^{-1}: S_x \to \Omega$ is given by $\varphi_r^{-1}(r') = T(r , r')$ which is continuous since :

$$\Phi_r^{-1} : S_x \xrightarrow{\cong} \{r\} \times S_x \xrightarrow{inc} \otimes^{m}_{i=1}Si \xrightarrow{T} \Omega$$

$$r' \longrightarrow (r , r') \longrightarrow (r , r') \longrightarrow T(r , r')$$

hence $\varphi_r : \Omega \to S_x$ is a homeomorphism and therefore $\varphi_r : \Omega \to S$ is an embedding , $\forall\, r \in S$.

(3-8) Definition [5] :

The groupoid $S\times S/\Omega = H$ of base $S/\Omega = K$ is called **Ehresmann groupoid.**

(3-9) Definition [1],[4] :

A topological groupoid (H , K) is said to act continuously on a topological space S if it acts on S and the maps:

(1) $\varphi : H*S \to S$ (2) $\mu : S \to K$

Are both continuous.

(3-10) Definition [1],[4]:

We recollection that a topological groupoid (H , K) acts principally on a topological space S if the action of H on S is free and transitive.

(3-11) Proposition :

Let $\theta^* : H\times\Omega\times H \to H$ be a law of continuous action of topological groupoid (H , K) on a topological space Z then :

(1) $\forall\, h \in H_{\sigma(x)}$, the map $\theta_h^* : Z_{\alpha(h)} \to Z_{\beta(h)}$, $\theta_h^*(x') = \theta^*(h , x' , h)$, $\forall x'\in Z_{\alpha(x)}$ is a homeomorphism for each $x \in Z$ where $Z_{\alpha(x)}$ and $Z_{\beta(x)}$ are subspace of Z.

(2) If $S : H' \to H$ be a morphism of topological groupoid over K then H' acts on Z (from left).

(3) If H acts freely on Z and S is injective map then H' acts freely on Z.

(3-12) Proposition:

Let δ^* : $H{*}\Omega{*}H \rightarrow H$ be a law of continuous action of topological groupoid (H , K) on a topological space Z then the pair ($H{*}\Omega{*}H$, H) is a topological groupoid where $H{*}\Omega{*}H$ has the subspace topology from $H\times\Omega\times H$ and the square :

$$\begin{array}{ccc} H{*}\Omega{*}H & \xrightarrow{P_1} & H \\ \downarrow{\sigma^*} & & \downarrow{\sigma} \\ \Omega\times H & \xrightarrow{\pi^2} & K\times K \end{array}$$

is commutative in topological groupoid

(3-13) Proposition:

Let φ^*: $H{*}\Omega{*}H \rightarrow H$ be a law of continuous action of topological groupoid (H , K) on a topological space Ω and let ($H{*}\Omega{*}H$, H) be associated topological groupoid then the map S^*: $H{*}\Omega{*}H \rightarrow H$, $S^*(h, x ,h) = h$ is a morphism of topological groupoid over π: $\Omega \rightarrow K$ (the map induced from action of H on Ω).

Proof: the following diagrams are commutative in S :

$$\begin{array}{ccc} H{*}\Omega{*}H & \xrightarrow{S{*}\times S^*} & H{*}\Omega \\ \downarrow{\beta^*}\ \downarrow{\alpha^*} & & \downarrow{\beta}\ \downarrow{\alpha} \\ \Omega\times\Omega & \xrightarrow{\pi} & K\times K \end{array}$$

and

$$\begin{array}{ccc} (H{*}\Omega{*}H) * (H{*}\Omega{*}H) & \xrightarrow{\gamma^*\times\gamma^*} & H{*}\Omega{*}H \\ \downarrow{S^*\times}\ \downarrow{S^*\times S^*} & & \downarrow{S^*\times S^*} \\ H{*}H{*}H & \xrightarrow{\gamma} & H{*}\Omega \end{array}$$

And S^* is continuous since its just the restriction of the first projection of $H\times\Omega\times H$ on the subspace $H{*}\Omega{*}H$ hence (S^* , π) is in ***TG***.

(3-14) Remark:

The groupoid ($H{*}\Omega{*}H$, Ω) is noun as **action groupoid** and the map S^*: $H{*}\Omega{*}H \rightarrow H$ is called **action morphism**.

(3-15) Proposition :

Let ($H\times\Omega$, $K\times\Omega$) be a transitive topological groupoid then $H\times\Omega$ acts principally on each α-fiber.

Proof : let (r , s) $\in K\times\Omega$, define μ^*: $(H\times\Omega){*}(H\times\Omega)_{(r,s)} \rightarrow (H\times\Omega)_{(r,s)}$

By $\mu^*((h , s) , (q , s)) = \delta[(h , s) , (q , s)]$, where $(H\times\Omega) * (H\times\Omega)_{(r,s)} \subset (H\times\Omega) * (H\times\Omega)$ is the fiber product of α and $\beta_{(r,s)}$ over $K\times\Omega$.

μ^* is a principal action since :

(1) $\forall$ $(q , s) \in (H\times\Omega)_{(r,s)}$, $\mu^*[\delta(\beta_{(r,s)}(q , s)) , (q , s)] = (q , s)$

(2) $\beta_{(r,s)}[\mu^*(h , s) , (q , s)] = \beta_{(r,s)}[\delta((h , s) , (q , s))] = \beta(h , s)$,

$\forall$ $[(h , s) , (q , s)] \in (H\times\Omega) * (H\times\Omega)_{(r,s)}$

(3) $\mu^*[(h , s)(h , s)', (q , s)] = \delta[\delta((h,s),(h,s)'),(q,s)] = \mu^*[(h,s) , \mu^*((h,s)',(q,s))]$

$\forall((h,s) , (h,s)') \in (H\times\Omega) * (H\times\Omega)$ and $((h,s)' , (q,s)) \in (H\times\Omega) * (H\times\Omega)_{(r,s)}$.

μ^* is continuous since it is restriction of δ on a subspace $(H\times\Omega) * (H\times\Omega)_{(r,s)}$.

Now let $(q , s) \in (H\times\Omega)_{(r,s)}$ s.t.

$\mu^*[(h , s) , (q , s)] = (q , s) \Longrightarrow \delta[(h , s) , (q , s)] = (q , s)$ then (h , s) is unity (by remark[$\forall$ h$\in$H, h has unique right unity $\omega(\mu(h))$ and unique left unity $\omega(\delta(h))$])[5] .

Hence μ^* is free action.

Let (q , s) , (t , s) $\in (H\times\Omega)_{(r,s)}$ then (q , s).(t , s)$^{-1} \in (H\times\Omega)$ and

μ^*[(q , s).(t , s)$^{-1}$, (t , s)] = δ [(q , s).(t , s)$^{-1}$, (t , s)] = (t , s) ,hence μ^* is transitive action.

Therefore μ^* is principal action of $(H\times\Omega)$ on $(H\times\Omega)_{(r,s)}$ $\forall$ (r , s) $\in (K\times\Omega)$.

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