# Statistical Simulation of Gamma-Rays Interactions : Sustainable Development and Monte Carlo Simulation Previewing

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**Abstract**

The human is the supreme aim of All the development plans. Because of an accompanier hazards of dealing with radioactive materials, the workers (i.e. staff, students, researchers and others) in various fields of nuclear physics need particular protection. In the present work, A computer simulation program based on Monte Carlo method was designed and written to be as virtual experimental system, instead of the real experimental system, concerning gamma rays radioactive source - detector system to mimic the statistical distributions of interactions of gamma-rays with active medium of the detector data. Clearly, the statistical distribution of present results describe as a Poisson distribution and Gaussian distribution at low and high gamma counted rate respectively. These behaviors of the present results were in agreement with theoretical concepts of statistical distributions and pervious published experimental work. So, the present simulation, as a virtual computer experiment, makes laboratory personnel safe from exposure to radiation, either from the use of radioactive sources in the laboratory's experimental or background of the laboratory.

**Key words:** statistical distributions, Gamma-Ray Interactions, Monte Carlo Simulation, Sustainable Development.

**المحاكاة الإحصائية لتفاعلات أشعة گاما: التنمية المستدامة**
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**المستخلص**

يعتبر الإنسان الهدف الأسمى في كافة خطط التنمية. وبسبب الأخطار المرافقة في التعامل مع المواد المشعة، فإن العاملين (تدريسين، طلبة، باحثين وآخرين) في مختلف حقول الفيزياء النووية بحاجة الى حماية من نمط خاص. في البحث الحالي، تم تصميم وكتابة برنامج محاكاة حاسوبي بالإستناد على طريقة مونت كارلو ليكون بمثابة نظام عملي إفتراضي بدلاً من النظام العملي الحقيقي المتعلق بنظام مصدر مشع لأشعة گاما وكاشف ليحاكي التوزيعات الإحصائية الناتجة من بيانات تفاعل أشعة گاما مع الوسط الفعّال للكاشف. أظهرت التوزيعات الإحصائية لنتائج البحث الحالي توزيع بوزون عند نسبة عد گاما قليلة وتوزيع كاوس عند نسبة عد عالية وهذا يتوافق مع المباديء النظرية للتوزيعات الإحصائية والنتائج العملية المنشورة. ولذلك فإن المحاكاة الحالية، كتجربة عملية إفتراضية، وفرت بيئة عمل آمنة من التعرض للإشعاع سواء من المصادر المشعة المستخدمة في التجربة العملية الحقيقية أو الخلفية الإشعاعية للمختبر.

## 1. Introduction:

The human is the supreme aim of All the development plans. Therefore, the workers (i.e. staff, students, researchers and others) in various fields of nuclear physics need particular protection. An accompanier hazards of dealing with radioactive materials require shield, least exposure time and largest distance. Sometimes, the nature of results oblige the workers to dissent the regulations.
Broadly, Computer simulation can be defined as: using a computer to imitate the operations of real world process or facility according to appropriately developed assumption taking the form of logical, statistical, or mathematical relationships which are developed and shaped into a model. [1]. The results can be manipulated by varying a set of input parameters to help an analyst understand the underlying system's dynamics. The model typically is

evaluated numerically over a simulated period of time and data is gathered to estimate real world system characteristics. Generally, the collected data is interpreted with statistics like any other experiment. So, we can say, a simulation is an experiment [2], or as computer experiments because they share much in common with laboratory experiments.
In the present work, A computer simulation program based on Monte Carlo method was designed and written to be as virtual experimental system, instead of the real experimental system, concerning gamma rays radioactive source - detector system and statistical distributions of interactions of gamma-rays with active medium of the detector.

## 2. The Scenario of Simulation:

The law of nuclear decay includes a statistical estimate of what happens to the radioactive sample that describes a purely random process.
Radiation activity is estimated for the rate of disintegration per minute can be calculated from the equation: [٣]

$$N = N_o e^{-\lambda t} \cdots\cdots\cdots (2.1)$$

This equation means that the average decay is proportional to the number of atoms $N$ at a time $t$, and $\lambda$ is the expression for the probability of decay of each atom, which is typical of the radioactive source. This expression describes the average behavior of the source.
The random system includes the emission of photons or particles from the source at different emission angles and their fall or non-fall on the detector window and thus its random movement inside the detector and the probability of its interaction or non-interaction with the detector material or its survival or failure to remain in the detector, all require the use of the Monte Carlo simulation to be able to describe all those operations mentioned above, called 'history of photon' as shown in Fig.1 whict include the configuration proposed for the simulation.

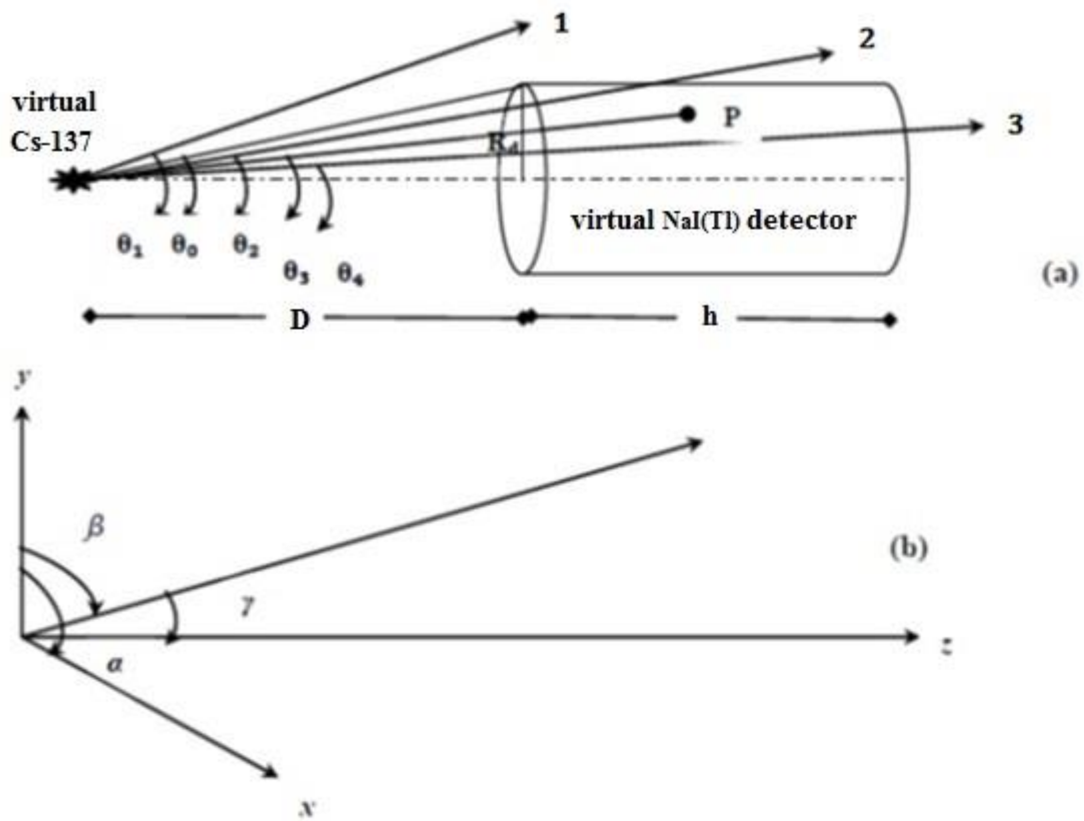


**Fig.1:** The suggested virtual experimental source-detector set up, based on real experimental set up, in: **a)** fixed axes system and **b)** Reference axes system.

The history of photon is defined during the following algorithm:

1. generate two random numbers $R_1$, $R_2$.
2. determine the random emitting point ($x_o$, $y_o$, $z_o$) of $N_o$ photons from the source.
3. generate three random numbers $R_1$, $R_2$, $R_3$.
4. cosα, cosβ, and cosγ are the direction cosines of photon which can be calculated.
5. determination of the location of photon in the system space by the following parametric equations:

$$x = x_o + t\cos\alpha \quad \cdots\cdots\cdots (2.2)$$

$$y = y_o + t\cos\beta \quad \cdots\cdots\cdots (2.3)$$

$$z = z_o + t\cos\gamma \quad \cdots\cdots\cdots (2.4)$$

where $t$ is the distance from the emission point of photon from the source to the point lies on the front face of the detector.

6. certain number ($N_{inc.}$) of photons will fall on the detector window at an angle $\theta$, where:

$$\theta < \theta_o : \theta_o = \tan^{-1}\frac{R_d}{D} \cdots\cdots\cdots (2.5)$$

with radius of spherical projects less than radius of front face of the detector, the tracks 2,3 and track of point p in Fig.1. otherwise go back to step1, as track1.

7. generate random numbers $R_4$.

8. The path length of photon within effective media of the detector, is:

$$p.l. = -\left(\frac{1}{\mu_t}\right)\ln(1 - R_4) \cdots\cdots\cdots (2.6)$$

where $\mu_t$ is total linear attenuation coefficient which is a function of material type and gamma energy.

9. call the set of equations in step5 to determine the location of the interaction point of photon in the system space within effective media of the detector, as point p in Fig.1., otherwise the photon penetrates the detector from side as track2 or from bottom as track3 and go back to step1.

10. If step 9 was achieved, which means the photon is counted. Then particular count $N_a$ is increasing by one.

11. repeat the pervious steps according to the number of emitted photons from the source $N_o$.

The calculated count rate values represents many of the averages counts which were distributed randomly about the real value of the average count. It was expressed by the arithmetic mean: [٤]

$$N_{av} = \frac{1}{k}(N_1 + N_2 + N_3 + \cdots\cdots\cdots + N_k)$$

$$N_{av} = \frac{1}{k}\sum_{i=1}^{k} N_i \cdots\cdots\cdots (2.7)$$

The accuracy of the results was determined by the deviation of individual for average, which is called the variance ($\sigma^2$) [٤]:

$$\sigma^2 = \sum_{i=1}^{k} (N_i - N_{av.})^2 \Big/ (k-1) \cdots\cdots\cdots (2.8)$$

Or standard deviation ($\sigma$): [4]

$$\sigma = \sqrt{\sigma^2} = \sqrt{\sum (N_i - N_{av.})^2 / (k-1)} \cdots\cdots\cdots (2.9)$$

the normal distribution as a function of the mean and standard deviation is one of the most commonly used probability distribution for various applications. The probability density function *f (N)* for the normal distribution given by[٤]:

$$f(N) = \frac{1}{\sqrt{2\pi}\,\sigma} e^{-(N_i - N_{av.})^2/2\sigma^2} \dots\dots\dots (2.10)$$

**3. Results and Discussion:**

The present computer program represents virtual experimental set up which was describes in Fig.1. To calculate the count rate, this program was carried out for virtual Cs-137 radioactive source with energy of 662 keV placed at 10 cm from (3″×3″) NaI(Tl) detector. For lowest statistical fluctuation, $10^7$ gamma photons was the virtual activity of the virtual Cs-137 source. The present program allows re-implementation of the program more than once. 5, 10, 50, 100 trails were carried out by the present program. Table-1 shows the results of 50 trials.

Table-1-: 50 trials of calculated count rate for virtual Cs-137 of $10^7$ gamma photons with 662 keV placed at 10 cm of virtual (3″×3″) NaI(Tl) detector.

| i: No. of Trials | $N_i$ counted photons | $N_i$-Nav. | ($N_i$-Nav.)2 | $N_i\pm\sigma$ | | *f(N$_{av}$)* |
|---|---|---|---|---|---|---|
| 1 | 12106 | -10 | 100 | 12106 | ±28.55 | 0.01314 |
| 2 | 12137 | 21 | 441 | 12137 | ±28.55 | 0.01066 |
| 3 | 12147 | 31 | 961 | 12147 | ±28.55 | 0.00775 |
| 4 | 12098 | -18 | 324 | 12098 | ±28.55 | 0.01145 |
| 5 | 12068 | -48 | 2304 | 12068 | ±28.55 | 0.00341 |
| 6 | 12155 | 39 | 1521 | 12155 | ±28.55 | 0.00550 |

| | | | | | | |
|---|---|---|---|---|---|---|
| 7 | 12125 | 9 | 81 | 12125 | ±28.55 | 0.01329 |
| 8 | 12144 | 28 | 784 | 12144 | ±28.55 | 0.00864 |
| 9 | 12141 | 25 | 625 | 12141 | ±28.55 | 0.00952 |
| 10 | 12097 | -19 | 361 | 12097 | ±28.55 | 0.01120 |
| 11 | 12074 | -42 | 1764 | 12074 | ±28.55 | 0.00474 |
| 12 | 12090 | -26 | 676 | 12090 | ±28.55 | 0.00923 |
| 13 | 12095 | -21 | 441 | 12095 | ±28.55 | 0.01066 |
| 14 | 12075 | -41 | 1681 | 12075 | ±28.55 | 0.00499 |
| 15 | 12079 | -37 | 1369 | 12079 | ±28.55 | 0.00604 |
| 16 | 12147 | 31 | 961 | 12147 | ±28.55 | 0.00775 |
| 17 | 12117 | 1 | 1 | 12117 | ±28.55 | 0.01396 |
| 18 | 12138 | 22 | 484 | 12138 | ±28.55 | 0.01038 |
| 19 | 12067 | -49 | 2401 | 12067 | ±28.55 | 0.00321 |
| 20 | 12168 | 52 | 2704 | 12168 | ±28.55 | 0.00267 |
| 21 | 12165 | 49 | 2401 | 12165 | ±28.55 | 0.00321 |
| 22 | 12065 | -51 | 2601 | 12065 | ±28.55 | 0.00284 |
| 23 | 12122 | 6 | 36 | 12122 | ±28.55 | 0.01366 |
| 24 | 12112 | -4 | 16 | 12112 | ±28.55 | 0.01383 |
| 25 | 12085 | -31 | 961 | 12085 | ±28.55 | 0.00775 |
| 26 | 12127 | 11 | 121 | 12127 | ±28.55 | 0.01297 |
| 27 | 12084 | -32 | 1024 | 12084 | ±28.55 | 0.00746 |
| 28 | 12099 | -17 | 289 | 12099 | ±28.55 | 0.01170 |
| 29 | 12080 | -36 | 1296 | 12080 | ±28.55 | 0.00631 |
| 30 | 12123 | 7 | 49 | 12123 | ±28.55 | 0.01355 |
| 31 | 12154 | 38 | 1444 | 12154 | ±28.55 | 0.00577 |
| 32 | 12148 | 32 | 1024 | 12148 | ±28.55 | 0.00746 |
| 33 | 12092 | -24 | 576 | 12092 | ±28.55 | 0.00981 |
| 34 | 12140 | 24 | 576 | 12140 | ±28.55 | 0.00981 |
| 35 | 12120 | 4 | 16 | 12120 | ±28.55 | 0.01383 |
| 36 | 12109 | -7 | 49 | 12109 | ±28.55 | 0.01355 |
| 37 | 12124 | 8 | 64 | 12124 | ±28.55 | 0.01343 |
| 38 | 12121 | 5 | 25 | 12121 | ±28.55 | 0.01375 |
| 39 | 12091 | -25 | 625 | 12091 | ±28.55 | 0.00952 |
| 40 | 12116 | 0 | 0 | 12116 | ±28.55 | 0.01397 |
| 41 | 12173 | 57 | 3249 | 12173 | ±28.55 | 0.00191 |
| 42 | 12119 | 3 | 9 | 12119 | ±28.55 | 0.01389 |
| 43 | 12109 | -7 | 49 | 12109 | ±28.55 | 0.01355 |

| | | | | | | |
|---|---|---|---|---|---|---|
| 44 | 12134 | 18 | 324 | 12134 | ±28.55 | 0.01145 |
| 45 | 12103 | -13 | 169 | 12103 | ±28.55 | 0.01259 |
| 46 | 12133 | 17 | 289 | 12133 | ±28.55 | 0.01170 |
| 47 | 12108 | -8 | 64 | 12108 | ±28.55 | 0.01343 |
| 48 | 12087 | -29 | 841 | 12087 | ±28.55 | 0.00834 |
| 49 | 12153 | 37 | 1369 | 12153 | ±28.55 | 0.00604 |
| 50 | 12136 | 20 | 400 | 12136 | ±28.55 | 0.01093 |
| $\sum$ | 605800 | | 39940 | | | |
| | $N_{av}$=12116 | $\sigma^2$ | 815 | | | |
| | | $\sigma$ | 28.54999 | | | |

Based on eq.(2.7), the average count rate $N_{av}$ was found to be equal to 12116 counts, whereas all fifty values were distributed randomly around this average. The accuracy of the result was determined by the individual deviations ($N_i$-$N_{av}$) as shown in Table-1. Therefore, according to eq.(2.8) and eq.(2.9) the variance $\sigma^2$ was equal to 15 and the standard deviation $\sigma$ was equal to 28.55. now, the count rate $N_i$ can be written as, for example, (12116±28.55). As a percentage, the standard deviation in a count to the value of this count $\left(\frac{28.55}{12116} \times 100\%\right)$ is 0.2%. Viz, the error in the calculated value was about 0.002 of these values. This was show on the accuracy of present simulation and computer program to mimic the interactions of gamma rays with the detector as the in real practice experiments.

The other important parameter is the number of events or experimentally, the activity of radioactive source. It effects on the random distribution of particular events around the average value as shown in Fig.2. As shown in Fig.(2)(a, b and c), the outcomes of present program with 50, 100 and 200 photons (experimentally, means low activity) distributed as the Poisson distribution with regression correlation of data $R^2$=1. While, in Fig.(2)(d) with 500 photons (experimentally, means mid activity),roughly, distributed as the Gaussian distribution with regression correlation of data $R^2$=0.986. Based on the central limit theorem [5], When we repeat an implementing of present program numerous times, the random variable representing the average or mean tends to have a normal distribution as the number of trials (emitted photons $N_o$) becomes large (see step11in item2). Clearly, Fig.(3) and Fig.(4) describe the distribution of present results as a Poisson distribution and Gaussian distribution at low and high gamma counted rate respectively. The behaviors of the present results were in agreement with the theoretical conceptions of statistical distributions [٣,٤] and published experimental work [6].

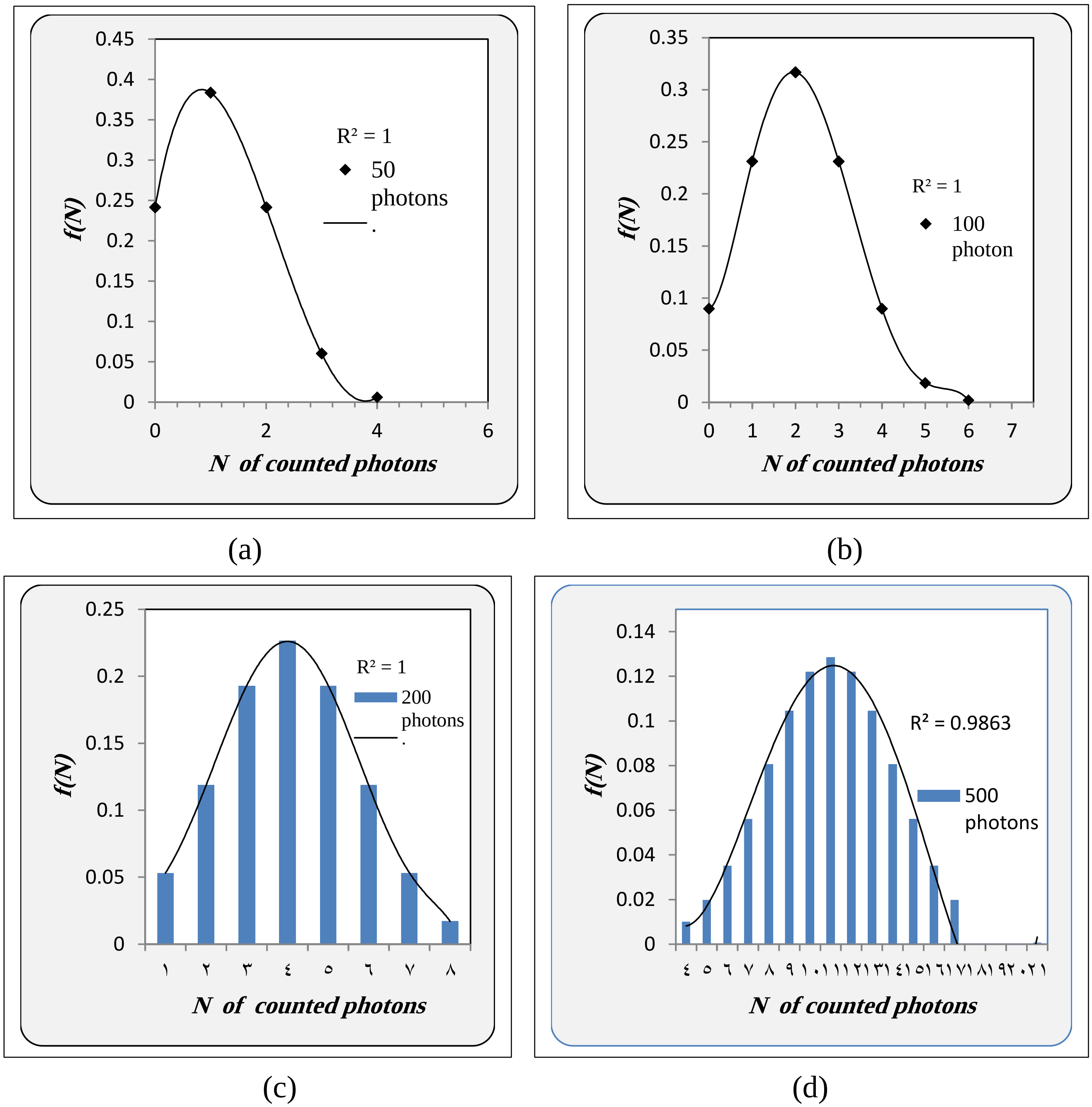


**Fig.2: Affected statistical distribution type by the events or number of counted gamma photons.**

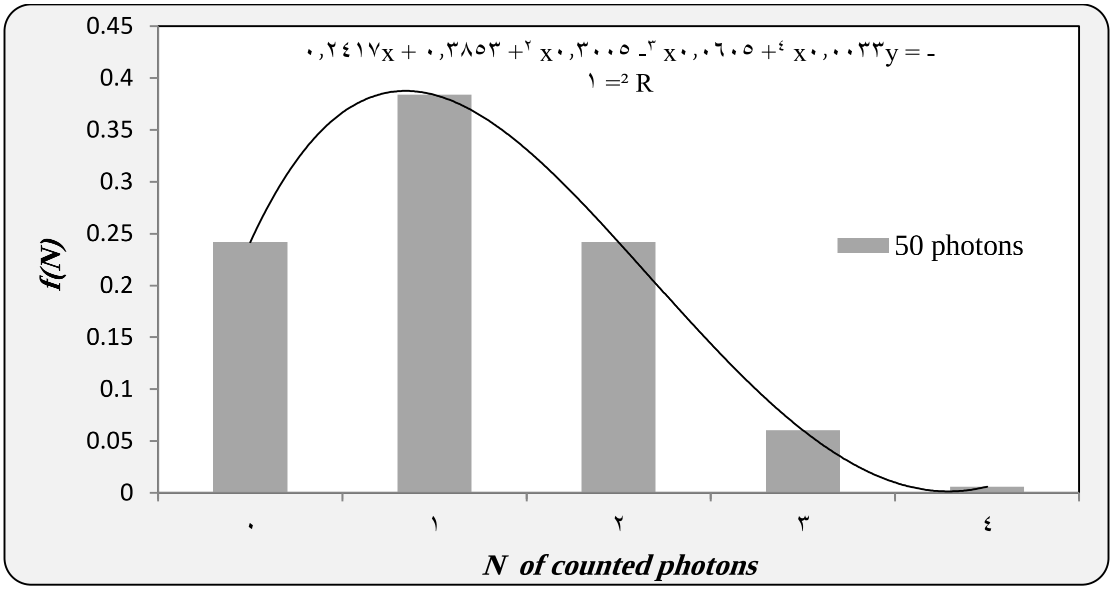


**Fig.3:** **Distribution of counted gamma photons at low activity of virtual radioactive source (50 photons) as a Poisson distribution.**

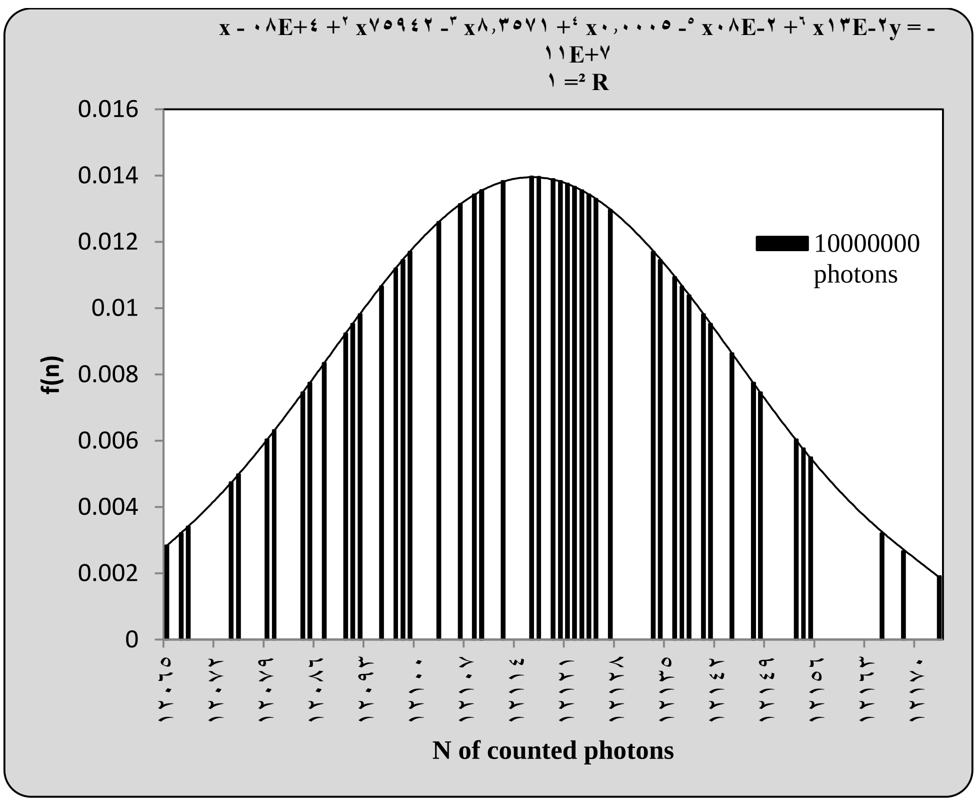


**Fig.4**: **Distribution of counted gamma photons at high activity of virtual radioactive source ($10^7$ photons) as a Gaussian (normal/bell) distribution.**

## 4. Conclusions:

1. The present simulation can be effective viable tool for mimic what happens in practice of the random behavior for each of the radioactive decay and the interactions of gamma rays with material.
2. The present simulation computer program was based on a Monte Carlo method which can be used as a virtual computer experiment instead of the real experimental set up for count statistics of nuclear radiation.
3. The present simulation, as a virtual computer experiment, makes laboratory personnel safe from exposure to radiation, either from the use of radioactive sources in the laboratory's experiments or the background of the laboratory it self.
4. The present simulation, as a virtual computer experiment, can be used as a safe training tool for workers (staff and students) in the field of radiation or nuclear physics laboratory.
5. The present simulation program can be performed to design different experimental geometries.

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