Calculations of Delta Mixing Ratios for γ-Transitions populated in $^{97}_{43}Tc_{54}$ Isotope using CST, a_2-Ratio and LSF Methods

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Abstract

In this research the three methods CST, a_2-Ratio and LSF were used to calculate the γ-mixing ratios for γ-transition populated in $^{97}_{43}\mathrm{Tc}_{54}$ energy levels from $^{97}_{42}\mathrm{Mo}_{55}(p,n\gamma)^{97}_{43}\mathrm{Tc}_{54}$ reaction. our results show good agreement with experimental data. The disagreement between (LSF) and experimental data in the value of delta mixing ratios was for γ-transition of 895.4KeV $\left(\frac{7^+}{2}-\frac{5^+}{2}\right)$ emitted from energy level 1219.9KeV and 583.2KeV $\left(\frac{7^-}{2}-\frac{5^-}{2}\right)$ emitted from energy level 1240.5KeV was due to statistical approximations used for calculation of $\rho_o^2(I_i)$ values.

حساب نسب الخلط لانتقالات كاما في النظير $^{97}_{43}Tc_{54}$ باستخدام الطريقة (CST) وطريقة نسبة a_2 وطريقة (LSF)

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المستخلص:

لقد تم في هذا البحث استخدام ثلاثة طرق نظرية (طريقة (CST) وطريقة نسبة a_2 وطريقة (LSF)) لحساب نسب الخلط لانتقالات كاما المنبعثة من مستويات الطاقة في النظير $^{97}_{43}Tc_{54}$ من التفاعل $^{97}_{42}Mo_{55}(p,n\gamma)^{97}_{43}Tc_{54}$.

اظهرت نتائج الحسابات الحالية توافقاً جيداً مع القيم العملية. ان الاختلاف بين قيم نسبة الخلط التي حصلنا علية بطريقة (LSF) والقيم العملية كان للانتقالات $895.4KeV\left(\frac{7^+}{2}-\frac{5^+}{2}\right)$ المنبعث من مستوي الطاقة $1219.9KeV$ و $583.2KeV\left(\frac{7^-}{2}-\frac{5^-}{2}\right)$ المنبعث من مستوي الطاقة $1240.5KeV$ وذلك بسبب التقريبات الاحصائية التي استخدمت في حساب قيمة $\rho_o^2(I_i)$.

Introduction

The multipole mixing ratio is defined as the ratio of the electric quadrupole moment E2 and magnetic dipole moment M1 matrix elements for gamma transition from an initial (I_i) to a final state (I_f) which is deduced experimentally from the analysis of the angular distribution of the emitted γ-ray [1]:

$$\delta = \frac{\left\langle I_f \parallel E2 \parallel I_i \right\rangle}{\left\langle I_f \parallel M1 \parallel I_i \right\rangle} \qquad (1)$$

It is observed that the admixture of mutlipole character are indicated by a connecting line E1-M2 and M1-E2, for the strong dependence of γ-ray transition probabilities on the multipole character.

The contribution to the γ-ray intensity from the second term is usually considerably less than from the first by the factor $(R/\lambda)^{2L}$ [2], where the wavelength of the emitted radiation λ is much greater than the dimension R of the excited nucleus, which means that E1>E2> E3… and M1>M2> M3 … [2, 3].

By comparing theoretically and experimental mixing ratios E2/M1 in terms of the reduced matrix elements [4], the simple assumptions on the M1 operator made by Warner [5] are not enable with the presence of both signs of $\delta(I \rightarrow I \pm 1)$ in the same nucleus.

The E2/M1 mixing ratios of γ-transition have been defined by Biedenlarn and Rose [6]; Rose and Brink [7]. Their differentiation of ions δ appears to have different signs for emission and absorption [8, 9].

Basic Considerations

Constant Statistical Tensor (CST) Method

In a certain nucleus, the magnetic substates population parameters, $p(m_i)$, as well as the statistical tensor coefficients $\rho_q^\lambda(I_i)$ of levels with the same spin value depend neither upon the energy of the level nor upon its parity [10]. The statistical tensor which is given by Pollti A. R. and Warburton E. K. [11] as follows:

$$\rho_q^\lambda(I_i) = \sum_{\substack{m_i=0 \\ \text{or } =1/2}}^{I_i} \rho_q^\lambda(I_i, m_i)\, p(m_i) \qquad (2)$$

where $p(m_i)$ is the population parameter, its normalized form is given by [12]:

$$\sum_{m_i=-I_i}^{I_i} p(m_i) = 1 \qquad (3)$$

Would be considered as constant for levels with the same I_i values. Therefore, by taking the δ=0 for pure, or considered to be pure transition in the angular distribution coefficient equation as follows [12]:

$$a_\lambda = (2I_i+1)^{\frac{1}{2}} \rho_q^\lambda(I_i) \frac{F_\lambda(I_f L L I_i) + 2\delta F_\lambda(I_f L L' I_i) + \delta^2 F_\lambda(I_f L' L' I_i)}{1+\delta^2} \qquad (4)$$

Thus we obtained:

$$\rho_o^2(I_i) = \frac{a_2}{(2I_i+1)^{\frac{1}{2}} F_2(I_f L L I_i)} \qquad (5)$$

a_2: is the experimental value of the angular distribution coefficient.

The γ-ray transitions from an initial nuclear state with angular momentum I_i and parity π_i to a final state (I_f, π_f), for allowed E2 and M1 transitions, for such γ-transition, the multipole mixing ratio δ which is defined as the ratio of electric quadrupole moment E2 and magnetic dripole moment M1 matrix elements [13].

The γ-ray angular distribution measurements are sensitive to interference effect between M1 and E2 amplitudes, and depend on the relative phase of the M1 and E2 matrix elements [14].

E2/M1 mixing ratios are deduced experimentally from an analysis of the emitted γ-ray where the (CST) method depends on the experimental data and does not depend on any nuclear model.

a_2-Ratio Method

In an electromagnetic radiation, the difference in angular momenta and relative parities of the nuclear states involved in the transition play an important role in determining the transition probability [15].

The δ-mixing ratio for gamma transitions can be calculated by many methods one of these methods is a_2-Ratio, where two or more of these transitions from the same initial state can be used, one of them is a pure transition in which I_f=0 such as 1^+-0^+ or 2^+-0^+ transition or it might be a pure transition E1, such as 2^--2^+, 3^--2^+, 3^--4^+, 5^--4^+, … etc or a pure transition E2, such as 4^+-2^+, 5^+-3^+, 6^+-4^+, … etc.

So that the other transition can be calculated by using the experimentally a_2-coefficient ratio of these transitions where the statistical tensor $\rho_o^{\lambda}(I_i)$ which is related to these a_2-coefficient would be the same.

This method has been successfully applied by Yohana et al., [10].

In such cases, the a_2-coefficient of the pure transition from a certain initial level is given by [12]:

$$a_2(I_i - I_{f_1}) = (2I_i + 1)^{1/2}\ \rho_o^2(I_i)\, F_2(I_{f_1} LLI_i) \tag{6}$$

and the other transition from the same level is given by [11, 12]:

$$a_2(I_i - I_{f_2}) = (2I_i + 1)^{\frac{1}{2}}\ \rho_o^2(I_i) \times \frac{F_2(I_{f_2} LLI_i) + 2\delta F_2(I_{f_2} LL'I_i) + \delta^2 F_2(I_{f_2} L'L'I_i)}{1+\delta^2} \tag{7}$$

Since the factor $(2I_i + 1)^{1/2}\ \rho_o^2(I_i)$ is the same for both transitions, so that we can obtain the following equation:

$$\frac{a_2(I_i - I_{f_2})}{a_2(I_i - I_{f_1})} = \frac{F_2(I_{f_2}LLI_i) + 2\delta F_2(I_{f_2}LL'I_i) + \delta^2 F_2(I_{f_2}L'L'I_i)}{(1+\delta^2)F_2(I_{f_1}LLI_i)} \tag{8}$$

Least Squares Fitting (LSF) Method

In comparing experimental angular distribution data with theoretical calculation (present work), the need often arises to try to extract the best values in an expansion of angular functions. Suppose that instead of measuring a set of n unknown quantities x_i directly, it is possible to measure a series of linear functions of these unknowns, if each of n measurement determines a set of coefficients, then these measurements will be related in general way by the sets of a linear equation [16]:

$$Y = A_o + A_1 x + A_2 x^2 + A_3 x^3 + \ldots \tag{9}$$

where x: is the initial state (I_i) and Y is the statistical tensor ($\rho_o^2(I_i)$).

The set with minimum χ^2 was used to calculate the $\rho_o^\lambda(I_i)$ values for all I_i values. The $\rho_o^\lambda(I_i)$ values were used to determine the δ-values of all γ-transitions whose angular distribution has been measured in the same manner.

This method is used when the statistical tensor $\rho_o^2(I_i)$ can not be calculated for the reasons that no pure γ-transition or transition considered to be pure can be obtained.

Finally the certain I_i values, are fitted to a polynomial series of the form:

$$\rho_o^2(I_i) = \sum_{j=0}^{n} A_j \, I_i^j \quad , n = 0, 1, 2, 3, 4 \tag{10}$$

Methods of Calculations

In the present work the calculations of constant statistical tensor (CST), a_2-Ratio and LSF methods of the used nuclei are depends totally on the F_2-coefficients of the angular momentum of γ-transitions of the initial and final energy levels, I_i, I_f and on the experimental data of a_2-coefficients of the spin sequence (I_i - I_f). Figure (1) shows the decay scheme of gamma-transitions of the nucleus used in this work. Table (1) listed all the used information's in the calculations of the mentioned method previously. **Note**: Some spins are written without parities as mentioned in the reference [17].

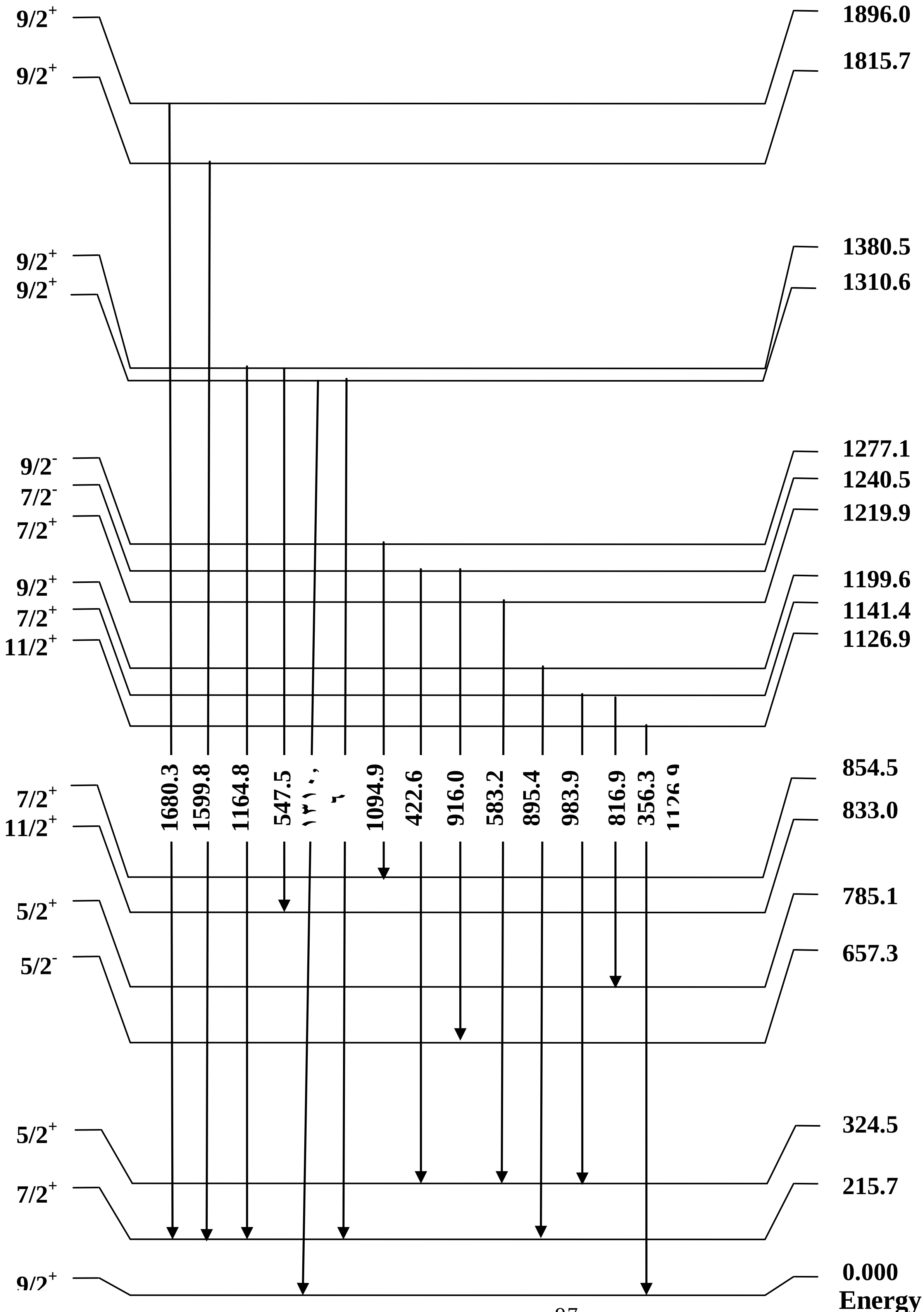

Figure (1): Decay scheme of $^{97}_{43}Tc_{54}$ *nucleus.*

Table (1): Energy of gamma transitions, spin sequences and angular distribution coefficients F_2 of $^{97}_{43}Tc_{54}$ nucleus.

Eγ (KeV) Ref.[17]	I_i^π	I_f^π	L	L'= L+1	$a_2(\Delta a_2)$ Ref.[17]	$a_2(I_i^\pi - I_f^\pi)$	F_2-Coefficients Ref. [15]		
							$F_2(I_fLLI_i)$	$F_2(I_fLL'I_i)$	$F_2(I_fL'L'I_i)$
1126.9	$\frac{11^+}{2}$	$\frac{9^+}{2}$	1	2	-0.50(4)	$-0.50(\frac{11^+}{2}-\frac{9^+}{2})$	$F_2(\frac{9^+}{2}\,11\,\frac{11^+}{2})=0.2876$ 2	$F_2(\frac{9^+}{2}\,12\,\frac{11^+}{2})=-0.92688$	$F_2(\frac{9^+}{2}\,22\,\frac{11^+}{2})=0.01580$
356.3	$\frac{7^+}{2}$	$\frac{5^+}{2}$	1	2	-0.11(4)	$-0.11(\frac{7^+}{2}-\frac{5^+}{2})$	$F_2(\frac{5^+}{2}\,11\,\frac{7^+}{2})=0.3273$ 3	$F_2(\frac{5^+}{2}\,12\,\frac{7^+}{2})=-0.94492$	$F_2(\frac{5^+}{2}\,22\,\frac{7^+}{2})=-0.07793$
816.9	$\frac{7^+}{2}$	$\frac{5^+}{2}$	1	2	-0.13(3)	$-0.13(\frac{7^+}{2}-\frac{5^+}{2})$	$F_2(\frac{5^+}{2}\,11\,\frac{7^+}{2})=0.3273$ 3	$F_2(\frac{5^+}{2}\,12\,\frac{7^+}{2})=-0.94492$	$F_2(\frac{5^+}{2}\,22\,\frac{7^+}{2})=-0.07793$
983.9	$\frac{9^+}{2}$	$\frac{7^+}{2}$	1	2	-0.29(2)	$-0.29(\frac{9^+}{2}-\frac{7^+}{2})$	$F_2(\frac{7^+}{2}\,11\,\frac{9^+}{2})=0.3027$ 7	$F_2(\frac{7^+}{2}\,12\,\frac{9^+}{2})=-0.93542$	$F_2(\frac{7^+}{2}\,22\,\frac{9^+}{2})=-0.01966$
895.4	$\frac{7^+}{2}$	$\frac{5^+}{2}$	1	2	-0.09(2)	$-0.09(\frac{7^+}{2}-\frac{5^+}{2})$	$F_2(\frac{5^+}{2}\,11\,\frac{7^+}{2})=$ 0.32733	$F_2(\frac{5^+}{2}\,12\,\frac{7^+}{2})=-0.94492$	$F_2(\frac{5^+}{2}\,22\,\frac{7^+}{2})=-0.07793$
583.2	$\frac{7^-}{2}$	$\frac{5^-}{2}$	1	2	-0.09(3)	$-0.09(\frac{7^-}{2}-\frac{5^-}{2})$	$F_2(\frac{5^-}{2}\,11\,\frac{7^-}{2})=0.327$ 33	$F_2(\frac{5^-}{2}\,12\,\frac{7^-}{2})=-0.94492$	$F_2(\frac{5^-}{2}\,22\,\frac{7^-}{2})=-0.07793$
916.0	$\frac{7^-}{2}$	$\frac{5^+}{2}$	1	2	-0.09(2)	$-0.09(\frac{7^-}{2}-\frac{5^+}{2})$	$F_2(\frac{5^+}{2}\,11\,\frac{7^-}{2})=0.3273$ 3	$F_2(\frac{5^+}{2}\,12\,\frac{7^-}{2})=-0.94492$	$F_2(\frac{5^+}{2}\,22\,\frac{7^-}{2})=-0.07793$
422.6	$\frac{9^-}{2}$	$\frac{7^+}{2}$	1	2	-0.25(5)	$-0.25(\frac{9^-}{2}-\frac{7^+}{2})$	$F_2(\frac{7^+}{2}\,11\,\frac{9^-}{2})=0.3027$ 7	$F_2(\frac{7^+}{2}\,12\,\frac{9^-}{2})=-0.93542$	$F_2(\frac{7^+}{2}\,22\,\frac{9^-}{2})=-0.01966$

Table (1): To be continued (2/2).

Eγ (KeV) Ref.[17]	I_i^{π}	$\mathbf{I_f^{\pi}}$	**L**	**L'= L+1**	**$a_2(\Delta a_2)$ Ref.[17]**	**$a_2(I_i^{\pi} - I_f^{\pi})$**	**F_2-Coefficients Ref. [15]**		
							$F_2(I_fLLI_i)$	**$F_2(I_fLL'I_i)$**	**$F_2(I_fL'L'I_i)$**
1094.9	$\frac{9^+}{2}$	$\frac{7^+}{2}$	1	2	-0.18(2)	$-0.18\ (\frac{9^+}{2}-\frac{7^+}{2})$	$F_2(\frac{7^+}{2}\,11\,\frac{9^+}{2})= 0.30277$	$F_2(\frac{7^+}{2}\,12\,\frac{9^+}{2})= -0.93542$	$F_2(\frac{7^+}{2}\,22\,\frac{9^+}{2})= -0.01966$
1310.6	$\frac{9^+}{2}$	$\frac{9^+}{2}$	1	2	-0.01(3)	$-0.01(\frac{9^+}{2}-\frac{9^+}{2})$	$F_2(\frac{9^+}{2}\,11\,\frac{9^+}{2})= -0.44039$	$F_2(\frac{9^+}{2}\,12\,\frac{9^+}{2})= -0.30151$	$F_2(\frac{9^+}{2}\,22\,\frac{9^+}{2})= 0.27524$
547.5	$\frac{9^+}{2}$	$\frac{11^+}{2}$	1	2	-0.18(3)	$-0.18(\frac{9^+}{2}-\frac{11^+}{2})$	$F_2(\frac{11^+}{2}\,11\,\frac{9^+}{2})= 0.16515$	$F_2(\frac{11^+}{2}\,12\,\frac{9^+}{2})= 0.76871$	$F_2(\frac{11^+}{2}\,22\,\frac{9^+}{2})= 0.27524$
1164.8	$\frac{9^+}{2}$	$\frac{7^+}{2}$	1	2	-0.26(2)	$-0.26(\frac{9^+}{2}-\frac{7^+}{2})$	$F_2(\frac{7^+}{2}\,11\,\frac{9^+}{2})= 0.30277$	$F_2(\frac{7^+}{2}\,12\,\frac{9^+}{2})= -0.93542$	$F_2(\frac{7^+}{2}\,22\,\frac{9^+}{2})= -0.01966$
1599.8	$\frac{9^+}{2}$	$\frac{7^+}{2}$	1	2	-0.18(4)	$-0.18(\frac{9^+}{2}-\frac{7^+}{2})$	$F_2(\frac{7^+}{2}\,11\,\frac{9^+}{2})= 0.30277$	$F_2(\frac{7^+}{2}\,12\,\frac{9^+}{2})= -0.93542$	$F_2(\frac{7^+}{2}\,22\,\frac{9^+}{2})= -0.01966$
1680.3	$\frac{9^+}{2}$	$\frac{7^+}{2}$	1	2	-0.27(4)	$-0.27\ (\frac{9^+}{2}-\frac{7^+}{2})$	$F_2(\frac{7^+}{2}\,11\,\frac{9^+}{2})= 0.30277$	$F_2(\frac{7^+}{2}\,12\,\frac{9^+}{2})= -0.93542$	$F_2(\frac{7^+}{2}\,22\,\frac{9^+}{2})= -0.01966$

Constant Statistical Tensor (CST) Method

The statistical tensor $\rho_9^{\lambda}(I_i)$, for the alignment of the initial state for all I_i-value of level having one or more pure gamma transitions or can be considered as a pure transition according to equation (6) (i.e. δ=0) can be calculated from the following reduced form of equation (7):

$$\rho_o^2(I_i) = \frac{a_2(I_i - I_f)}{(2I_i + 1)^{1/2} F_2(I_f LLI_i)} \qquad (11)$$

By using this equation for the daughter nucleus ${}^{97}_{43}Tc_{54}$ the values of $\rho_o^2(I_i)$ are listed in table (2). Therefore, angular distribution coefficients $a_2(I_i^{\pi}\text{-}I_f^{\pi})$ could be calculated for selected gamma transitions for ${}^{97}_{43}Tc_{54}$ nucleus by using equation (7). The results are shown in tables (3).

Table (2): Values of Constant Statistical Tensor (CST) of selected gamma transitions obtained for ${}^{97}_{43}Tc_{54}$ nucleus.

E_γ (KeV) Ref. [17]	I_i^{π}-I_f^{π}	$\rho_o^2(I_i^{\pi})$ for pure transition or considered to be pure
		$\rho_o^2(I_i^{\pi}) = \frac{a_2(I_i - I_f)}{(2I_i + 1)^{1/2} F_2(I_f LLI_i)}$
1126.9	$\frac{11^+}{2}$-$\frac{9^+}{2}$	$\rho_o^2(\frac{11^+}{2}) = \frac{a_2\left(\frac{11^+}{2} - \frac{9^+}{2}\right)}{(2I_i+1)^{1/2} F_2\left(\frac{9}{2}11\frac{11}{2}\right)} = \frac{a_2\left(\frac{11^+}{2} - \frac{9^+}{2}\right)}{(12)^{1/2} x0.28762}$
816.9 *356.3* *583.2*	$\frac{7^-}{2}$-$\frac{5^+}{2}$	$\rho_o^2(\frac{7^-}{2}) = \frac{a_2\left(\frac{7^-}{2} - \frac{5^+}{2}\right)^*}{(2I_i+1)^{1/2} F_2\left(\frac{5}{2}11\frac{7}{2}\right)} = \frac{a_2\left(\frac{7^-}{2} - \frac{5^+}{2}\right)}{(8)^{1/2} x0.32733}$
422.6 *9839* *1094.9*	-$\frac{9^-}{2}$ $\frac{7^+}{2}$	$\rho_o^2(\frac{9^-}{2}) = \frac{a_2\left(\frac{9^-}{2} - \frac{7^+}{2}\right)^*}{(2I_i+1)^{1/2} F_2\left(\frac{7}{2}11\frac{9}{2}\right)} = \frac{a_2\left(\frac{9^-}{2} - \frac{7^+}{2}\right)}{(10)^{1/2} x0.30277}$
547.5	$\frac{9^+}{2}$-$\frac{11^+}{2}$	$\rho_o^2(\frac{9^+}{2}) = \frac{a_2\left(\frac{9^+}{2} - \frac{11^+}{2}\right)}{(2I_i+1)^{1/2} F_2\left(\frac{11}{2}11\frac{9}{2}\right)} = \frac{a_2\left(\frac{9^+}{2} - \frac{11^+}{2}\right)}{(10)^{1/2} x0.16515}$
1310.2	-$\frac{9^+}{2}$ $\frac{9^+}{2}$	$\rho_o^2(\frac{9^+}{2}) = \frac{a_2\left(\frac{9^+}{2} - \frac{9^+}{2}\right)}{(2I_i+1)^{1/2} F_2\left(\frac{9}{2}11\frac{9}{2}\right)} = \frac{a_2\left(\frac{9^+}{2} - \frac{9^+}{2}\right)}{(10)^{1/2} x0.44039}$

Table (3): Values of a_2 (I_i^{π} - I_f^{π}) for selected gamma transitions for $^{97}_{43}Tc_{54}$ nucleus by using equation (7).

E_γ(KeV) Ref. [17]	$a_2(I_i^{\pi}-I_f^{\pi})=(2I_i+1)^{1/2}\rho_o^2(I_i)\dfrac{F_2(I_fLLI_i)+2\delta F_2(I_fLL'I_i)+\delta^2F_2(I_fL'L'I_i)}{1+\delta^2}$
1126.9	$a_2\left(\frac{11^+}{2}-\frac{9^+}{2}\right)=3.4\rho_o^2\left(\frac{11}{2}\right)\dfrac{0.28762-1.85376\delta+0.01580\delta^2}{1+\delta^2}$
356.3 *816.9* *583.2*	$a_2\left(\frac{7^-}{2}-\frac{5^+}{2}\right)=2.8\rho_o^2\left(\frac{7}{2}\right)\dfrac{0.32733-1.88984\delta-0.07793\delta^2}{1+\delta^2}$
983.9 *422.6* *1094.9*	$a_2\left(\frac{9^-}{2}-\frac{7^+}{2}\right)=3.1\rho_o^2\left(\frac{9}{2}\right)\dfrac{-0.93542-0.03932\delta-0.440398\delta^2}{1+\delta^2}$
1310.2	$a_2\left(\frac{9^-}{2}-\frac{9^+}{2}\right)=3.1\rho_o^2\left(\frac{9}{2}\right)\dfrac{-0.44039-0.603028\delta+0.27524\delta^2}{1+\delta^2}$
547.5	$a_2\left(\frac{9^+}{2}-\frac{11^+}{2}\right)=3.1\rho_o^2\left(\frac{9}{2}\right)\dfrac{0.16515+1.53742\delta+0.27524\delta^2}{1+\delta^2}$

The a_2-ratio Method

In this method the δ-mixing ratios needs to be determined by using double $a_2(I_i$-$I_f)$ coefficients from the same initial level I_i to different levels I_f. One of them represents the experimental a_2-coefficients reported for □-transition whose δ-values are to be calculated and the other represents the experimental a_2-coefficient reported for the pure transition or can be considered as a pure □-transition.

The product nuclei that have at least two □-transitions whose angular distributions have been measured its angular momentum L is taken to be:

$$L_{min} = |I_i - I_f| \quad \text{and } L' = L + 1, \quad \text{where } L \neq 0$$

For all possible □-transitions in the case of the odd-even $^{97}_{43}Tc_{54}$ nucleus. The calculations of a_2-Ratio is independent neither on energy level nor on the □-transitions energy, so that by using F_2 and a_2 coefficients we can get more values of δ-mixing ratios by this method.

Table (4) listed the values of δ-mixing ratios according to equation (8) of a_2-Ratio method.

The δ-values of selected □-transitions whose angular distributions have been measured are calculated in the same manner for the selected reactions.

Table (4): a_2-Ratios for selected gamma- transitions of $^{97}_{43}Tc_{54}$ and δ-mixing ratios by using equation (8).

E_i (KeV) Ref. [17]	E_γ (KeV) Ref. [17]	$\frac{a_2(I_i^\pi - I_{f_2}^\pi)}{a_2(I_i^\pi - I_{f_1}^\pi)} = \frac{F_2(I_{f_2}LLI_i) + 2\delta F_2(I_{f_2}LL'I_i) + \delta^2 F_2(I_{f_2}L'L'I_i)}{F_2(I_{f_1}LLI_i)(1+\delta^2)}$
1141.4	356.3 816.9	$\frac{a_2\left(\frac{7^+}{2}-\frac{5^+}{2}\right)}{a_2\left(\frac{7^+}{2}-\frac{5^+}{2}\right)} = \frac{0.32733 - 1.889846 - 0.07793\delta^2}{0.32733(1+\delta^2)}$
1141.4	816.9 356.3	$\frac{a_2\left(\frac{7^+}{2}-\frac{5^+}{2}\right)}{a_2\left(\frac{7^+}{2}-\frac{5^+}{2}\right)} = \frac{0.32733 - 1.889846 - 0.07793\delta^2}{0.32733(1+\delta^2)}$
1199.6 *1277.1*	983.9 422.6	$\frac{a_2\left(\frac{9^+}{2}-\frac{7^+}{2}\right)}{a_2\left(\frac{9^-}{2}-\frac{7^+}{2}\right)} = \frac{0.30277 - 1.870846 - 0.01966\delta^2}{0.30277(1+\delta^2)}$
1219.9 *1240.5*	895.4 583.2	$\frac{a_2\left(\frac{7^+}{2}-\frac{5^+}{2}\right)}{a_2\left(\frac{7^-}{2}-\frac{5^-}{2}\right)} = \frac{0.32733 - 1.889846 - 0.07793\delta^2}{0.32733(1+\delta^2)}$
1240.5	583.2 916.0	$\frac{a_2\left(\frac{7^-}{2}-\frac{5^-}{2}\right)}{a_2\left(\frac{7^-}{2}-\frac{5^+}{2}\right)} = \frac{0.32733 - 1.889846 - 0.07793\delta^2}{0.32733(1+\delta^2)}$
1240.5	916.0 583.2	$\frac{a_2\left(\frac{7^-}{2}-\frac{5^+}{2}\right)}{a_2\left(\frac{7^-}{2}-\frac{5^-}{2}\right)} = \frac{0.32733 - 1.889846 - 0.07793\delta^2}{0.32733(1+\delta^2)}$
1277.1 *1199.6*	422.6 983.9	$\frac{a_2\left(\frac{9^-}{2}-\frac{9^+}{2}\right)}{a_2\left(\frac{9^+}{2}-\frac{7^+}{2}\right)} = \frac{-0.44039 - 0.603026 - 0.27524\delta^2}{0.30277(1+\delta^2)}$
1310.6	1094.9 1310.6	$\frac{a_2\left(\frac{9^+}{2}-\frac{7^+}{2}\right)}{a_2\left(\frac{9^+}{2}-\frac{9^+}{2}\right)} = \frac{0.30277 - 1.870846 - 0.01966\delta^2}{-0.44039(1+\delta^2)}$
1310.6	1310.6 1094.9	$\frac{a_2\left(\frac{9^+}{2}-\frac{9^+}{2}\right)}{a_2\left(\frac{9^+}{2}-\frac{7^+}{2}\right)} = \frac{-0.44039 - 0.60302\delta - 0.27524\delta^2}{0.30277(1+\delta^2)}$
1380.5	547.5 1164.8	$\frac{a_2\left(\frac{9^+}{2}-\frac{11^+}{2}\right)}{a_2\left(\frac{9^+}{2}-\frac{7^+}{2}\right)} = \frac{0.16515 + 1.537426 - 0.27524\delta^2}{0.30277(1+\delta^2)}$
1815.5 *1896.0*	1599.8 1680.3	$\frac{a_2\left(\frac{9^+}{2}-\frac{7^+}{2}\right)}{a_2\left(\frac{9^-}{2}-\frac{7^+}{2}\right)} = \frac{0.30277 - 1.870846 - 0.01966\delta^2}{0.30277(1+\delta^2)}$
1986.0 *1380.5*	1680.3 547.5	$\frac{a_2\left(\frac{9^+}{2}-\frac{7^+}{2}\right)}{a_2\left(\frac{9^+}{2}-\frac{11^+}{2}\right)} = \frac{0.30277 - 1.870846 - 0.01966\delta^2}{0.16515(1+\delta^2)}$

Least Squares Fitting (LSF) Method

In this method the statistical tensor $\rho_o^2(I_i)$ values were calculated for levels with different I_i-values, and fitted to a polynomial series as given in equation (10) for n=4 which given as follows:

$$\rho_o^2(I_i) = A_o + A_1 I_i + A_2 I_i^2 + A_3 I_i^3 + A_4 I_i^4 \qquad (12)$$

The set of A-parameters that gave minimum Chi-squared (χ^2_{min}) was used to calculate the $\rho_o^2(I_i)$ for all I_i values.

The δ-mixing ratios of selected γ-transitions were calculated depending on the (CST) values obtained by this method.

Results and Discussions

Determination of □-mixing ratio of □-transitions from levels of $^{97}_{43}\mathrm{Tc}_{54}$ populated in The $^{97}_{42}\mathrm{Mo}_{55}(\alpha,n\gamma)^{97}_{43}\mathrm{Tc}_{54}$ Reaction

1: Using CST Method

Table (5) contains the results of the present calculations of δ-mixing ratio for gamma transitions in comparisons with experimental data reported in ref. [17].

The comparison shows in general a good agreement between the present □-values with those of ref. [17].

The disagreements in δ-values occur in the following transitions:

1- The 547.5KeV $\left(\frac{9^+}{2}-\frac{11^+}{2}\right)$ transition emitted from energy level 1380.5KeV.

2- The 1164.8KeV $\left(\frac{9^+}{2}-\frac{7^+}{2}\right)$ transition emitted from energy level 1380.5KeV.

3- The 983.9KeV $\left(\frac{9^+}{2}-\frac{7^+}{2}\right)$ transition emitted from energy level 1199.6KeV.

4- The 422.6KeV $\left(\frac{9^+}{2}-\frac{7^+}{2}\right)$ transition emitted from energy level 1277.1KeV.

Due to the statistical approximations in considering them as a pure transitions in the calculation of the weighted average of $\rho_o^2(I_i)$.

Table (5): Values of δ-mixing ratios according to the three methods and adopted values of $^{97}_{43}Tc_{54}$ nucleus.

E_i (KeV) Ref. [17]	E_γ (KeV) Ref. [17]	$I_i^\pi - I_f^\pi$	$a_2(\Delta a_2)$ $a_4(\Delta a_2)$ Ref. [17]	$\rho_q^\lambda\ (I_i)$: $\rho_o^2(I_i)$ (pw)	$\rho_q^\lambda\ (I_i)$: Weighted average (pw)	δ-mixing Ratios: Exp. Ref.[17]	δ-mixing Ratios (pw): CST	(pw): a_2-Ratio	(pw): a_2-Ratio	(pw): LSF	(pw): Adopted	Multipolarity (pw)
1126.9	1126.9	$\frac{11^+}{2}-\frac{9^+}{2}$	-0.50(4) 0.01(4)	1.740(139)	1.74000 (139)	$0.84^{+0.5}_{-0.4}$	0.60 (5) or $18.56^{+1.90}_{-1.59}$	——	——	0.44 (5) or 3.64 (5)	0.44(1)	M1+(16.1±0.7)%E2
1141.4	356.3	$\frac{7^+}{2}-\frac{5^+}{2}$	-0.11(4) 0.0(5)	-0.336(122)	-0.31149 (2716)	$0.36^{+0.17}_{-0.12}$	$0.28^{+0.44}_{-0.0[illegible]}$	M1	0.14(18)	0.20(7) or 12.64(4)	0.19(4)	M1+(3.4±1.6)%M1
	816.9	$\frac{7^+}{2}-\frac{5^+}{2}$	-0.13(3) 0.0(3)	-0.397(61)		$0.78^{+0.93}_{-0.44}$	0.30(22)	0.0340)	M1	0.15(7) or 12.35(4)	0.16(3)	M1+(3.4±1.2)%E2
1199.6	983.9	$\frac{9^+}{2}-\frac{7^+}{2}$	-0.29(2) 0.0(3)	-0.957(66)	-0.64419 (0.03083)	$0.97^{+0.3}_{-0.4}$	0.1(5) or 13.9(9)	M1	0.5(50)	0.17(4) or 1.9(9)	0.12(3)	M1+(1.4±0.8)%E2
1219.9	895.4	$\frac{7^+}{2}-\frac{5^+}{2}$	-0.09(2) 0.0(3)	-0.274(61)	-0.31149 (2716)	$0.81^{+07}_{-0.4}$	0.26(22) or 21.63 (11)	M1	0.0 (11) or 25(24)	0.16 (4) or 13.9(2)	0.14(3)	M1+(1.9±0.9)%E2
1240.5	583.2	$\frac{7^-}{2}-\frac{5^-}{2}$	-0.09(3) 0.0(4)	-0.274(91)		$0.81^{+0.58}_{-0.37}$	0.26(32) or 21.68 (22)	M1	0.0 (16) or 24.23(24.0)	0.16(4) or 13.9(8)	0.02(1)	M1+(0.039±0.05)%E2
	916.0	$\frac{7^-}{2}-\frac{5^+}{2}$	-0.09(2) 0.0(3)	-0.274(61)		$0.81^{+1.0}_{-0.51}$	0.26(1) or 28.05 (17)	0.0(0.11) or 24.23(24)	E1	0.16(7) or 13.9(22)	0.25(6)	E1+(5±3.7)%M2
1277.1	422.6	$\frac{9^-}{2}-\frac{7^+}{2}$	-0.25(5) 0.01(6)	-0.825(1.651)	-0.64419 (0.03083)	$0.38^{+0.11}_{-0.08}$ or $1.66^{+028}_{-0.31}$	0.1(2)	E1	0.5(2)	0.22(6) or 2.5(3)	0.23(5)	E1+(5±2.2)%M2

Table (5): To be continued 2/2.

E_i (Ke) Ref. [14]	E□ (KeV) Ref. [14]	$I_i^{\pi} - I_f^{\pi}$	a_2(□a_2) a_4(□a_2) Ref. [14]	$\rho_q^{\lambda}(I_i)$		□-mixing Ratios						Multipolarity (pw)
				$\rho_o^2(I_i)$ (pw)	Weighted average (pw)	Exp. Ref. [14]	(pw)					
							CST	a_2-Ratio		LSF	Adopted	
13106	10949	$\frac{9^+}{2} - \frac{7^+}{2}$	-0.18(2) 0.0(3)	-0.594 (66)	-0.64419 (0.03083)	$0.14^{+0.05}_{-0.04}$ or $3.10^{+0.51}_{-0.4}$	0.12(5)	M1	I.Root	0.13 (4) or 3.7(5)	0.12(0)	M1+1.4 %E2
13106	13106	$\frac{9^+}{2} - \frac{9^+}{2}$	-0.01(3) 0.0(4)	-0.022 (68)		$0.63^{+0.14}_{-0.11}$	0.58(7) or 2.80(11)	1.09(4) I.Root	M1	0.15 (2) or 40.45 (1)	0.35(1)	M1+(10.9±0.5)% E2
13805	547.5	$\frac{9^+}{2} - \frac{11^+}{2}$	-0.18(3) 0.003(0)	-1.085 (181)		$-4.0^{+1.1}_{-1.7}$ or $-0.33^{+0.081}_{-0.10}$	-7.49(49) or -0.061(8)	M1	-5.5(1.2) or 0.02(5)	-7.4(5) or -0.21 (3)	-7.3(3)	M1+(98.1±0.1)% E2
	11648	$\frac{9^+}{2} - \frac{7^+}{2}$	-0.26(2) 0.004(30)	-0.858 (66)	-0.64419 (0.03083)	$-0.31^{+0.01}_{-0.04}$ or $1.96^{+0.18}_{-0.12}$	0.10 (0) or $-1.53^{++1.05}_{-1.22}$	0.05* (11) or 0.95(9)	M1	0.22 (2) or 2.44(3)	0.10(0)	M1+0.99 %E2
18155	15998	$\frac{9^+}{2} - \frac{7^+}{2}$	-0.18(4) 0.00(5)	-0.594 (132)		$0.141^{+0.07}_{-0.06}$ or $3.08^{+0.74}_{-0.62}$	0.12(9) or $-7.42^{+33.70}_{-13.17}$	M1	0.05(3)	0.13 (2) or 3.7(3)	0.10(1)	M1+(0.9±0.2)%E2
18960	16803	$\frac{9^+}{2} - \frac{7^+}{2}$	-0.27(4) 0.0(5)	-0.980 (132)		0.21±0.11 or $2.48^{+1.1}_{-0.06}$	0.10(11) or $-1.48^{+2.00}_{-2.74}$	M1	0.05(5)	0.24 (2) or 2.30(7)	0.21(1)	M1+(4.2±0.3)%E2

2: Using a_2-Ratio Method

The energy level of $^{97}_{43}Tc_{54}$ and the related γ-transitions whose a_2-coefficients have been used to calculate the corresponding δ-values by this method are presented in table (5).

The δ-values which can be calculated by a_2-Ratio are in agreement with the values in ref. [17] except the following:

1- The 1094.9 KeV $\left(\frac{9^+}{2}-\frac{7^+}{2}\right)$ transition and the 1310.6KeV $\left(\frac{9^+}{2}-\frac{9^+}{2}\right)$ transition emitted from the same energy level of 1310.6KeV.

The imaginary root obtained for the 1094.9KeV transition assuming a pure M1 transition for the second one confirm that the disagreement is also occurs for the 1310.6KeV transition when we assume a pure M1 transition for the 1094.9KeV respectively.

2- The 547.5KeV $\left(\frac{9^+}{2}-\frac{11^+}{2}\right)$ transition and 1164.8KeV $\left(\frac{9^+}{2}-\frac{7^+}{2}\right)$ transition emitted from the same energy level 1380.5KeV.
This indicates that both transitions are not pure, but we considered them as a pure transitions in the calculation of $\rho_o^2(I_i)$. Also because the experimental a_2-coefficients differ with two values of delta mixing ratios ($-4.0^{+1.1}_{-1.07}$ and $-0.33^{+0.08}_{-0.10}$for 547.5Kev transition and $-0.31^{+0.01}_{-0.04}$ and $1.96^{+0.18}_{-0.12}$ for 1164.8KeV transition respectively) due to some circumstances in measurements.

3- The 583.2KeV $\left(\frac{7^-}{2}-\frac{5^-}{2}\right)$ transition and 916.0KeV$\left(\frac{7^-}{2}-\frac{5^+}{2}\right)$ transition emitted from the same energy level 1240.5KeV, which consistent with pure M1 and E1 respectively.
The adopted δ-values are still correct since they are lying within the range of the experimental data.

3: Using LSF-Method

The $\rho_o^2(I_i)$values presented in table (5), by fitting equation (10), using (LSF.for) program.

The fitting equation parameters A_o, A_1, A_2, A_3 and A_4, the χ^2_{min} values and the weighted average of $\rho_o^2(I_i)$ values for each I_i are present in table (6).

Table (6): The calculated $\rho_o^2(I_i)$ values using (LSF) method at minimum chi squared and the A_j coefficients for each initial level in $^{97}_{43}Tc_{54}$ nucleus.

I_i	A_o	A_1	A_2	A_3	A_4	χ^2	$\rho_o^2(I_i)$
$\frac{7}{2}$	-0.264429E+00	0.178426E-17	-0.254236E-01	-0.206236E-19	0.000000	0.724465E+02	-0.575869E+00
$\frac{9}{2}$	-0.583291E+00	0.163590E-16	-0.317972E-01	0.597001E-20	0.000000	0.484378E+03	-0.122718E+01
$\frac{11}{2}$	0.162864E+01	-0.242344E-16	0.57507E-01	0.597340E-19	0.000000	0.137284E+03	0.336864E+01

The $\rho_o^2(I_i)$ values were then used to calculate the δ-values for all γ-transitions, whose angular distributions have been measured.

The δ-values calculated using these $\rho_o^2(I_i)$ values are also presented in table (6). The comparison of δ-values calculated by this method with experimental values and other method (CST and a_2-ratios) show a good agreement with most values.

The disagreements in the values of δ are in the following transitions.

1- The 816.9KeV $\left(\frac{7^+}{2}-\frac{5^+}{2}\right)$ transition emitted from energy level 1141.4KeV.

2- The 983.9KeV $\left(\frac{9^+}{2}-\frac{7^+}{2}\right)$ transition emitted from energy level 1199.6KeV.

3- The 1310.6KeV $\left(\frac{9^+}{2}-\frac{9^+}{2}\right)$ transition emitted from energy level 1310.6KeV.

4- The 547.5 KeV $\left(\frac{9^+}{2}-\frac{11^+}{2}\right)$ transition emitted from energy level 1380.5KeV.

Because there are no pure transition emitted from these levels in the calculations of the weighted average of $\rho_o^2(I_i)$, but they are considered to be pure.

The disagreement is also in the following:

1- The 895.4 KeV $\left(\frac{7^+}{2}-\frac{5^+}{2}\right)$ transition emitted from energy level 1219.9KeV.

2- The 583.2 KeV $\left(\frac{7^-}{2}-\frac{5^-}{2}\right)$ transition and 916.0 KeV $\left(\frac{7^-}{2}-\frac{5^+}{2}\right)$

transition both of them emitted from the same energy level 1240.5KeV.

In cases 1 and 2 the δ-values are acceptable with δ-values of CST method due to overlapping within the error between them, but the discrepancy occurs between our results and experimental data for these transitions due to statistical approximations of $\rho_o^2(I_i)$ values.

Conclusions

1- The CST, a_2-Ratio and LSF methods have been applied for the first time for the selected reactions in the present work to calculate the δ-values of γ-transitions. The comparison of these values with the experimental data. Generally, they show a good agreement, and providing a good estimation in comparison with the accuracy between experimental and theoretical (calculated) results.

2- The a_2-Ratio method is accurate and simple in calculating the δ-values for gamma-transitions from levels that have at least two γ-transitions one of them is pure or may be considered to be pure transitions, we confirm that the δ-values obtained by using this method are independent neither on the energy of level, nor on its parity.

3- The LSF method has been used to calculate $\rho_o^2(I_i)$ for each initial state in the nuclear reaction of the present work, so that we get an equation for each (I_i) instead of one equation for each nuclear reaction. The results were in good agreement as good as (CST) and a_2-Ratio methods. The smallest value of (χ^2) was chosen from these equations.

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