On Fuzzy b-hyper Connected Space

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Abstract

This paper is devoted to introduce the notion of fuzzy extremely disconnected space in fuzzy topological spaces on fuzzy sets , and study some theorems and properties on fuzzy b- extremely disconnected space and **fuzzy b-hyper connected space**.

حول الفضاء الضبابي المتصل(b-hyper)

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المستخلص:

في هذا البحث قدمنا مفهوم الفضاء الضبابي غير المتصل (extremely) في الفضاء التبولوجي الضبابي وعلى مجموعة ضبابية ودراسة بعض النظريات والخواص حول الفضاء الضبابي غير المتصل (b- extremely) والفضاء الضبابي المتصل (b-hyper) .

Introduction:

The concept of fuzzy set was introduced by Zadeh in his classical paper [12] in 1965. The fuzzy topological space on fuzzy set was introduced by Chakrabarty and Ahsanullal [2] in 1992, Fatteh and Bassan [5] in 1985 introduce the notions of fuzzy connected and disconnected spaces , Benchalli and Jenifer [1] has introduced the concepts of fuzzy b-open and fuzzy b-closed . The purpose of this paper is to introduce and study two stronger forms of fuzzy disconnectedness .

Preliminaries

A fuzzy set $\tilde{A}$ in a universe set X is characterization by a membership function $\mu_{\tilde{A}}:X\rightarrow I$, which an associated with each point x in X a real number in closed interval I = [0 ,1] .The collection of all fuzzy subset in X will be denote by I^X [12].

$p(\tilde{A}) =\{ \tilde{B}: \tilde{B} \in I^x$ and $\tilde{B} \subseteq \tilde{A} \}$ which $p(\tilde{A})$ is called fuzzy power set [2].

For any two fuzzy sets $\tilde{B}$, $\tilde{C}$ in X we write , $\forall x \in X$

$\tilde{B} \subseteq \tilde{C}$ if $\mu_{\tilde{B}}(x) \leq \mu_{\tilde{C}}(x)$, $\tilde{B} = \tilde{C}$ if $\mu_{\tilde{B}}(x) = \mu_{\tilde{C}}(x)$, $\tilde{B}^c$ is the complement of $\tilde{B}$ with membership function $\mu_{\tilde{B}^c}(x) = 1- \mu_{\tilde{B}}(x)$, $\tilde{D} = \tilde{B} \cap \tilde{C}$ if $\mu_{\tilde{D}}(x) = \min\{\mu_{\tilde{B}}(x) , \mu_{\tilde{C}}(x)\}$ and $\tilde{E} =\tilde{B} \cup \tilde{C}$ if $\mu_{\tilde{E}}(x) =\max\{\mu_{\tilde{B}}(x), \mu_{\tilde{C}}(x)\}$[3,11].

The support of a fuzzy set $\tilde{B}$ in $\tilde{A}$ will be denoted by

$Supp(\tilde{B}) = \{x \in X : \mu_{\tilde{B}}(x) > 0 \}$ [6] .

Finally a fuzzy point x_r in X is a fuzzy set with membership function

$\mu_{x_r}(x) = r$, if $x = v$ where $0 < r \leq 1$ and $\mu_{x_r}(x) = 0$, if $x \neq v$, such that v is called the support of x_r and r the value of x_r [9] , $x_r \in \tilde{A}$ iff $\mu_{x_r}(x) \leq \mu_{\tilde{A}}(x)$ and $x_r \notin \tilde{A}$ iff $\mu_{x_r}(x) > \mu_{\tilde{A}}(x)$ [8] .

A family $\tilde{\tau}$ of fuzzy sets of Ã in X is called a fuzzy topology on Ã if $\tilde{\varphi}$ and Ã belong to $\tilde{\tau}$ and $\tilde{\tau}$ is closed with respect to arbitrary union and finite intersection [2].The members of $\tilde{\tau}$ are called fuzzy open sets and their complements are fuzzy closed sets [2]. A fuzzy set $\tilde{B}$ which is both fuzzy open and fuzzy closed is called fuzzy clopen set [4] .We shall denote a fuzzy topological space on fuzzy set (fts. for short) by $(\tilde{A}, \tilde{\tau})$. Let $\tilde{B}$ be a fuzzy set in fts $(\tilde{A}, \tilde{\tau})$, then we define $cl(\tilde{B}) = \cap\{ \tilde{C}_i : \tilde{C}_i^c \in \tilde{\tau}, \tilde{B} \subseteq \tilde{C}_i \}$ and $int(\tilde{B}) = \cup\{ \tilde{C}_i : \tilde{C}_i \in \tilde{\tau}, \tilde{C}_i \subseteq \tilde{B} \}$ [2]. A fuzzy point x_r is said to be quasi coincident with a fuzzy set $\tilde{B}$ in Ã denoted by $\tilde{B}\ q\ x_r$ if there exists $x \in X$ such that $\mu_{\tilde{B}}(x) + \mu_{x_r}(x) > \mu_{\tilde{A}}(x)$ and denote by $\tilde{B}\ \tilde{\bar{q}}\ x_r$ if $\mu_{\tilde{B}}(x) + \mu_{x_r}(x) \leq \mu_{\tilde{A}}(x)$, $\forall x \in X$ [2] . A fuzzy set $\tilde{B}$ is said to be quasi coincident with a fuzzy set $\tilde{C}$ in Ã denoted by $\tilde{B}\ q\ \tilde{C}$ if there exists $x \in X$ such that $\mu_{\tilde{B}}(x) + \mu_{\tilde{C}}(x) > \mu_{\tilde{A}}(x)$ and denote by $\tilde{B}\ \tilde{\bar{q}}\ \tilde{C}$ if $\mu_{\tilde{B}}(x) + \mu_{\tilde{C}}(x) \leq \mu_{\tilde{A}}(x)$, $\forall x \in X$ [2] . $\tilde{B}\ \tilde{\bar{q}}\ \tilde{B}^c$ [10].

1. Basic Definition and Some Results

Definition 1.١ [1] :

A fuzzy set $\tilde{B}$ of fts $(\tilde{A}, \tilde{\tau})$ is said to be

- Fuzzy b-open set if $\tilde{B} \subseteq int(cl(\tilde{B})) \cup cl(int(\tilde{B}))$
- Fuzzy b-closed set if $int(cl(\tilde{B})) \cap cl(int(\tilde{B})) \subseteq \tilde{B}$

The family of fuzzy b-open sets is denoted by $FBO(\tilde{A})$ and the complement of a fuzzy b-open set is a fuzzy b-closed set .

Definition 1.2 [7] :

If $\tilde{B}$ is a fuzzy set in fts $(\tilde{A}, \tilde{\tau})$, then :

- The b-closure of $\tilde{B}$ is denoted by $(bcl(\tilde{B}))$ and defined by
  $bcl(\tilde{B}) = \cap\{ \tilde{C}_i : \tilde{C}_i^c \in FBO(\tilde{A}), \tilde{B} \subseteq \tilde{C}_i \}$.
- The b-interior of $\tilde{B}$ is denoted by $(bint(\tilde{B}))$ and defined by
  $bint(\tilde{B}) = \cup\{ \tilde{C}_i : \tilde{C}_i \in FBO(\tilde{A}), \tilde{C}_i \subseteq \tilde{B} \}$.

Definition 1.3 :

A fuzzy set $\tilde{B}$ of fts $(\tilde{A}, \tilde{\tau})$ is said to be :

- Fuzzy b-regular open set if $\tilde{B} = bint(bcl(\tilde{B}))$
- Fuzzy b-regular closed set if $\tilde{B} = bcl(bint(\tilde{B}))$.

Theorem 1.4 [7] :

If $\tilde{B}$, $\tilde{C}$ are a fuzzy set in fts $(\tilde{A}, \tilde{\tau})$, then

- $int(\tilde{B}) \subseteq bint(\tilde{B}) \subseteq \tilde{B} \subseteq bcl(\tilde{B}) \subseteq cl(\tilde{B})$.
- $bint(\tilde{B} \cap \tilde{C}) = bint(\tilde{B}) \cap bint(\tilde{C})$.

Theorem 1.5 :

If $\tilde{B}$ is a fuzzy set in fts $(\tilde{A}, \tilde{\tau})$, then ;

1. $bint(\tilde{B}) = (bcl(\tilde{B}^c))^c$.
2. $bcl(\tilde{B}) = (bint(\tilde{B}^c))^c$.

Proof : (1) Since $bint(\tilde{B}) \subseteq \tilde{B}$ and $bint(\tilde{B})$ is a fuzzy b-open set .

Then $\tilde{B}^c \subseteq (bint(\tilde{B}))^c$ and $bcl(\tilde{B}^c) \subseteq (bint(\tilde{B}))^c$, hence

$bint(\tilde{B}) \subseteq (bcl(\tilde{B}^c))^c$ ....(*)

Since $\tilde{B}^c \subseteq bcl(\tilde{B}^c)$ and $bcl(\tilde{B}^c)$ is a fuzzy b-closed set , then

$(bcl(\tilde{B}^c))^c \subseteq \tilde{B}$, hence $(bcl(\tilde{B}^c))^c \subseteq bint(\tilde{B})$ ....(**)

From (*) and (**) we get $bint(\tilde{B}) = (bcl(\tilde{B}^c))^c$.

(2) Obvious .

Theorem 1.6 [7] :

If $\tilde{B}$ is a fuzzy set in fts $(\tilde{A}, \tilde{\tau})$, then ;

1. $\tilde{B}$ is a fuzzy b-closed set iff $\tilde{B} = bcl(\tilde{B})$.
2. $\tilde{B}$ is a fuzzy b-open set iff $\tilde{B} = bint(\tilde{B})$.

Theorem 1.7 :

If $(\tilde{A}, \tilde{\tau})$ is a fuzzy topological space , then :

1. The b-closure of a fuzzy b-open set is a fuzzy b-regular closed set .
2. The b-interior of a fuzzy b-closed set is a fuzzy b-regular open set .

Proof :(1) Let $\tilde{B}$ be a fuzzy b-open set in $(\tilde{A}, \tilde{\tau})$ and

Since $bint(bcl(\tilde{B})) \subseteq bcl(\tilde{B})$, then $bcl(bint(bcl(\tilde{B}))) \subseteq bcl(\tilde{B})$ ....(*)

Since $\tilde{B} \subseteq bcl(\tilde{B})$ and $\tilde{B}$ is a fuzzy b-open set , then

$\tilde{B} \subseteq bint(bcl(\tilde{B}))$, hence $bcl(\tilde{B}) \subseteq bcl(bint(bcl(\tilde{B})))$…………(**)

From (*) and (**) we get $bcl(bint(bcl(\tilde{B}))) = bcl(\tilde{B})$.

(2) Obvious .

Theorem 1.8 :

A fuzzy set $\tilde{B}$ of fts $(\tilde{A}, \tilde{\tau})$ is a fuzzy b-regular closed (b-regular open) set iff $\tilde{B}$ is both fuzzy b-open set and fuzzy b-closed set .

Proof : Obvious.

Proposition 1.9:

If $\tilde{B}$ is a fuzzy open set and $\tilde{C}$ is a fuzzy b-open set in fts $(\tilde{A}, \tilde{\tau})$, then

$\tilde{B} \cap \tilde{C}$ is a fuzzy b-open set in $(\tilde{A}, \tilde{\tau})$.

Proof **:** To prove $\tilde{B} \cap \tilde{C} = bint(\tilde{B} \cap \tilde{C})$.

Since $\tilde{B} = int(\tilde{B})$ and $\tilde{C} = bint(\tilde{C})$, then

$int(\tilde{B}) \cap bint(\tilde{C}) \subseteq bint(\tilde{B}) \cap bint(\tilde{C}) = bint(\tilde{B} \cap \tilde{C})$.

Hence $\tilde{B} \cap \tilde{C} \subseteq bint(\tilde{B} \cap \tilde{C})$ …………..(*)

and since $bint(\tilde{B} \cap \tilde{C}) \subseteq \tilde{B} \cap \tilde{C}$ ……………(**)

Then from (*) and (**) we get $\tilde{B} \cap \tilde{C} = bint(\tilde{B} \cap \tilde{C})$.

Thus $\tilde{B} \cap \tilde{C}$ is a fuzzy b-open set in $\tilde{A}$.

Corollary 1.10 :

If $\tilde{B}$ is a fuzzy open subspace in $(\tilde{A}, \tilde{\tau})$ and $\tilde{C}$ is a fuzzy b-open set in $(\tilde{A}, \tilde{\tau})$, then $\tilde{C} \cap \tilde{B}$ is a fuzzy b-open set in $\tilde{B}$.

Proof : Obvious .

Proposition 1.11 :

If $\tilde{B}$ is a fuzzy open subspace of $(\tilde{A}, \tilde{\tau})$ and $\tilde{C}$ is a fuzzy b-open set in $\tilde{B}$. Then there exists a fuzzy b-open set $\tilde{D}$ in $(\tilde{A}, \tilde{\tau})$ such that $\tilde{C} = \tilde{D} \cap \tilde{B}$.

Proof : Obvious .

Definition 1.12 [7] :

A fuzzy set $\tilde{B}$ in fts $(\tilde{A}, \tilde{\tau})$ is said to be fuzzy quasi neighborhood (fuzzy b-quasi neighborhood) of a fuzzy point x_r in $\tilde{A}$, if there exists a fuzzy open set (fuzzy b-open set) $\tilde{C}$ in $(\tilde{A}, \tilde{\tau})$ such that $x_r \, q \, \tilde{C}$ and $\tilde{C} \subseteq \tilde{B}$.

Theorem 1.13 [7] :

If $\tilde{B}$ is a fuzzy set and x_r fuzzy point in fts $(\tilde{A}, \tilde{\tau})$, then $x_r \in bcl(\tilde{B})$ if and only if for every fuzzy b-quasi neighborhood of x_r is quasi coincident with $\tilde{B}$.

Proposition 1.14 [7] :

If $\tilde{B}$ is a fuzzy set and $\tilde{C}$ is a fuzzy b-open set in fts $(\tilde{A}, \tilde{\tau})$, then $\tilde{C} \tilde{\bar{q}} \tilde{B}$ iff $\tilde{C} \tilde{\bar{q}} \, bcl(\tilde{B})$.

Definition 1.15 [9] :

Let (Ã , $\tilde{\tau}$) be any fts and B̃ be any fuzzy set of Ã . Define

$\tilde{\tau}_{\tilde{B}}$ = { B̃ ∩ C̃ : C̃ ∈ $\tilde{\tau}$ } . Then it is well known that $\tilde{\tau}_{\tilde{B}}$ is a fuzzy topology in B̃ and the fts (B̃ , $\tilde{\tau}_{\tilde{B}}$) is called a fuzzy subspace of (Ã , $\tilde{\tau}$) .

2. On Fuzzy b-Separation Axioms :

Definition 2.1 :

A fts (Ã , $\tilde{\tau}$) is said to be :

- Fuzzy b-$\tilde{T}_0$ space if for each distinct fuzzy points x_r , y_s ∈ Ã , there exists fuzzy b-open set B̃ in (Ã, $\tilde{\tau}$) such that x_r ∈ B̃, B̃ $\tilde{\bar{q}}$ y_s or y_s ∈ B̃, B̃ $\tilde{\bar{q}}$ x_r.
- Fuzzy b-$\tilde{T}_1$ space if for each distinct fuzzy points x_r , y_s ∈ Ã , there exists two fuzzy b-open sets B̃, C̃ in (Ã, $\tilde{\tau}$) such that x_r ∈ B̃, B̃ $\tilde{\bar{q}}$ y_s and y_s ∈ C̃, C̃ $\tilde{\bar{q}}$ x_r.
- Fuzzy b-$\tilde{T}_2$ space if for each distinct fuzzy points x_r , y_s ∈ Ã , there exists two fuzzy b-open sets B̃, C̃ in (Ã , $\tilde{\tau}$) such that x_r ∈ B̃ , y_s ∈ C̃ and B̃ $\tilde{\bar{q}}$ C̃.
- Fuzzy b-regular space if for each x_r ∈ Ã and each fuzzy closed set B̃ in (Ã , $\tilde{\tau}$) such that x_r $\tilde{\bar{q}}$ B̃ , there exists two fuzzy b-open sets C̃ , D̃ in (Ã , $\tilde{\tau}$) such that x_r ∈ C̃ , B̃ ⊆ D̃ and C̃ $\tilde{\bar{q}}$ D̃ .
- Fuzzy b-normal space if for each two fuzzy closed sets $\tilde{B}_1$ and $\tilde{B}_2$ in (Ã , $\tilde{\tau}$) such that $\tilde{B}_1$ $\tilde{\bar{q}}$ $\tilde{B}_2$, there exists two fuzzy b-open sets C̃ , D̃ in (Ã , $\tilde{\tau}$) such that $\tilde{B}_1$ ⊆ C̃ , $\tilde{B}_2$ ⊆ D̃ and C̃ $\tilde{\bar{q}}$ D̃ .

Theorem 2.2 :

If (Ã , $\tilde{\tau}$) is a fuzzy b-$\tilde{T}_{i=0,1,2}$ space and B̃ is a fuzzy open subset of (Ã , $\tilde{\tau}$) , then (B̃ , $\tilde{\tau}_{\tilde{B}}$) is a fuzzy b-$\tilde{T}_{i=0,1,2}$ space .

Proof : Obvious .

Theorem 2.3 :

If $(\tilde{A}, \tilde{\tau})$ is a fts , then the following statements are equivalents .

1. $(\tilde{A}, \tilde{\tau})$ is a fuzzy b-regular space .
2. For each $x_r \in \tilde{A}$ and each fuzzy closed set $\tilde{B}$ in $(\tilde{A}, \tilde{\tau})$ such that $x_r \tilde{\bar{q}} \tilde{B}$, there exists two fuzzy b-open sets $\tilde{C}$, $\tilde{D}$ in $(\tilde{A}, \tilde{\tau})$ such that $x_r \in \tilde{C}$, $\tilde{B} \subseteq \tilde{D}$ and $\tilde{C} \tilde{\bar{q}}$ bcl$(\tilde{D})$.
3. For each $x_r \in \tilde{A}$ and each fuzzy open set $\tilde{B}$ in $(\tilde{A}, \tilde{\tau})$ such that $x_r \in \tilde{B}$, there exists a fuzzy b-open set $\tilde{C}$ in $(\tilde{A}, \tilde{\tau})$ such that $x_r \in \tilde{C} \subseteq$ bcl$(\tilde{C}) \subseteq \tilde{B}$.
4. For each $x_r \in \tilde{A}$ and each fuzzy closed set $\tilde{B}$ in $\tilde{A}$ such that $x_r \tilde{\bar{q}} \tilde{B}$, there exists two fuzzy b-open sets $\tilde{C}$, $\tilde{D}$ in $(\tilde{A}, \tilde{\tau})$ such that $x_r \in \tilde{C}$, $\tilde{B} \subseteq \tilde{D}$ and bcl$(\tilde{C}) \tilde{\bar{q}}$ bcl$(\tilde{D})$.
5. For each fuzzy set $\tilde{B}$ and each fuzzy closed set $\tilde{C}$ in $(\tilde{A}, \tilde{\tau})$ such that $\tilde{B} \tilde{\bar{q}} \tilde{C}$, there exists two fuzzy b-open sets $\tilde{D}_1$, $\tilde{D}_2$ in $(\tilde{A}, \tilde{\tau})$ such that $x_r \in \tilde{D}_1$, $\tilde{B} \subseteq \tilde{D}_2$ and $\tilde{D}_1 \tilde{\bar{q}} \tilde{D}_2$.
6. For each fuzzy set $\tilde{B}_1$ and each fuzzy open set $\tilde{B}_2$ in $(\tilde{A}, \tilde{\tau})$ such that $\tilde{B}_1 \subseteq \tilde{B}_2$, there exists fuzzy b-open set $\tilde{C}$ in $(\tilde{A}, \tilde{\tau})$ such that $\tilde{B}_1 \subseteq \tilde{C} \subseteq$ bcl$(\tilde{C}) \subseteq \tilde{B}_2$.

Proof :

(1)→(2) Let $x_r \in \tilde{A}$ and $\tilde{B}$ be a fuzzy closed set in $(\tilde{A}, \tilde{\tau})$ such that $x_r \tilde{\bar{q}} \tilde{B}$, then there exists two fuzzy b-open sets $\tilde{C}$, $\tilde{D}$ in $(\tilde{A}, \tilde{\tau})$ such that $x_r \in \tilde{C}$, $\tilde{B} \subseteq \tilde{D}$ and $\tilde{C} \tilde{\bar{q}} \tilde{D}$, hence by proposition 1.14 $\tilde{C} \tilde{\bar{q}}$ bcl$(\tilde{D})$.

(2)→(3) Let $x_r \in \tilde{A}$ and $\tilde{B}$ be a fuzzy open set in $(\tilde{A}, \tilde{\tau})$ such that $x_r \in \tilde{B}$, then $x_r \tilde{\bar{q}} \tilde{B}^c$ and there exists two fuzzy b-open sets $\tilde{C}$, $\tilde{D}$ in $(\tilde{A}, \tilde{\tau})$ such that $x_r \in \tilde{C}$, $\tilde{B}^c \subseteq \tilde{D}$ and $\tilde{C} \tilde{\bar{q}}$ bcl$(\tilde{D})$, hence $\tilde{C} \tilde{\bar{q}} \tilde{D}$.

This implies that $\tilde{C} \subseteq \tilde{D}^c$ and bcl$(\tilde{C}) \subseteq$ bcl$(\tilde{D}^c) = \tilde{D}^c$

Therefore $x_r \in \tilde{C} \subseteq$ bcl$(\tilde{C}) \subseteq \tilde{B}$.

(3)→(4) Let $x_r \in \tilde{A}$ and $\tilde{B}$ be a fuzzy closed set in $(\tilde{A}, \tilde{\tau})$ such that

$x_r \bar{\tilde{q}}\ \tilde{B}$, then $x_r \in \tilde{B}^c$ and there exists fuzzy b-open set $\tilde{C}$ in $(\tilde{A}, \tilde{\tau})$ such that

$x_r \in \tilde{C} \subseteq bcl(\tilde{C}) \subseteq \tilde{B}^c$, hence $\tilde{B} \subseteq bint(\tilde{C}^c)$.

Since $\tilde{C}$ is fuzzy b-open set and let $\tilde{D} = bint(\tilde{C}^c)$, then $\tilde{C}\ \bar{\tilde{q}}\ bcl(\tilde{D})$,

by proposition 1.14 and every fuzzy b-regular closed is fuzzy b-open set we get $bcl(\tilde{C})\ \bar{\tilde{q}}\ bcl(\tilde{D})$.

(4)→(5) Let $\tilde{B}$ be a fuzzy set and $\tilde{C}$ be a fuzzy closed set in $(\tilde{A}, \tilde{\tau})$ such that $\tilde{B}\ \bar{\tilde{q}}\ \tilde{C}$,

then for each $x_r \in \tilde{B}$, $x_r \bar{\tilde{q}}\ \tilde{C}$ and there exists two fuzzy b-open sets $\tilde{C}_1$, $\tilde{D}_2$ in

$(\tilde{A}, \tilde{\tau})$ such that $x_r \in \tilde{C}_1$, $\tilde{B} \subseteq \tilde{D}_2$ and $bcl(\tilde{C}_1)\ \bar{\tilde{q}}\ bcl(\tilde{D}_2)$.

Since $x_r \in \tilde{C}_1$, then $\tilde{B} = \cup\, x_{ri} \subseteq \cup\, \tilde{C}_{1i}$, let $\tilde{D}_1 = \cup\, \tilde{C}_{1i}$ and $\tilde{D}_1\ \bar{\tilde{q}}\ \tilde{D}_2$.

(5)→(6) Let $\tilde{B}_1$ be a fuzzy set and $\tilde{B}_2$ be a fuzzy open set in $(\tilde{A}, \tilde{\tau})$ such that

$\tilde{B}_1 \subseteq \tilde{B}_2$, then $\tilde{B}_1\ \bar{\tilde{q}}\ \tilde{B}_2^c$ and there exists two fuzzy b-open sets $\tilde{C}$, $\tilde{D}$ in

$(\tilde{A}, \tilde{\tau})$ such that $\tilde{B}_1 \subseteq \tilde{C}$, $\tilde{B}_2^c \subseteq \tilde{D}$ and $\tilde{C}\ \bar{\tilde{q}}\ \tilde{D}$, hence $\tilde{C} \subseteq \tilde{D}^c$.

This implies that $bcl(\tilde{C}) \subseteq bcl(\tilde{D}^c) = \tilde{D}^c$

Therefore $\tilde{B}_1 \subseteq \tilde{C} \subseteq bcl(\tilde{C}) \subseteq \tilde{B}_2$.

(6)→(1) Let $x_r \in \tilde{A}$ and $\tilde{B}$ be a fuzzy closed set in $(\tilde{A}, \tilde{\tau})$ such that

$x_r \bar{\tilde{q}}\ \tilde{B}$, then $x_r \in \tilde{B}^c$ and there exists fuzzy b-open set $\tilde{C}$ in $(\tilde{A}, \tilde{\tau})$ such that

$x_r \in \tilde{C} \subseteq bcl(\tilde{C}) \subseteq \tilde{B}^c$, $\tilde{B} \subseteq bint(\tilde{C}^c)$ and $\tilde{C}\ \bar{\tilde{q}}\ bint(\tilde{C}^c)$.

Hence $(\tilde{A}, \tilde{\tau})$ is a fuzzy b-regular space .

Theorem 2.4 :

If $(\tilde{A}, \tilde{\tau})$ is a fts , then the following statements are equivalents .

1. $(\tilde{A}, \tilde{\tau})$ is a fuzzy b-normal space .
2. For each two fuzzy closed sets $\tilde{B}_1$, $\tilde{B}_2$ in $(\tilde{A}, \tilde{\tau})$ such that $\tilde{B}_1 \tilde{\bar{q}} \tilde{B}_2$, there exists two fuzzy b-open sets $\tilde{C}_1$, $\tilde{C}_2$ in $(\tilde{A}, \tilde{\tau})$ such that $\tilde{B}_1 \subseteq \tilde{C}_1$, $\tilde{B}_2 \subseteq \tilde{C}_2$ and $bcl(\tilde{C}_1) \tilde{\bar{q}} \tilde{C}_2$.
3. For each fuzzy closed set $\tilde{B}_1$ and each fuzzy open set $\tilde{B}_2$ in $(\tilde{A}, \tilde{\tau})$ such that $\tilde{B}_1 \subseteq \tilde{B}_2$, there is a fuzzy b-open set $\tilde{C}$ in $(\tilde{A}, \tilde{\tau})$ such that $\tilde{B}_1 \subseteq \tilde{C} \subseteq bcl(\tilde{C}) \subseteq \tilde{B}_2$.

Proof : (1)→(2) Let $\tilde{B}_1$, $\tilde{B}_2$ be a fuzzy closed sets in $(\tilde{A}, \tilde{\tau})$ such that

$\tilde{B}_1 \tilde{\bar{q}} \tilde{B}_2$, then there exists two fuzzy b-open sets $\tilde{C}_1$, $\tilde{C}_2$ in $(\tilde{A}, \tilde{\tau})$ such that

$\tilde{B}_1 \subseteq \tilde{C}_1$, $\tilde{B}_2 \subseteq \tilde{C}_2$ and $\tilde{C}_1 \tilde{\bar{q}} \tilde{C}_2$. Hence by proposition 1.14 $bcl(\tilde{C}_1) \tilde{\bar{q}} \tilde{C}_2$.

(2)→(3) Let $\tilde{B}_1$ be a fuzzy closed set and $\tilde{B}_2$ be a fuzzy open set in $(\tilde{A}, \tilde{\tau})$ such

that $\tilde{B}_1 \subseteq \tilde{B}_2$, then $\tilde{B}_2^c \tilde{\bar{q}} \tilde{B}_1$ and there exists two fuzzy b-open sets $\tilde{C}$, $\tilde{D}$ in $(\tilde{A}, \tilde{\tau})$

such that $\tilde{B}_1 \subseteq \tilde{C}$, $\tilde{B}_2^c \subseteq \tilde{D}$ and $bcl(\tilde{C}) \tilde{\bar{q}} \tilde{D}$, hence $\tilde{C} \tilde{\bar{q}} \tilde{D}$ ……………(*)

by (*) we get $\tilde{C} \subseteq \tilde{D}^c$ then $bcl(\tilde{C}) \subseteq bcl(\tilde{D}^c) = \tilde{D}^c$

Therefore $\tilde{B}_1 \subseteq \tilde{C} \subseteq bcl(\tilde{C}) \subseteq \tilde{B}_2$.

(3)→(1) Let $\tilde{B}_1$, $\tilde{B}_2$ be a fuzzy closed sets in $(\tilde{A}, \tilde{\tau})$ such that

$\tilde{B}_1 \tilde{\bar{q}} \tilde{B}_2$, then $\tilde{B}_1 \subseteq \tilde{B}_2^c$ and there exists fuzzy b-open set $\tilde{C}$ in $(\tilde{A}, \tilde{\tau})$ such that

$\tilde{B}_1 \subseteq \tilde{C} \subseteq bcl(\tilde{C}) \subseteq \tilde{B}_2^c$, hence $\tilde{B}_2 \subseteq bint(\tilde{C}^c)$.

Since $bint(\tilde{C}^c)$ is fuzzy b-open set and $bint(\tilde{C}^c) \subseteq \tilde{B}_2^c$, then there exists two fuzzy

b-open sets $\tilde{C}$, $bint(\tilde{C}^c)$ such that $\tilde{B}_1 \subseteq \tilde{C}$, $\tilde{B}_2 \subseteq bint(\tilde{C}^c)$ and $\tilde{C} \tilde{\bar{q}} bint(\tilde{C}^c)$.

Hence $(\tilde{A}, \tilde{\tau})$ is a fuzzy b-normal space .

3. Fuzzy b-extremely Disconnected Space

Definition 3.1 :

A fts $(\tilde{A}, \tilde{\tau})$ is said to be fuzzy b-extremely disconnected space iff the b-closure of every fuzzy b-open set is a fuzzy b-open set

i.e if $\tilde{B} \in FBO(\tilde{A}) \rightarrow bcl(\tilde{B}) \in FBO(\tilde{A})$.

Example 3.2 :

Let X = {a , b , c} , $\tilde{A}$ = {(a , 0.5) , (b , 0.5) , (c , 0.5)} ,

$\tilde{B}_{\text{١}}$ = {(a , 0.5) , (b , 0.0) , (c , 0.0)} , $\tilde{B}_{\text{٢}}$ = {(a , 0.0) , (b , 0.5) , (c , 0.0)} ,

$\tilde{B}_{\text{٣}}$ = {(a , 0.5) , (b , 0.5) , (c , 0.0)} , $\tilde{B}_4$ = {(a , 0.5) , (b , 0.0) , (c , 0.5)},

$\tilde{B}_5$ = {(a , 0.0) , (b , 0.5) , (c , 0.5)} and $\tilde{B}_6$ = {(a , 0.0) , (b , 0.0) , (c , 0.5)} be a fuzzy sets in $\tilde{A}$ and $\tilde{\tau} = \{\tilde{\varphi}, \tilde{A}, \tilde{B}_{\text{١}}, \tilde{B}_2, \tilde{B}_3, \tilde{B}_4, \tilde{B}_5, \tilde{B}_6\}$ be a fuzzy topology on $\tilde{A}$, then the $FBO(\tilde{A}) = \tilde{\tau}$ and $(\tilde{A}, \tilde{\tau})$ is a fuzzy b-extremely disconnected space .

Theorem 3.3 :

A fts $(\tilde{A}, \tilde{\tau})$ is a fuzzy b- extremely disconnected space iff for every pair of fuzzy b-open sets $\tilde{B}$, $\tilde{C}$ in $(\tilde{A}, \tilde{\tau})$ such that $\tilde{B}\ \tilde{\bar{q}}\ \tilde{C}$, then $bcl(\tilde{B})\ \tilde{\bar{q}}\ bcl(\tilde{C})$.

Proof: Let $\tilde{B}$ and $\tilde{C} \in FBO(\tilde{A})$ such that

$\tilde{B}\ \tilde{\bar{q}}\ \tilde{C}$, then by proposition 1.14 $bcl(\tilde{B})\ \tilde{\bar{q}}\ \tilde{C}$.

Since $(\tilde{A}, \tilde{\tau})$ is a fuzzy b- extremely disconnected space .

Then $bcl(\tilde{B}) \in FBO(\tilde{A})$.

Hence by proposition 1.14 $bcl(\tilde{B})\ \tilde{\bar{q}}\ bcl(\tilde{C})$.

Conversely : Let $\tilde{B} \in FBO(\tilde{A})$, then $\tilde{B}$ and $bint(\tilde{B}^c) \in FBO(\tilde{A})$

such that $\tilde{B}\ \tilde{\bar{q}}\ bint(\tilde{B}^c)$.

By hypothesis $bcl(\tilde{B})\ \tilde{\bar{q}}\ bcl(bint(\tilde{B}^c))$, hence

$bcl(\tilde{B}) \subseteq (bcl(bint(\tilde{B}^c)))^c = bint(bcl(\tilde{B}))$ ....(*)

Since $bint(bcl(\tilde{B})) \subseteq bcl(\tilde{B})$ ....(**)

From (*) , (**) we get $bint(bcl(\tilde{B})) = bcl(\tilde{B})$.

Hence $bcl(\tilde{B}) \in FBO(\tilde{A})$.

Theorem 3.4 :

A fts $(\tilde{A} , \tilde{\tau})$ is a fuzzy b-extremely disconnected space iff $bint(bcl(\tilde{B})) = bcl(\tilde{B})$, $\forall\ \tilde{B} \in FBO(\tilde{A})$.

Proof:
Let $\tilde{B} \in FBO(\tilde{A})$ and $(\tilde{A} , \tilde{\tau})$ is a fuzzy b-extremely disconnected space.

Then $bcl(\tilde{B}) \in FBO(\tilde{A})$, hence $bint(bcl(\tilde{B})) = bcl(\tilde{B})$.

Conversely : Let $\tilde{B} \in FBO(\tilde{A})$, then $bint(bcl(\tilde{B})) = bcl(\tilde{B})$

Hence $bcl(\tilde{B}) \in FBO(\tilde{A})$.

Therefore $(\tilde{A} , \tilde{\tau})$ is a fuzzy b-extremely disconnected space .

Corollary 3.5 :

A fts $(\tilde{A} , \tilde{\tau})$ is a fuzzy b-extremely disconnected space iff

$bcl(bint(\tilde{B})) = bint(\tilde{B})$, $\forall\ \tilde{B}^c \in FBO(\tilde{A})$.

Proof: Let $\tilde{B}^c \in FBO(\tilde{A})$, then $bint(bcl(\tilde{B}^c)) = bcl(\tilde{B}^c)$.

Hence $(bint(bcl(\tilde{B}^c)))^c = (bcl(\tilde{B}^c))^c$.

Therefore $bcl(bint(\tilde{B})) = bint(\tilde{B})$.

Conversely : Let $\tilde{B} \in$ FBO($\tilde{A}$) , then $\tilde{B}^c$ is a fuzzy b-closed set in $\tilde{A}$ and by hypotheses we get bcl(bint($\tilde{B}^c$)) = bint($\tilde{B}^c$) and (bcl(bint($\tilde{B}^c$)))^c = (bint($\tilde{B}^c$))^c .

Hence bint(bcl($\tilde{B}$)) = bcl($\tilde{B}$) .

Therefore ($\tilde{A}$, $\tilde{\tau}$) is a fuzzy b- extremely disconnected space .

Theorem 3.6 :

Every fuzzy open subspace of fuzzy b-extremely disconnected space is A fuzzy b-extremely disconnected space .

Proof: Let $\tilde{C}_1$ be a fuzzy b-open set in ($\tilde{B}$, $\tilde{\tau}_{\tilde{B}}$) , then there exists fuzzy b-open set $\tilde{C}_2$ in ($\tilde{A}$, $\tilde{\tau}$) such that $\tilde{B} \cap \tilde{C}_2 = \tilde{C}_1$.

Since $\tilde{B} \cap \tilde{C}_2 \in$ FBO($\tilde{A}$) and ($\tilde{A}$, $\tilde{\tau}$) is a fuzzy b-extremely disconnected space.

Then bcl($\tilde{B} \cap \tilde{C}_2$) $\in$ FBO($\tilde{A}$) .

Hence bcl($\tilde{C}_1$) $\in$ FBO($\tilde{B}$) .

Theorem 3.7 :

For any fts ($\tilde{A}$, $\tilde{\tau}$) the following statement are equivalent :-

1. ($\tilde{A}$, $\tilde{\tau}$) is a fuzzy b-extremely disconnected space .
2. For each fuzzy b-closed set $\tilde{B}$, then bint($\tilde{B}$) is a fuzzy b-closed set .
3. For each fuzzy b-open set $\tilde{B}$ we have $\mu_{bcl(\tilde{B})}(x) + \mu_{bcl(bint(\tilde{B}^c))}(x) = \mu_{\tilde{A}}(x)$.
4. For every pair of fuzzy b-open sets $\tilde{B}$, $\tilde{C}$ with $\mu_{bcl(\tilde{B})}(x) + \mu_{\tilde{C}}(x) = \mu_{\tilde{A}}(x)$ we have $\mu_{bcl(\tilde{B})}(x) + \mu_{bcl(\tilde{C})}(x) = \mu_{\tilde{A}}(x)$.

Proof: (1) → (2) Let B̃ be a fuzzy b-closed set , then $\tilde{B}^c$ is a fuzzy b-open set .

Since (Ã , $\tilde{\tau}$) is a fuzzy b-extremely disconnected space , then bcl($\tilde{B}^c$) is a fuzzy b-open set , hence $(bcl(\tilde{B}^c))^c$ is a fuzzy b-closed set .

Since $(bcl(\tilde{B}^c))^c$ = bint(B̃) , then bint(B̃) is a fuzzy b-closed set .

(2) → (3) Let B̃ be a fuzzy b-open set , then $\tilde{B}^c$ is a fuzzy b-closed set

$\mu_{bcl(\tilde{B})}(x) + \mu_{bcl(bint(\tilde{B}^c))}(x) = \mu_{bcl(\tilde{B})}(x) + \mu_{bint(\tilde{B}^c)}(x)$ (by (2))

$= \mu_{bcl(\tilde{B})}(x) + \mu_{(bcl(\tilde{B}))^c}(x) = \mu_{\tilde{A}}(x)$.

(3) → (4) Let B̃, C̃ be a fuzzy b-open sets such that $\mu_{bcl(\tilde{B})}(x) + \mu_{\tilde{C}}(x) = \mu_{\tilde{A}}(x)$,

then $\tilde{C} = (bcl(\tilde{B}))^c = bint(\tilde{B}^c)$ ………….(*)

Since $\mu_{bcl(\tilde{B})}(x) + \mu_{bcl(bint(\tilde{B}^c))}(x) = \mu_{\tilde{A}}(x)$, then

From (*) we get $\mu_{bcl(\tilde{B})}(x) + \mu_{bcl(\tilde{C})}(x) = \mu_{\tilde{A}}(x)$.

(4) → (1) Let B̃ be a fuzzy b-open set and let $\tilde{C} = (bcl(\tilde{B}))^c$, then

$\mu_{bcl(\tilde{B})}(x) + \mu_{\tilde{C}}(x) = \mu_{\tilde{A}}(x)$ and by (4) $\mu_{bcl(\tilde{B})}(x) + \mu_{bcl(\tilde{C})}(x) = \mu_{\tilde{A}}(x)$, hence

$bcl(\tilde{C}) = (bcl(\tilde{B}))^c = bint(\tilde{B}^c)$.

Since bcl(C̃) is a fuzzy b-closed set , then bint($\tilde{B}^c$) is a fuzzy b-closed set .

Hence bcl(B̃) is a fuzzy b-open set .

Theorem 3.8 :

In a fuzzy b-extremely disconnected space (Ã , $\tilde{\tau}$) is a fuzzy b-$\tilde{T}_2$ space iff for every distinct fuzzy points x_r , y_s ∈ Ã, there exists two fuzzy b-open sets B̃ , C̃ in

(Ã , $\tilde{\tau}$) such that $x_r \in$ B̃ , $y_s \in$ C̃ and bcl(B̃) $\tilde{\bar{q}}$ bcl(C̃) .

Proof :

Let x_r , $y_s \in$ Ã , then there exists two fuzzy b-open sets B̃ , C̃ in (Ã , $\tilde{\tau}$) such that $x_r \in$ B̃ , $y_s \in$ C̃ and bcl(B̃) $\tilde{\bar{q}}$ bcl(C̃) .

Since B̃ ⊆ bcl(B̃) and C̃ ⊆ bcl(C̃) , then B̃ $\tilde{\bar{q}}$ C̃ .

Therefore (Ã , $\tilde{\tau}$) is a fuzzy b-$\tilde{T}_2$ space .

Conversely : Let x_r , $y_s \in$ Ã , then there exists two fuzzy b-open sets B̃ , C̃ in (Ã , $\tilde{\tau}$) such that $x_r \in$ B̃ , $y_s \in$ C̃ and B̃ $\tilde{\bar{q}}$ C̃ .

Since C̃ is a fuzzy b-open set , then by proposition 1.14 bcl(B̃) $\tilde{\bar{q}}$ C̃ .

Since (Ã , $\tilde{\tau}$) is a fuzzy b-extremely disconnected space , then bcl(B̃) is a fuzzy b-open set and by by proposition 1.14 we get bcl(B̃) $\tilde{\bar{q}}$ bcl(C̃).

Theorem 3.9 :

In a fuzzy b-extremely disconnected space (Ã , $\tilde{\tau}$) is fuzzy b-regular space iff for every fuzzy point $x_r \in$ Ã and for every fuzzy closed set B̃ in (Ã , $\tilde{\tau}$) such that x_r $\tilde{\bar{q}}$ B̃ , there exists two fuzzy b-open sets $\tilde{C}_1$, $\tilde{C}_2$ in (Ã , $\tilde{\tau}$) such that $x_r \in$ $\tilde{C}_1$, B̃ ⊆ $\tilde{C}_2$ and bcl($\tilde{C}_1$) $\tilde{\bar{q}}$ bcl($\tilde{C}_2$) .

Proof: Let $x_r \in$ Ã and B̃ be a fuzzy closed set in (Ã , $\tilde{\tau}$) such that x_r $\tilde{\bar{q}}$ B̃ , there exists two fuzzy b-open sets $\tilde{C}_1$, $\tilde{C}_2$ in (Ã , $\tilde{\tau}$) such that $x_r \in$ $\tilde{C}_1$.

Since $\tilde{C}_1$ is a fuzzy b-open set , then by proposition 1.14 $\tilde{C}_1$ $\tilde{\bar{q}}$ bcl($\tilde{C}_2$) .

Since (Ã , $\tilde{\tau}$) is a fuzzy b-extremely disconnected space , then bcl($\tilde{C}_2$) is a fuzzy b-open set and by proposition 1.14 we get bcl($\tilde{C}_1$) $\tilde{\bar{q}}$ bcl($\tilde{C}_2$) .

Conversely : Let $x_r \in \tilde{A}$ and $\tilde{B}$ be a fuzzy closed set in $(\tilde{A}, \tilde{\tau})$

$x_r \tilde{\bar{q}}\ \tilde{B}$, by hypotheses there exists two fuzzy b-open sets $\tilde{C}_1$, $\tilde{C}_2$ in $(\tilde{A}, \tilde{\tau})$

such that $x_r \in \tilde{C}_1$, $\tilde{B} \subseteq \tilde{C}_2$ and $bcl(\tilde{C}_1)\ \tilde{\bar{q}}\ bcl(\tilde{C}_2)$.

Hence $\tilde{C}_1 \tilde{\bar{q}}\ \tilde{C}_2$ .Therefore $(\tilde{A}, \tilde{\tau})$ is fuzzy b-regular space .

Theorem 3.10 :

If $(\tilde{A}, \tilde{\tau})$ is a fuzzy b-extremely disconnected fuzzy b-normal space , $\tilde{B}$ is a fuzzy closed subspace in $(\tilde{A}, \tilde{\tau})$ and $\tilde{C}$ is a fuzzy clopen subspace in $\tilde{B}$, then there exists fuzzy b-open set $\tilde{D}$ in $(\tilde{A}, \tilde{\tau})$ such that $\tilde{D} \cap \tilde{B} = \tilde{C}$.

Proof: Since $\tilde{C}$ is a fuzzy open set in $\tilde{B}$, then here exists a fuzzy open set $\tilde{C}_1$ in $(\tilde{A}, \tilde{\tau})$ such that $\tilde{C}_1 \cap \tilde{B} = \tilde{C}$.

Since $\tilde{C}$ is a fuzzy closed set and $\tilde{C}_1$ is a fuzzy open set in the fuzzy b-normal space $(\tilde{A}, \tilde{\tau})$ such that $\tilde{C} \subseteq \tilde{C}_1$, therefore by theorem 2.4

there exists a fuzzy b-open set $\tilde{C}_2$ in $(\tilde{A}, \tilde{\tau})$ such that

$\tilde{C} \subseteq \tilde{C}_2 \subseteq bcl(\tilde{C}_2) \subseteq \tilde{C}_1$

Since $(\tilde{A}, \tilde{\tau})$ is a fuzzy b-extremely disconnected space , then $bcl(\tilde{C}_2)$ is a fuzzy b-open set and let $bcl(\tilde{C}_2) = \tilde{D}$. Thus $bcl(\tilde{C}_2) \cap \tilde{B} \subseteq \tilde{C}_1 \cap \tilde{B} = \tilde{C}$……(*) .

Since $\tilde{C} \subseteq bcl(\tilde{C}_2)$ and $\tilde{C}_1 \subseteq \tilde{B}$, then $\tilde{C} \subseteq bcl(\tilde{C}_2) \cap \tilde{B}$……….(**)

From (*) and (**) it follows that $\tilde{C} = \tilde{D} \cap \tilde{B}$.

4. Fuzzy b-hyper Connected Space

Definition 4.1 :

A fuzzy topological space $(\tilde{A}, \tilde{\tau})$ is said to be fuzzy b-hyper connected space if

$\forall\, \tilde{B} \in FBO(\tilde{A}) \rightarrow bcl(\tilde{B}) = \tilde{A}$.

Example 4.2 :

Let $X = \{a, b, c\}$, $\tilde{A} = \{(a, 0.5), (b, 0.5), (c, 0.5)\}$,

$\tilde{B}_1 = \{(a, 0.5), (b, 0.0), (c, 0.0)\}$, $\tilde{B}_2 = \{(a, 0.5), (b, 0.5), (c, 0.0)\}$,

$\tilde{B}_3 = \{(a, 0.5), (b, 0.0), (c, 0.5)\}$ be a fuzzy sets in $\tilde{A}$, $\tilde{\tau} = \{\tilde{\varphi}, \tilde{A}, \tilde{B}_1\}$ be a fuzzy topology on $\tilde{A}$, then the $FBO(\tilde{A}) = \{\tilde{\varphi}, \tilde{A}, \tilde{B}_1, \tilde{B}_2, \tilde{B}_3\}$, and $(\tilde{A}, \tilde{\tau})$ is a fuzzy b-hyper connected space .

Proposition 4.3 :

Every fuzzy b-hyper connected space is a fuzzy b-extremely disconnected space.

Proof: Obvious .

Remark 4.4 :

The converse of proposition 4.3 is not true in general as shown in the following example.

Example 4.5 :

The space $(\tilde{A}, \tilde{\tau})$ in the example 3.2 is a fuzzy b-extremely disconnected space but not fuzzy b-hyper connected space.

Theorem 4.6 :

Every fuzzy open subspace of fuzzy b-hyper connected space is a fuzzy b-hyper connected space .

Proof: Let $\tilde{C}_1$ be a fuzzy b-open set in $(\tilde{B}, \tilde{\tau}_{\tilde{B}})$, then

there exists fuzzy b-open set $\tilde{C}_2$ in $(\tilde{A}, \tilde{\tau})$ such that $\tilde{B} \cap \tilde{C}_2 = \tilde{C}_1$.

Since $\tilde{B} \cap \tilde{C}_2 \in$ FBO($\tilde{A}$) and $(\tilde{A}, \tilde{\tau})$ is a fuzzy b-hyper connected space .

Then bcl($\tilde{B} \cap \tilde{C}_2$) = $\tilde{A}$.

Since bcl($\tilde{C}_1$) = bcl($\tilde{C}_1$) ∩ $\tilde{B}$ = $\tilde{A} \cap \tilde{B} = \tilde{B}$.

Hence bcl($\tilde{C}_1$) = $\tilde{B}$.

Lemma 4.7 :

Let $(\tilde{A}, \tilde{\tau}_1)$ *and* $(\tilde{A}, \tilde{\tau}_2)$ be two fuzzy topological spaces such that

$(FBO(\tilde{A}))_1 \subseteq (FBO(\tilde{A}))_2$, then $(bcl(\tilde{B}))_2 \subseteq (bcl(\tilde{B}))_1$.

Proof:

Let $x_r \notin (bcl(\tilde{B}))_1$, then there exists fuzzy b-quasi neighborhood $\tilde{C}$ of x_r in $(FBO(\tilde{A}))_1$ such that $\tilde{B} \,\tilde{\bar{q}}\, \tilde{C}$.

Since $\tilde{C}$ is a fuzzy b-quasi neighborhood of x_r .

Then , there exists $\tilde{D} \in (FBO(\tilde{A}))_1$ such that $x_r \, q \, \tilde{D}$ and $\tilde{D} \subseteq \tilde{C}$.

Hence $\tilde{D} \in (FBO(\tilde{A}))_2$ and $\tilde{C}$ b-quasi neighborhood of x_r in $(FBO(\tilde{A}))_2$.

Therefore , $x_r \notin (bcl(\tilde{B}))_2$.

Proposition 4.8 :

Let $(\tilde{A}, \tilde{\tau}_1)$ *and* $(\tilde{A}, \tilde{\tau}_2)$ be two fuzzy topological spaces such that $\tilde{\tau}_1 \subseteq \tilde{\tau}_2$ and

$(FBO(\tilde{A}))_1 \subseteq (FBO(\tilde{A}))_2$, if $(\tilde{A}, \tilde{\tau}_2)$ is a fuzzy b-hyper connected space then

$(\tilde{A}, \tilde{\tau}_1)$ *is* a fuzzy b-hyper connected space.

Proof: Let $\tilde{B} \in (FBO(\tilde{A}))_1$, then $\tilde{B} \in (FBO(\tilde{A}))_2$.

Since $(\tilde{A}, \tilde{\tau}_2)$ is a fuzzy b-hyper connected space , then $(bcl(\tilde{B}))_2 = \tilde{A}$.

Since $(bcl(\tilde{B}))_2 \subseteq (bcl(\tilde{B}))_1$, then $\tilde{A} \subseteq (bcl(\tilde{B}))_1$ but $(bcl(\tilde{B}))_1 \subseteq \tilde{A}$.

Therefore $(\tilde{A}, \tilde{\tau}_1)$ *is* a fuzzy b-hyper connected space.

Remark 4.9 :

The converse of proposition 4.8 is not true in general as shown in the following example.

Example 4.10 :

Let X = {a , b , c} , $\tilde{A}$ = {(a , 0.5) , (b , 0.5) , (c , 0.5)} ,

$\tilde{B}_1$ = {(a , 0.5) , (b , 0.0) , (c , 0.0)} , $\tilde{B}_2$ = {(a , 0.0) , (b , 0.5) , (c , 0.5)} ,

$\tilde{B}_3$ = {(a , 0.5) , (b , 0.5) , (c , 0.0)} , $\tilde{B}_4$ = {(a , 0.5) , (b , 0.0) , (c , 0.5)},

$\tilde{B}_5$ = {(a , 0.0) , (b , 0.5) , (c , 0.0)} and $\tilde{B}_6$ = {(a , 0.0) , (b , 0.0) , (c , 0.5)} be a fuzzy sets in $\tilde{A}$, $\tilde{\tau}_1 = \{\tilde{\varphi}, \tilde{A}, \tilde{B}_1\}$ and $\tilde{\tau}_2 = \{\tilde{\varphi}, \tilde{A}, \tilde{B}_1, \tilde{B}_2\}$ be a fuzzy topology on

$\tilde{A}$, then the $(FBO(\tilde{A}))_1 = \{\tilde{\varphi}, \tilde{A}, \tilde{B}_1, \tilde{B}_3, \tilde{B}_4\}$,

$(FBO(\tilde{A}))_2 = \{\tilde{\varphi}, \tilde{A}, \tilde{B}_1, \tilde{B}_3, \tilde{B}_4, \tilde{B}_5, \tilde{B}_6\}$ and $(\tilde{A}, \tilde{\tau}_1)$ is a fuzzy b-hyper connected space but $(\tilde{A}, \tilde{\tau}_2)$ is not fuzzy b-hyper connected space.

References

[1] : Benchalli , S.S and Jenifer Karnel , " On Fuzzy b-Open Sets In Fuzzy Topological Space", J.Computer and Mathematical Sciences No.1, Vol.127-134 , (2010) .

[2] : Chakrabarty , M. K. and Ahsanullah , T. M. G. “Fuzzy Topology On Fuzzy Sets And Tolerance Topology “ , Fuzzy sets and Systems 45,103-108 , (1992) .

[3] : Dubois , D. and Prade H., “Fuzzy Sets and Systems; Theory and Applications”, Mathematics in Science and Engineering Vol.144, (1980).

[4] : Erdal Ekici , “Generalization of Some Fuzzy Functions” , Institute of Mathematics Academia Sinica No.3 , Vol.33 , (2005) .

[5] : Fatteh , U. V. and Bassan , D.S. ,“Fuzzy Connectedness and its stronger forms“ Journal of Mathematical Analysis and Applications , III (1985) , 44-464 .

[6] : Kider , J. R. “Fuzzy Locally Convex-Algebras “ Ph . D. Thesis , School of Applied Sciences , Univ. Technology , (2004).

[7] : Luay, A.A. and Amed , S.A.O., “Fuzzy γ-Open Sets And Fuzzy γ-Closed Sets”, American Journal of Scientific Research, 27, pp.62-67, (2011) .

[8] : Mahmuod , F.S. , Fath , . Alla M.A and Abd.Ellah, S.M. ,"Fuzzy Topology On Fuzzy Sets: Fuzzy Semi Continuity And Fuzzy Separation Axioms", APPl. Math. and Comp.153, 127-140, (2004) .

[9] : Rekha srivastava, S. N. L. and Arun K. srivastava “ Fuzzy Hausdorff

Topological Spaces " J. Math. Anal. And Appl . 81, 497-506, (1981) .
[10] : Shahla , H.K, "On Fuzzy Δ-Open Sets In Fuzzy Topological Spaces ", M.Sc Thesis ,College of Science, Salahaddin Univ, (2004) .

[11]: Wong, C. K., "Fuzzy Points and Local Properties of Fuzzy Topology", J. Math. Anal. Appl., Vol.46, pp. 316-328, (1974).

[12] : Zadeh, L.A.," Fuzzy sets", Inform. Contr. 8, 338-353, (1965).