Energy bands (g,β,γ) and energy ratios for even-even Yb (A=168-172)

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Abstract

In the present work, the interacting boson model (IBM-1) was used in the calculations of the energy ratios, energy levels and energy bands for even-even isotopes of Yb(A=168-172). The dynamical symmetries for the isotopes under study have been determined, which is found to be SU(5)-SU(3).The well-deformed even-even nuclei in general populate in high-spin states, so that the low-lying bands will extinctions to a high spin also. The energy ratios of Yb (A=168-172) is the $\frac{E(4_1^+)}{E(2_1^+)}$, $\frac{E(6_1^+)}{E(2_1^+)}$ and $\frac{E(8_1^+)}{E(2_1^+)}$ will give a big values. The energy spectrum according to the sequences of energy bands (g, β, γ)give very good agreement of the level sequence of each band with the typical sequence of ground band (0^+, 2^+, 4^+,……), β – band (0^+, 2^+, 4^+,……), and γ – band (2^+, 3^+,4^+, 5^+,……). There are good agreement between the present results and the experimental data.

حزم الطاقه (g,β,γ) و نسب الطاقه للنواة الزوجيه-زوجيه $Yb(A=168-172)$

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المستخلص

في البحث الحالي استخدم نموذج البوزونات المتفاعله الاول(IBM-1) لحساب نسب الطاقة وحزم الطاقه للنظائر الزوجية – زوجية ضمن الاعداد $Yb(A=168-172)$. واظهرت النتائج بان النظائر تنتمي الى التناظرات الديناميكيه $SU(5)-SU(3)$. النواة الزوجيه- زوجيه المشوهه كلياً $Yb(A=168-172)$ بصورة عامه تكون في حالة دوران عاليه ولهذا تمتد الحزم الواطئه الى حالات دورانيه عاليه ونسب الطاقه $\frac{E(4_1^+)}{E(2_1^+)}$ ، $\frac{E(6_1^+)}{E(2_1^+)}$ و $\frac{E(8_1^+)}{E(2_1^+)}$ للنظائر $Yb\ (A=168-172)$ تكون قيمتها كبيرة.

ان طيف الطاقه وفقاً لتتابع حزم الطاقه $(g,\ \beta,\ \gamma)$. قد اعطى توافقا في تسلسل مستويات الطاقه لكل حزمه مع التسلسل المثا لي للحزمه الارضيه $(0^+,\ 2^+,\ 4^+,......)$ و حزمه بيتا $(0^+,\ 2^+,\ 4^+,......)$ و لحزمه كاما $(2^+,\ 3^+,4^+,\ 5^+,......)$ وهناك توافق جيد بين القيم التي حصلنا عليها و القيم العمليه المتوفرة.

1. Introduction

A study of nuclear physics centers around tow main problems, first one hopes to understand the properties of the force which holds the nuclear together. Second one attempts to describe the behavior of systems of many particles**.** A transition between excited states occurs by emission of electromagnetic radiation (gamma rays) completely analogous to light emission from atoms. The main difference is that, where as atomic states are separated by energies of the order of an electron volt [1].

The underlying physical picture one has in mind the spherical shell model, here the particles move in the average due to all others. This average field is producing the single-particle level, with quantum numbers l, j where l is the orbital angular momentum and j is the total angular momentum. The nature of average potential is such that gives a rise to a shell structure.

Major shell closures occur at certain nucleon numbers. These magic numbers are usually taken as 2,8,20,28,50,82,126,184. Sometimes, one also considers 'partially magic' numbers where smaller but non- negligible gaps occur in the single particle spectrum. In order to calculate properties of given even-even nuclei, diagonlization of the residual interaction in the space provided by average field is necessary. This is an extremely difficult (and in some cases impossible) problem. One thus seeks truncation of the shell- model space. First we usually assume that the closed shells corresponding to the magic number are inert. Second, assume that the important particle configurations in even-even nuclei are those in which identical particles are paired together states with total angular momentum and parity $I^{\pi}=0^{+}$ and $I^{\pi}=2^{+}$. Finally, the treatments of each pair as a boson of all these assumptions can be slightly relaxed and there exist already several calculations with active closed shells with $I^{\pi}=4^{+}$ pair and with pairs explicitly treated as fermions. If one retains all three approximations one is led to consider a system of interacting boson of two types, proton bosons and neutron bosons.

2. Basic consideration

The most commonly used form of IBM-1 Hamiltonian is [2, 3]:

$$\hat{H} = \in \hat{n}_d + a_0(\hat{p}.\hat{p}) + a_1(\hat{I}.\hat{I}) + a_2(\hat{Q}.\hat{Q}) + a_3(\hat{T}_3.\hat{T}_3) + a_4(\hat{T}_4.\hat{T}_4)$$

(1)

Where $\in = \in_d - \in_s$ is the boson energy.

The operators:

$$\left.\begin{array}{ll}
\hat{n}_d = (\hat{d}^{\dagger}.\hat{\tilde{d}}) & \text{the boson number operator} \\
\hat{p} = \frac{1}{2}(\hat{\tilde{d}}.\hat{\tilde{d}}) - \frac{1}{2}(\hat{\tilde{s}}.\hat{\tilde{s}}) & \text{the pairing bosons operator} \\
\hat{L} = \sqrt{10}\left[\hat{d}^{\dagger}.\hat{\tilde{d}}\right]^{(1)} & \text{the boson number operator} \\
\hat{Q} = \left[\hat{d}^{\dagger} \times \hat{\tilde{s}}) + (\hat{s}^{\dagger} \times \hat{\tilde{d}})\right]^{(2)} - \frac{1}{2}\sqrt{7}\left[\hat{d}^{\dagger} \times \hat{\tilde{d}}\right]^{(2)} & \text{the quadrupole operator} \\
\hat{T}_3 = \left[\hat{d}^{\dagger} \times \hat{\tilde{d}}\right]^{(3)} & \text{the octupole operator} \\
\hat{T}_4 = \left[\hat{d}^{\dagger} \times \hat{\tilde{d}}\right]^{(4)} & \text{the hexadecaploe operator}
\end{array}\right\} \quad (2)$$

and a_0 ,a_1,a_2,a_3,a_4 are the phenomenological parameter .

The electromagnetic transition rates, B(E2) values of this chain and the quadrupole moments (Q_I) are described by [(4)]:

$$B\,(E2;\, I+2 \rightarrow I) = \alpha_2^2 \left[\frac{I+2}{I}\right]\left[\frac{2N-I}{2}\right] \tag{3}$$

$$Q_I = \beta_2 \left[\left(\frac{16\pi}{70}\right)^{\frac{1}{2}} I\right] \tag{4}$$

In particular, for I=0, or 2

$$B\,(E2;\, 2_1^+ - 0_1^+) = \alpha_2^2 N \tag{5}$$

$$Q_{2_1^+} = \beta_2 \left[\frac{32\pi}{35}\right]^{1/2} \tag{6}$$

The basic condition for the observation of a SU(5) symmetry in the electromagnetic transition is [(4)] :

$$\frac{B(E2;4_1^+ - 2_1^+)}{B(E2;2_1^+ - 0_1^+)} = \frac{B(E2;2_2^+ - 2_1^+)}{B(E2;2_1^+ - 0_1^+)} = \frac{B(E2;0_2^+ - 2_1^+)}{B(E2;2_1^+ - 0_1^+)} = 2\left[\frac{(N-1)}{N}\right] < 2 \tag{7}$$

Where the necessary conditions for the observation of the SU(3) symmetry are[(4)]:

$$\frac{B(E2;4_1^+ \rightarrow 2_1^+)}{B(E2;2_1^+ \rightarrow 0_1^+)} = \frac{10(N-1)(2N+5)}{7 \quad N(2N+3)} \langle \frac{10}{7} \tag{8}$$

$$\frac{B(E2;2_2^+ \rightarrow 2_1^+)}{B(E2;2_1^+ \rightarrow 0_1^+)} = \frac{B(E2;0_2^+ \rightarrow 2_1^+)}{B(E2;2_1^+ \rightarrow 0_1^+)} = 0 \tag{9}$$

3. Transitional Regions in IBM-1

SU(3)-SU(5) transitional dynamical symmetry

This transitional region includes the two groups, SU(3) and SU(5). The SU(3) has to be broken with $\in n_d$ term. The general form of Hamiltonian operator of this region can be given as [4, 5, 6]:

$$\hat{H} = \in \hat{n}_d + a_1 \hat{I}.\hat{I} + a_2 \hat{Q}.\hat{Q} \tag{10}$$

The solution of the equation (10) depends on the ratio of ($\in/a_2$), when the ratio ($\in/a_2$) is large the eigenfunction of $\hat{H}$ are those appropriate to the limiting SU(5). Also the B(E2) values are affected by the ratio ($\in/a_2$).

The B(E2) ratios (Branching Ratios) can be given by [(4,5)]:

$$R=\frac{B(E2;2_2^+\rightarrow 0_1^+)}{B(E2;2_2^+\rightarrow 2_1^+)} \qquad (11)$$

Where: R= 0 in SU(5) region
R=7/10 in SU(3) region

4. Results and Desiccations

4.1. Energy Bands spectrum (g, β, γ):

The behaviors of the structure of each nucleus considered in this work which is deduced by studying the dynamical symmetry and energy spectrum according to the sequences of energy bands (g, β, γ). Yb (A=168-172) rotational spectra occur in even-even nuclei with permanent qurdrupole deformation in lowest part of the experimental spectrum of the nucleus A=(168,170,172) at the highest angular momentum the expected degeneracy's are broken having as a result vibrational spectrum. A transition from vibrational to rotational character is observed as one move away from closed major shells.

The Yb (A=168-170) are clearly rotational. The rotational character becoming more profound as the number of valance neutrons increases. The SU(3) limit of IBM-1. The fallowing discrepancies between theory and experiment exist. We have seen that in this limit the γ_1 band and β_1 band belong to the same SU(3) irrep, (2N-4,2). As a result, the levels of the tow bands are predicted to be degenerated. In this limit the ground state band belongs to the (2N,0) irrep, while the β_1 and γ_1 bands belong to the (2N-4,2) irrep $\hat{T}^{(E2)}$ transition operator can connect only states within the same irrep as a result, B(E2) for $\gamma_1 \rightarrow \beta_1$ transitions, while the $\gamma_1 \rightarrow$ ground and the $\beta_1 \rightarrow$ ground transitions should be forbidden.

The (β) vanishes for spherical shapes, while it is different from zero for deformed shape . (γ) vanishes for prolate shapes and has the value $\frac{\pi}{3}$ for oblate shapes. Nuclei in transitional region are through to be triaxial γ-unstable. Continuously changing their shape from prolate to oblate and vice versa. Figures (1→3) show the energy levels for the selected isotopes where they obey the typical energy bands spectrum. Noticed that there are very good agreement of the level sequence of each band with the typical sequence of ground band (0^+, 2^+, 4^+,……), (β – band) (0^+, 2^+, 4^+,……), and (γ – band) (2^+, 3^+,4^+, 5^+,……). There are good agreement between the present results and the experimental values of the energy bands.

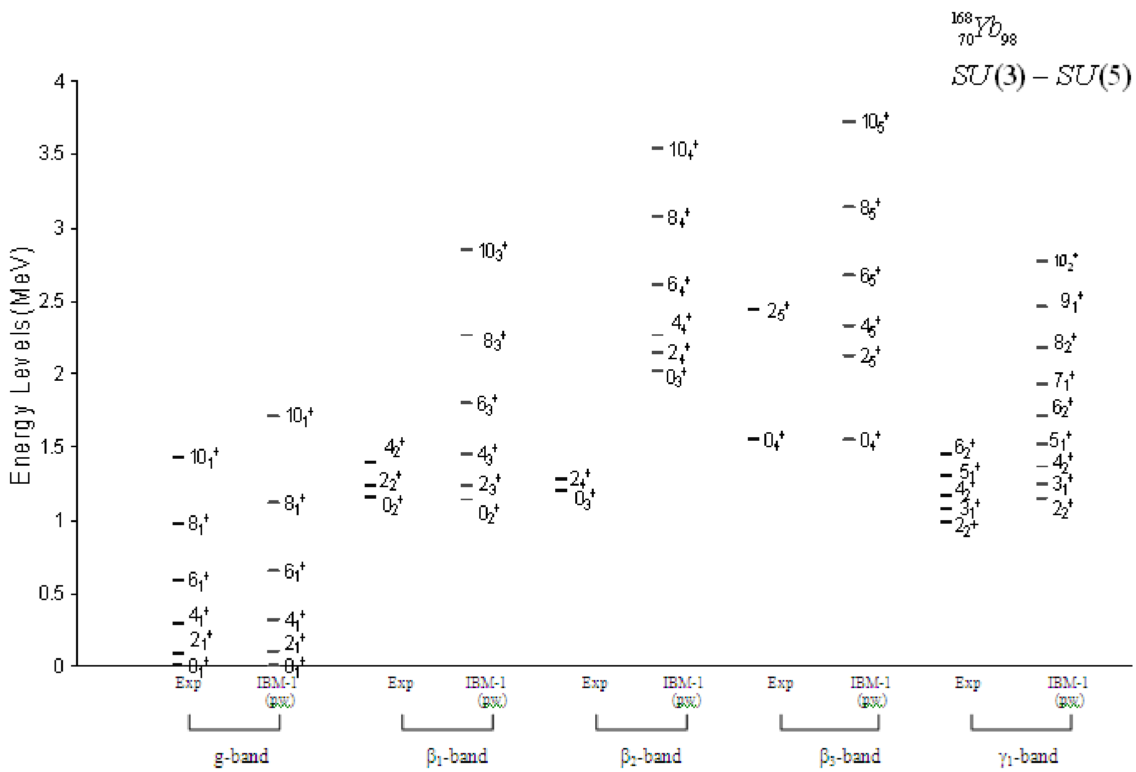

Figure (1):Comparison between calculated IBM (PW); and experimental energy bands states(g, β, γ -bands) in $^{168}_{70}\boldsymbol{Yb}_{98}$ isotope of the dynamical symmetry SU(3)-SU(5).

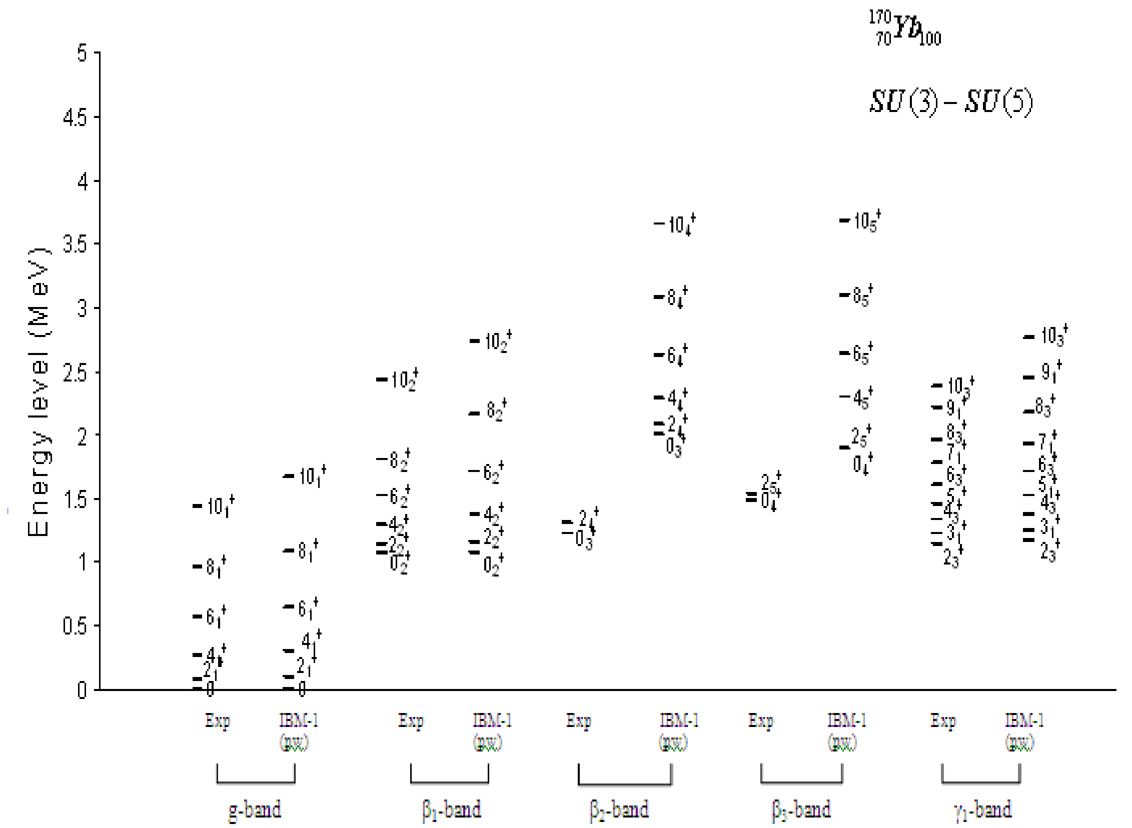

Figure (2):Comparison between calculated IMB (PW); and experimental energy bands states (g, β, γ-bands) in $^{170}_{70}Yb_{100}$ isotope of the dynamical symmetry SU(3)-SU(5).

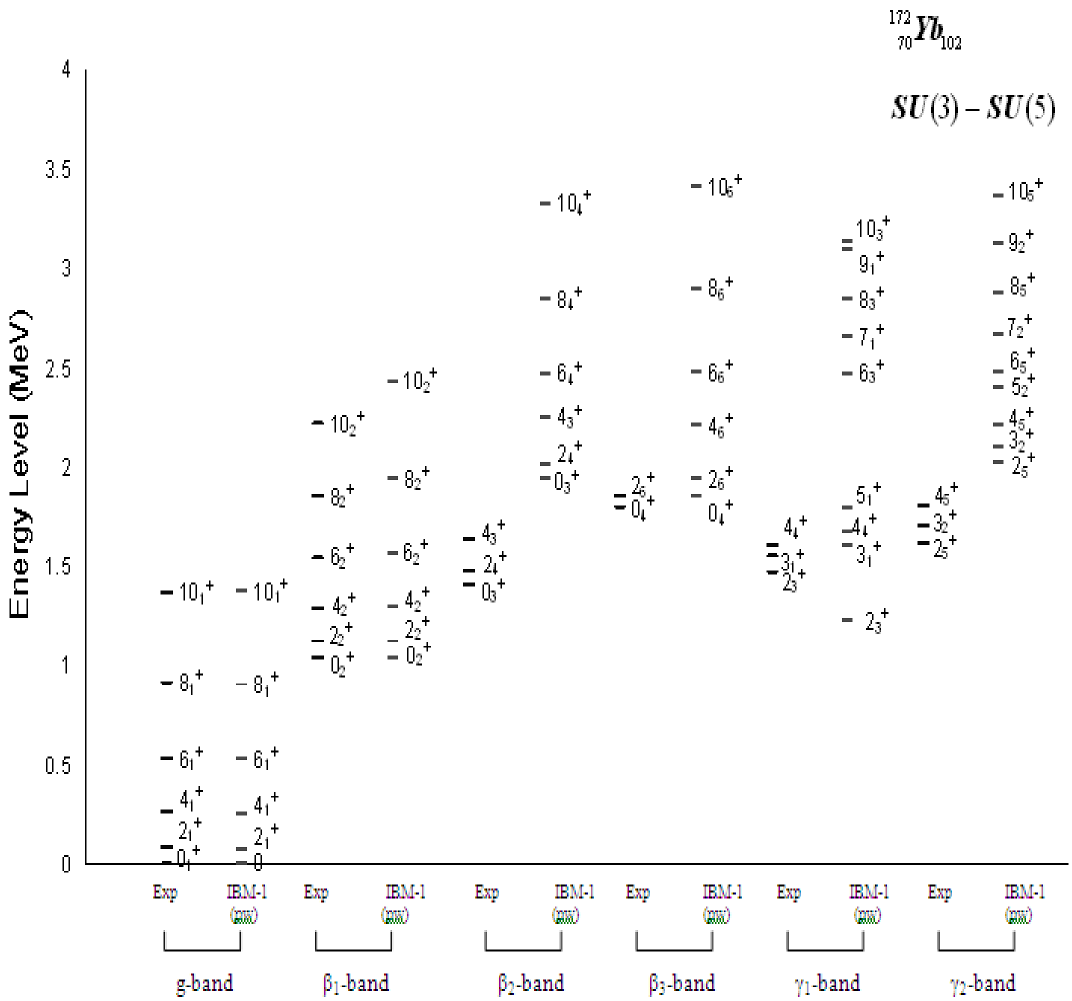

Figure (3):Comparison between calculated IMB (PW); and experimental energy bands states (g,β,γ-bands) in $^{172}_{70}Yb_{102}$ isotope of the dynamical symmetry SU(3)-SU(5).

4.2. Energy Ratio

The Yb (A=168- 172) isotopes are characterized by high deformed due to increasing the neutron number. The deformation is caused by the distorting influence of particles (holes) outside a closed shell. Near closed shells, the nuclear ground state wave-function represents a linear combination of spherical and deformed shapes. As we go away from the closed shells, the "deformed" part of the wave function increases and the "spherical" part decreases. When the number of particles (holes) reaches a critical value, the "spherical" part vanishes and the spherical deformed phase transition sets. Table 1 tabulated the corresponding energy ratios for each limit.

Table (1): The energy ratios of corresponding limits [7].

Limit	$E(4_1^+) / E(2_1^+)$	$E(6_1^+) / E(2_1^+)$	$E(8_1^+) / E(2_1^+)$
SU(5)	2.3	3	4
O(6)	2.5	4.5	7
SU(3)	3.33	7	12

The predicted ratios $\frac{E(4_1^+)}{E(2_1^+)}$ is 3.33 in remarkable agreement with the symmetries of nuclear levels for 150 <A< 190. Nuclei Yb (A=168-172) show structures most characteristic of rotations of a non spherical system. The well-deformed even-even nuclei Yb(A=168-172) in general populate high-spin states, so that an extinctions of the low-lying bands to a high spin and the level spacing shows that the energy ratios of Yb (A=168-172) highest energy ratios of $\frac{E(4_1^+)}{E(2_1^+)}$, $\frac{E(6_1^+)}{E(2_1^+)}$ and $\frac{E(8_1^+)}{E(2_1^+)}$. Table 2 shows the comparison between the calculated theoretical and experimental energy ratios for each limit. The comparison shows good agreement. Also figure (4) shows the relation between the energy ratios as a function of number of neutron (N) for the even-even Yb (A=168-172) isotopes. The comparison is in good agreement.

Table (2): Theoretical energy ratios compared with the experimental data for chosen even-even isotopes.

Isotopes	$E(4_1^+) / E(2_1^+)$		$E(6_1^+) / E(2_1^+)$		$E(8_1^+) / E(2_1^+)$	
	EXP.	IBM-1 (pw)	EXP.	IBM-1 (pw)	EXP.	IBM-1 (pw)
${}^{168}_{70}Yb_{98}$	3.3058[8]	3.3058	6.7044[8]	6.7044	11.1111[8]	11.8819
${}^{170}_{70}Yb_{100}$	3.2918[9]	3.3525	6.8030[9]	6.9938	11.4317[9]	11.9795
${}^{172}_{70}Yb_{102}$	3.0330[10]	3.3325	6.7386[10]	6.9942	11.5761[10]	11.9803

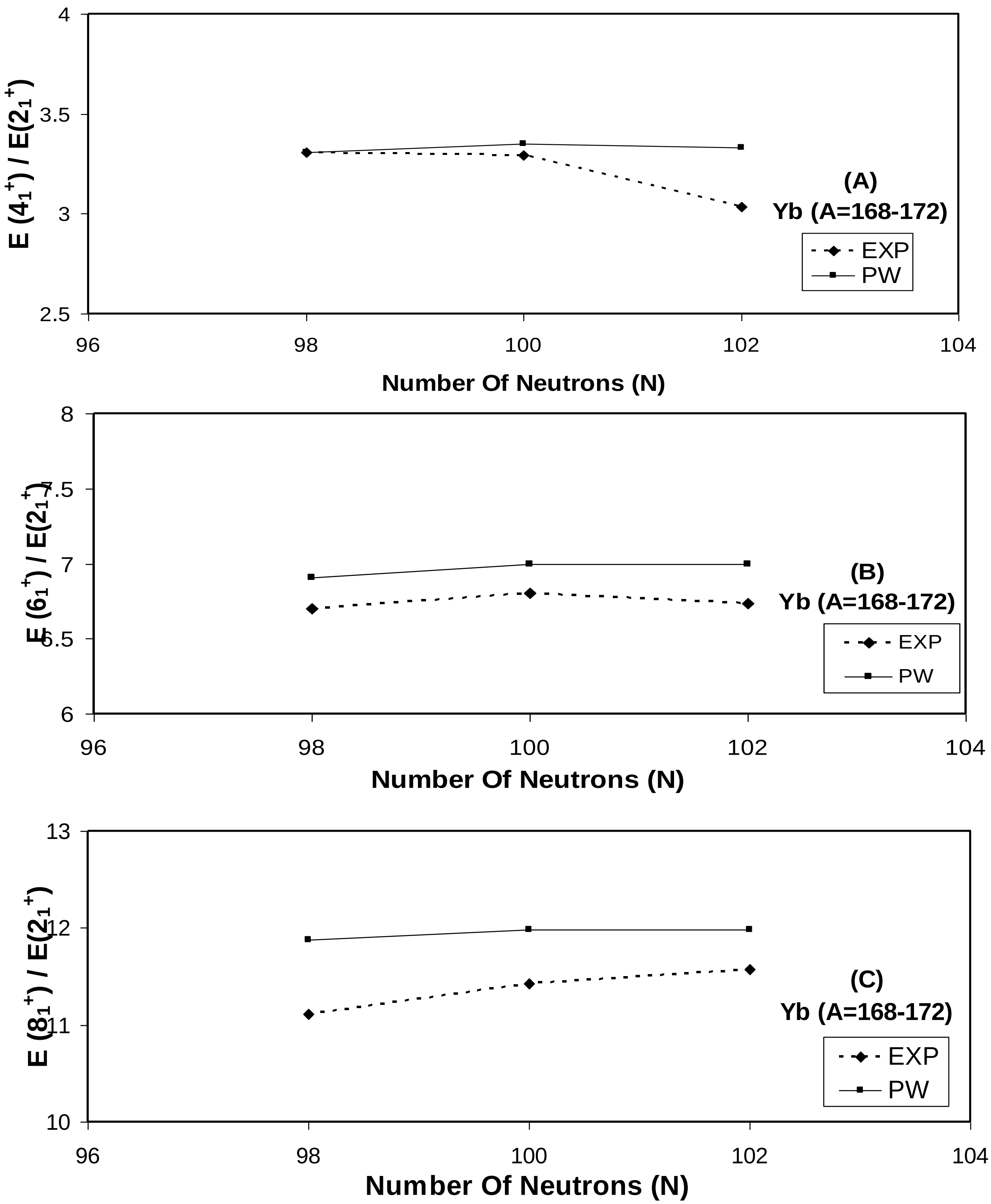

Figure (4-A,B,C): The relation between the energy ratios as a function of number of neutron (N) for the even-even Yb (A=168-172) isotopes.

5. References

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