Common Random Fixed Points of Commuting Random Operators

Saheb K.Alsaidy
Department of Mathematics, college of Science, AL-Mustansirya University
Dr.sahebk@yahoo.com

Salwa Salman Abed

Department of Mathematics, college of Pure Sciences, Ibn AL-Haithem, Baghdad University
salwaalbundi@yahoo.com
Yusra Jarallah Ajeel

Department of Mathematics, college of Science, AL-Mustansirya University
Ysrajara@yahoo.com

Abstract:
In this paper ,we prove three common random fixed point theorems for commuting operators h-nonexpansive and continuous operators defined on a non-starshaped subset of a p-normed space X. Also hypotheses conclude compactness condition , demi-closeness or Opial condition.
Keywords: p-normed spaces , common random fixed point, random best approximation.

النقاط الصامدة العشوائية المشتركة لمؤثرات عشوائية إبداليه

د.صاحب كحيط الساعدي

الجامعة المستنصرية, كلية العلوم, قسم الرياضيات

د.سلوى سلمان عبد

جامعة بغداد, كلية التربية ابن الهيثم, قسم الرياضيات

يسرى جارالله عجيل

الجامعة المستنصرية, كلية العلوم, قسم الرياضيات

المستخلص

في هذا البحث نثبت ثلاث مبرهنات للنقطة الصامدة العشوائية المشتركة لمؤثرات h–لاممتدة ومستمرة إبداليه معرفة على مجموعة جزئية غير نجمية من فضاء p–المعياري كذلك الفرضيات تتضمن شرط التراص و demi–الانغلاق وشرط Opial .

1.Introduction and **Preliminaries**

The random fixed point theorems for contraction mappings in separable complete metric space were proved by Spacek [22], Han's [4,5]. Recently, many researchers are interested in this subject as Bharucha-Reid[2], Itoh[8] proved several random fixed point theorems and gave their applications to random differential equations in Banach spaces. Nashine [14] establishes the existence of random fixed point as random best approximations with respect to compact and weakly compact domain .

In this paper, we prove some random common fixed point theorems for h-nonexpansive random operators defined on non-star-shaped subset of a p-normed space, which extending the previous work by Nashine [15].

The following classes are needed:

2^X is the classes of all subsets of X ,

CB(X) is the classes of all bounded closed subsets of X,

K(X)is the classes of all nonempty compact subsets of X ,

$K_w(X)$is the classes of all nonempty weakly compact subsets of X,

$\bar{A}$ is the closure of a set A ,

w$-\overline{(A)}$ is the weakly closure of a set A .

We need the following definitions and facts:

Definition (1.1):[14]

Let X be a linear space and $\| \|_p$be a real valued function on X with $0 < p \leq 1$. The pair (X, $\| \|_p$) is called a p-normed space if for all x , y in X and scalars λ **:**

i. $\|x\|_p \geq 0$ and $\|x\|_p = 0$ iff $x = 0$

ii. $\|\lambda x\|_p = |\lambda|^p \|x\|_p$

iii. $\|x + y\|_p \leq \|x\|_p + \|y\|_p$

Every p-normed space X induces a metric space with $d(x,y) = \|x - y\|_p$,for all x , y in X . If $p = 1$,we have the concept of a normed space**.** Since a p-normed space is not necessarily locally convex space then continuous dual $X^{`}$ of p-normed space X need not separate the point of X.

<u>Definition (1.2)</u>:[13]

The space X is said to be **Opial** if for every sequence$< x_n >$ in X ,$x_n \to x$ weakly as $n \to \infty$ and $x \neq y$implying that $\lim_{n\to\infty} \inf \|x_n - x\|_p \leq \lim_{n\to\infty} \inf \|x_n - y\|_p$,for all $y \in X$.

<u>Definition (1.3)</u>:[21]

Let X be a p-normed space, A subset A of X is called **starshaped** if there exists a point q$\in A$ such that $\lambda x + (1-\lambda)q \in A$ for all $x \in A$ and $0 \leq \lambda$ in this case q is called the starcenter of .

<u>Definition (1.4)</u>: [23]

A self mapping h of a p-normed space X is said to be **affine** if for all x, y in X and for any λ, $0 \leq \lambda \leq 1$, $h(\lambda x + (1-\lambda) y) = \lambda hx + (1-\lambda) hy$. And h is called q-affine if there is $q \in X$ such that $h(kx + (1-k)q = \lambda hx + (1-\lambda)q$, for all $\lambda \in [0,1]$ and all $\in X$.

<u>Definition (1.5)</u>: [14]

Let A be a subset of a p-normed space X for $x_\circ \in X,$ define the set $p_A(x_\circ) = \{z \in A: \|x - z\|_p = d(x, A)\}$, then an element $z \in p_A(x_\circ)$ is called a **best approximation** of $x_\circ$ in A .

<u>Definition (1.6)</u>: [13]

Let X be a p-normed space, $A \subseteq X$ and $G : A \longrightarrow X$ be a mapping, G is called ***demiclosed*** of $x \in A$, if for every sequence $< x_n >$ in A such that x_n weakly converges to x $(x_n \rightharpoonup x)$ and $G(x_n)$ converges to $y \in X$ $(G(x_n) \longrightarrow y)$ then $y = G(x)$.And G is demiclosed on A if it is demiclosed of each x in A.

Definition (1.7): [13]

Let X be a p-normed space and $h, G: X \longrightarrow X$ be a mapping then G is said to be ***h*** ***–contraction*** if there exists $k \in [0,1]$such that

$\|Gx - Gy\|_p \leq k\|hx - hy\|_p$ ....(2.1) for all x , y in X

If $k = 1$ in (2. 1), then G is called ***h* - nonexpansive mapping** .

Definition (1.8): [13]

A pair (h, G) of self mappings of a metric space X is said to be **Commute** if $hGx = Ghx$ for all $x \in X$.

Throughout this paper the order pair (Ω,Σ) denote to the measurable space with sigma algebra Σ of subsets of Ω .

Definition(1.9):[20]

A mapping $F: \Omega \rightarrow 2^X$is called measurable (or, weakly measurable) if, for any closed (respectively,open) subset B of X , $F^{-1}(B) = \{\omega \in \Omega: F(\omega) \cap B \neq \emptyset\} \in \Sigma$.

Definition (1.10): [17]

A mapping $\delta: \Omega \rightarrow X$ is called a measurable selector of a measurable mapping F:Ω→ 2^X if δ measurable and $\delta(\omega) \in F(\omega)$ for each $\omega \in \Omega$.

Definition (1.11):[9]

A mapping $h: \Omega \times X \rightarrow X$(or a multivalued $G: \Omega \times X \rightarrow CB(X)$) is called a random operator if for any $x \in X, h(.,x)$(respectively G(. ,x)) is measurable .

Definition (1.12):[18]

A measurable mapping $\delta: \Omega \rightarrow A$ is called random fixed point of a random operator $h: \Omega \times X \rightarrow X$ (or a multivalued $G: \Omega \times X \rightarrow CB(X)$ if for every $\omega \in \Omega$, $\delta(\omega) = h(\omega, \delta(\omega))$ (respectively$\delta(\omega) \in G(\omega, \delta(\omega))$.

Definition (1.13):[1]

A measurable mapping $\delta: \Omega \to A$ is called common random fixed point of a random operator $h: \Omega \times A \to X$ and $G: \Omega \times A \to A$ if for all $\omega \in \Omega$

$\delta(\omega) = h(\omega, \delta(\omega)) = G(\omega, \delta(\omega))$.

Definition (1.14):[19]

A random operator $h: \Omega \times A \to X$ is called continuous (weakly continuous) if for each $\omega \in \Omega, h(\omega, .)$ is continuous (weakly continuous).

Definition (1.15):[16]

Let X be a p-normed space a random operator , $G: \Omega \times X \longrightarrow X$ is ***h*-nonexpansive** if for each $\omega \in \Omega$, $G(\omega, .)$ is $h(\omega, .)$ - nonexpansive .

Definition (1.16): [18]

A random operator $h: \Omega \times X \to X$ is said to be **affine** if for each $\omega \in \Omega$, $h(\omega, .): X \to X$ is affine .

Definition (1.17):[15]

Let X be a metric space . A random operators $h, G: \Omega \times X \longrightarrow X$ are said to be **commute** if $h(\omega, .)$ and $G(\omega, .)$ are commute for each $\omega \in \Omega$.

In the following we give a generalization for starshaped properties for $\emptyset \neq A \subseteq$ X.

Definition (1.18):

Let X be a p-normed space , $A \subseteq$ X and $G: \Omega \times X \to X$ be a random operator we say that A has property (a_1) if

i. G : $\Omega \times A \to A$

ii. $(1 - k_n)q + k_n G(\omega, x) \in A$, for some q $\in A$ and a fixed real sequence $< k_n >$ converging to 1 and for each $x \in A$ and for each $\omega \in \Omega$.

Remark(1.19):

Any q-starshaped set has the property (a_1) w.r.t any random operator : $\Omega \times A \rightarrow A$, but the converse is not true in general .

In the following we give a generalization for affine properties for random operator

Definition (1.20):

Let X be a p-normed space , $A \subseteq X$ and A has property (a_1) w.r.t a random operator $G: \Omega \times X \rightarrow X$, q $\in A$ and sequence $< k_n >$. A random operator h: $\Omega \times X \rightarrow X$ is said to be have property (a_2) on A with property (a_1) if

$$h(\omega , (1 - k_n)q + k_n\, G(\omega, x)) = (1 - k_n)\, h(\omega, q) + k_n h(\omega, G(\omega, x))$$

for all $x \in A \; and \; n \in N \; and \; \omega \in \Omega$.

2. Random Common Fixed Point

Theorem (2.1):[15]

Let X be a compact metric space . Let $h, G: \Omega \times X \rightarrow X$ be two commuting random operators such that $G(\omega, X) \subseteq h(\omega, X)$ for all $\omega \in \Omega$. If h is continuous and $d(G(\omega, x), G(\omega, y)) \leq d(h(\omega, x), h(\omega, y))$ for each $x, y \in X$ and each $\omega \in \Omega$ whenever $h(\omega, x) \neq h(\omega, y)$, then h and G have a common random fixed point .

Now , by using Theorem (2.1) we prove the following :

Theorem(2.2):

Let A be a nonempty compact subset of a p-normed space . Let $h: \Omega \times X \rightarrow X$ be a continuous random operator and has property (a_2) with $h(\omega, A) = A$ and $h(\omega, q) = q$ for all $\omega \in \Omega$, $G: \Omega \times X \rightarrow X$ be an h-nonexpansive random operator commute with h . If A has property (a_1) w.r.t G and G is continuous random operator, then h and G have a common random fixed point .

Proof :

Since A has property (a_1) w.r.t , then $(1-k_n)q+k_nG(\omega,x)\in A$, for some q ∊ A and a fixed real sequence $< k_n >$converging to 1 $(0<k_n<1)$, for all $x\in A$ and for all $\omega\in\ \Omega$.

For each $n\geq 1$, defined the random operators G_n by

$G_n(\omega,x)=(1-k_n)q+k_nG(\omega,x)$, for all $x\in A$ and all $\omega\in\ \Omega$.

It is clear that , $G_n:\Omega\times A\to A$.

Since $G_n(\omega,A)\subseteq A=h(\omega,A)$, then $G_n(\omega,A)\subseteq h(\omega,A)$ for all $\omega\in\Omega$.

Since G commute with h , h has property (a_2) and $h(\omega,q)=q$ for all $\omega\in\Omega$ then for each $x\in A$ and all $\omega\in\ \Omega$

$$\begin{aligned}G_n(\omega,h(\omega,x))&=(1-k_n)q+k_nG(\omega,h(\omega,x))\\&=(1-k_n)h(\omega,q)+k_nh(\omega,G(\omega,x))\\&=h(\omega,(1-k_n)q+k_nG(\omega,x))\\&=h\big(\omega,G_n(\omega,x)\big)\end{aligned}$$

Therefore each G_n is commute with h .

Since G is h -nonexpansive , then

$$\begin{aligned}&\left\|G_n(\omega,x)-\ G_n(\omega,y)\right\|_p\\&=\|(1-k_n)q+k_nG(\omega,x)-(1-k_n)q-k_nG(\omega,y)\|_p\\&=|k_n|^p\|G(\omega,x)-G(\omega,y)\|_p\\&\leq|k_n|^p\|h(\omega,x)-h(\omega,y)\|_p\\&<\|h(\omega,x)-h(\omega,y)\|_p\end{aligned}$$

For all $x\,,y\in A$ and all $\omega\in\ \Omega$, thus each G_n is a random h-nonexpansive .

Hence , by theorem (2.1) , there is a common random fixed point $\delta_n:\Omega\to A$ of G_n and h such that $\delta_n(\omega)=h\big(\omega,\ \delta_n(\omega)\big)=G_n(\omega,\ \delta_n(\omega))$.

For each n define $Q_n:\Omega\to k(A)$ by $Q_n(\omega)=\overline{\{\delta_\iota(\omega):\iota\geq n\}}$,

Define $Q:\Omega\to k(A)$ by $Q(\omega)=\cap_{n=1}^{\infty}Q_n(\omega)$.

so Q is measurable [7] and Q has measurable selector $\delta:\Omega\to A$ [12] .

Since A is compact and $\{\delta_n(\omega)\}$ sequence in A

Then there is a subsequence $\{\delta_m(\omega)\}$ of $\{\delta_n(\omega)\}$ converges to $\delta(\omega)$.

Having $\delta_m(\omega) = G_m(\omega, \delta_m(\omega)) = h(\omega, \delta_m(\omega))$.

As h and G are continuous and $k_n \to 1$, we have $G_m(\omega, \delta_m(\omega))$ converges to $G(\omega, \delta(\omega))$ and $h(\omega, \delta_m(\omega))$ converges to $h(\omega, \delta(\omega))$. Consequently

$$\lim_{m\to\infty} \delta_m(\omega) = \lim_{m\to\infty} G_m\big(\omega, \delta_m(\omega)\big) = \lim_{m\to\infty} h\big(\omega, \delta_m(\omega)\big)$$

$$\delta(\omega) = G(\omega, \delta(\omega)) = h(\omega, \delta(\omega))$$

Hence $\delta: \Omega \to A$ is a common random fixed point of h and G.■

As a consequence of theorem (2.2) we have the following

Corollary (2.3):

Let A be a nonempty compact subset of a p-normed space . Let $h: \Omega \times X \to X$ be a continuous random operator and has property (a_2) with $h(\omega, A) = A$ and $h(\omega, q) = q$ for all $\omega \in \Omega$, $G: \Omega \times X \to X$ be an h-nonexpansive random operator commute with h . If A is q-starshaped and G is continuous random operator , then h and G have a common random fixed point .

Theorem(2.4):

Let A be a nonempty separable weakly compact subset of a complete p-normed space . Let $h: \Omega \times X \to X$ be a continuous and weakly continuous random operator and has property (a_2) with $h(\omega, A) = A$ and $h(\omega, q) = q$ for all $\omega \in \Omega$, $G: \Omega \times X \to X$ be an h -nonexpansive random operator commute with h . If A has property (a_1) w.r.t G , and $(h - G)(\omega, .)$ is demiclosed at zero for all $\omega \in \Omega$, then h and G have a common random fixed point .

Proof :

Since A has property (a_1) w.r.t , then

$(1-k_n)q + k_n G(\omega,x) \in A$, for some q $\in A$ and a fixed real sequence $< k_n >$converging to 1 $(0 < k_n < 1)$, for all $x \in A$ and for all $\omega \in \Omega$.

For each $n \geq 1$, defined the random operators G_n by

$G_n(\omega,x) = (1-k_n)q + k_n G(\omega,x)$, for all $x \in A$ and all $\omega \in \Omega$.

It is clear that , $G_n : \Omega \times A \to A$.

Since $G_n(\omega,A) \subseteq A = h(\omega,A)$, then $G_n(\omega,A) \subseteq h(\omega,A)$ for all $\omega \in \Omega$.

Since G commute with h , h has property (a_2) and $h(\omega,q) = q$ for all $\omega \in \Omega$ then for each $x \in A$ and all $\omega \in \Omega$

$$\begin{aligned} G_n(\omega,h(\omega,x)) &= (1-k_n)q + k_n G(\omega,h(\omega,x)) \\ &= (1-k_n)h(\omega,q) + k_n h(\omega,G(\omega,x)) \\ &= h(\omega,(1-k_n)q + k_n G(\omega,x)) \\ &= h\big(\omega,G_n(\omega,x)\big) \end{aligned}$$

Therefore each G_n is commute with h .

Since G is h-nonexpansive , then

$$\begin{aligned} &\left\| G_n(\omega,x) - G_n(\omega,y) \right\|_p \\ &\quad = \|(1-k_n)q + k_n G(\omega,x) - (1-k_n)q - k_n G(\omega,y)\|_p \\ &\quad = |k_n|^p \|G(\omega,x) - G(\omega,y)\|_p \\ &\quad \leq |k_n|^p \|h(\omega,x) - h(\omega,y)\|_p \\ &\quad < \|h(\omega,x) - h(\omega,y)\|_p \end{aligned}$$

For all $x , y \in A$ and all $\omega \in \Omega$, thus each G_n is a random h -nonexpansive. Since A is separable and weakly compact ,then the weak topology on A is a metric topology[3] which implies that A is hausdorff [10] then A is strongly closed , this implies A is a complete metric space [11] Thus all the conditions of Theorem (2.1) are satisfied .

Hence there is a common random fixed point $\delta_n : \Omega \to A$ of G_n and h such that $\delta_n(\omega) = h\big(\omega,\ \delta_n(\omega)\big) = G_n(\omega,\ \delta_n(\omega))$.

Now ,for each n define $Q_n : \Omega \to k_w(A)$ by $Q_n(\omega) = w - \overline{\{\delta_\iota(\omega) : \iota \geq n\}}$.

Define $Q: \Omega \rightarrow k_w(A)$ by $Q(\omega) = \cap_{n=1}^{\infty} Q_n(\omega)$.

Since A is complete p-normed space ,then Q is weakly measurable[7] so Q has measurable selector $\delta: \Omega \rightarrow A$ [12] .

By weak compactness of A , we have that a subsequence $\{\delta_m(\omega)\}$ of $\{\delta_n(\omega)\}$ converges weakly to $\delta(\omega)$.

Having $\delta_m(\omega) = G_m(\omega, \delta_m(\omega)) = h(\omega, \delta_m(\omega))$.

Now ,from weakly continuity of h , we have

$$h(\omega, \delta(\omega)) = h\left(\omega, \lim_{m\rightarrow\infty} \delta_m(\omega)\right) = \lim_{m\rightarrow\infty} h(\omega, \delta_m(\omega)) = \lim_{m\rightarrow\infty} \delta_m(\omega)$$
$$= \delta(\omega)$$

Hence $h(\omega, \delta(\omega)) = \delta(\omega)$.

Since $G(\omega, \delta_m(\omega)) = \frac{G_m(\omega,\delta_m(\omega))-(1-k_m)q}{k_m}$ and $\delta_m(\omega) = h\big(\omega, \delta_m(\omega)\big) = G_m(\omega, \delta_m(\omega))$, we have

$$(h-G)(\omega, \delta_m(\omega)) = h(\omega, \delta_m(\omega)) - G(\omega, \delta_m(\omega)$$
$$= \delta_m(\omega) - \frac{G_m(\omega,\delta_m(\omega))-(1-k_m)q}{k_m}$$
$$= \frac{\delta_m(\omega)k_m - G_m(\omega,\delta_m(\omega))+(1-k_m)q}{k_m}$$
$$= \frac{\delta_m(\omega)k_m - \delta_m(\omega)+(1-k_m)q}{k_m}$$
$$= \frac{-\delta_m(\omega)(1-k_m)+(1-k_m)q}{k_m}$$
$$= \frac{(1-k_m)(q-\delta_m(\omega))}{k_m}$$

Therefore , $(h-G)\big(\omega, \delta_m(\omega)\big) = \left(\frac{1}{k_m} - 1\right)(q - \delta_m(\omega))$

Thus $\left\|(h-G)\big(\omega, \delta_m(\omega)\big)\right\|_p = \left|\frac{1}{k_m} - 1\right|^p \|(q - \delta_m(\omega))\|_p \leq \left|\frac{1}{k_m} - 1\right|^p \left[\| \delta_m(\omega)\|_p + \|q\|_p\right]$

Since A is bounded by A is weakly compact , and $\delta_m(\omega) \in A$ implies $\langle\|\delta_m(\omega)\|_p\rangle$ is bounded and so by the fact that $k_m \rightarrow 1$, we have

$\left\|(h-G)\left(\omega,\delta_m(\omega)\right)\right\|_p \to 0$

Since $\delta_m(\omega) \to \delta(\omega)$, then $\delta_m(\omega) \rightharpoonup \delta(\omega)$.

Since $\delta_m(\omega) \rightharpoonup \delta(\omega)$, $(h-G)(\omega,\delta_m(\omega)) \to 0$ and $(h-G)(\omega,.)$ is demiclosed at 0 , then $(h-G)(\omega,\delta(\omega)) = 0$

This implies $h(\omega,\delta(\omega)) - G(\omega,\delta(\omega)) = 0$

Hence $h(\omega,\delta(\omega)) = G(\omega,\delta(\omega))$

Thus $h\left(\omega,\delta(\omega)\right) = G\left(\omega,\delta(\omega)\right) = \delta(\omega)$.

Therefore $\delta: \Omega \to A$ is a common random fixed of h and G . ■

As a special case of theorem (2.4) we have the following

Corollary (2.5):

Let A be a nonempty separable weakly compact subset of a complete p-normed space . Let $h: \Omega \times X \to X$ be a continuous and weakly continuous random operator and has property (a_2)with $h(\omega, A) = A$ and $h(\omega,q) = q$ for all $\omega \in \Omega$, $G: \Omega \times X \to X$ be an h -nonexpansive random operator commute with h . If A is q-starshaped, and $(h-G)(\omega,.)$ is demiclosed at zero for all $\in \Omega$, then h and G have a common random fixed point .

Another common random fixed point theorem will be given for opial$'s$ spaces :

Theorem(2.6):

Let A be a nonempty separable weakly compact subset of a complete p-normed opial space X . Let $h: \Omega \times X \to X$ be a continuous and weakly continuous random operator and has property (a_2) with $h(\omega, A) = A$ and $h(\omega,q) = q$ for all $\omega \in \Omega$,

$G: \Omega \times X \to X$ be an h -nonexpansive random operator commute with h . If A has property (a_1) w.r.t G, then h and G have a common random fixed point .

Proof :

As in theorem (2.4), $h\big(\omega,\delta(\omega)\big)=\delta(\omega)$ and $\big\|(h-G)\big(\omega,\delta_m(\omega)\big)\big\|_p \to 0$

As $\to \infty$.

Now, since X is opial space , if $h(\omega,\delta(\omega)) \neq G(\omega,\delta(\omega))$,then

$$\lim_{m\to\infty} inf \parallel h\big(\omega,\delta_m(\omega)\big)-h\big(\omega,\delta(\omega)\big)\|_p$$

$$< \lim_{m\to\infty} inf \parallel h\big(\omega,\delta_m(\omega)\big)-G\big(\omega,\delta(\omega)\big)\|_p$$

$$= \lim_{m\to\infty} inf \parallel h\big(\omega,\delta_m(\omega)\big)-G\big(\omega,\delta(\omega)\big)-G(\omega,\delta_m(\omega)+G(\omega,\delta_m(\omega)\|_p$$

$$\leq \lim_{m\to\infty} inf \parallel h\big(\omega,\delta_m(\omega)\big)-G(\omega,\delta_m(\omega)\|_p + \lim_{m\to\infty} inf\|G\big(\omega,\delta_m(\omega)\big) - G\big(\omega,\delta(\omega)\big)\|_P$$

Since G is h-nonexpansive and $\parallel h\big(\omega,\delta_m(\omega)\big)-G(\omega,\delta_m(\omega)\|_p \to 0$, then

$$\lim_{m\to\infty} inf \parallel h\big(\omega,\delta_m(\omega)\big)-h\big(\omega,\delta(\omega)\big)\|_p$$

$< \lim_{m\to\infty} inf \parallel h\big(\omega,\delta_m(\omega)\big)-h\big(\omega,\delta(\omega)\big)\|_p$ which is contraction .

Hence $(\omega)=h(\omega,\delta(\omega)) = G(\omega,\delta(\omega))$.

Therefore $\delta:\Omega\to A$ is a common random fixed of h and G . ■

As a consequence of theorem (2.6) we have the following

<u>Corollary(2.7)</u>:

Let A be a nonempty separable weakly compact subset of a complete p-normed opial space . Let $h:\Omega\times X \to X$ be a continuous and weakly continuous random operator and has property (a_2) with $h(\omega,A)=A$ and $h(\omega,q)=q$ for all $\omega\in\Omega$, $G:\Omega\times X \to X$ be an h-nonexpansive random operator commute with h . If A is q-starshaped then h and G have a common random fixed point .

<u>3.*Random Best Approximation*</u>

The following lemma is needed:

Lemma (3.1):[6]

Let A be a subset of a p-normed space . Then , for any $x \in X$, $P_A(x) \subseteq \partial A$ (the boundary of A) .

As an application of Theorem (2.2),(2.4) and (2.6) , we have the following result on random best approximation .

Theorem (3.2):

Let X be a p-normed space. Let $h, G: \Omega \times X \to X$ be two random operators and A a nonempty subset of X such that $G(\omega,.): \partial A \to A$ and $x_\circ = h(\omega, x_\circ) = G(\omega, x_\circ)$ for all $\omega \in \Omega$. Let $P_A(x_\circ)$ be a nonempty has property (a_1) w.r.t G with $h(\omega, q) = q$ and $h(\omega, P_A(x_\circ)) = P_A(x_\circ)$ for all $\omega \in \Omega$ and let G is h -nonexpansive random operator and h has property (a_2) , continuous on $P_A(x_\circ)$ and $h(\omega, G(\omega, x)) = G(\omega, h(\omega, x))$ for all $x \in P_A(x_\circ)$ and all $\omega \in \Omega$ then there exists a measurable map $\delta: \Omega \to P_A(x_\circ)$ such that $\delta(\omega) = h(\omega, \delta(\omega)) = G(\omega, \delta(\omega))$ for each $\omega \in \Omega$ if one of the following conditions is satisfied :

i. $P_A(x_\circ)$ is compact and G is continuous on $P_A(x_\circ) \cup \{x_\circ\}$;
ii. $P_A(x_\circ)$ is separable weakly compact subset of a complete p-normed space X , h weakly continuous random operator , $(h - G)(\omega,.)$ is demiclosed at zero for all $\omega \in \Omega$;
iii. $P_A(x_\circ)$ is separable weakly compact subset of a complete p- normed space X , h weakly continuous random operator and X is opial space .

Proof :

Let $= P_A(x_\circ)$, $y \in M$, since $h(\omega, M) = M$ for all $\omega \in \Omega$, so $h(\omega, y) \in M$. Also by lemma (3.1) $\in \partial A$. As $G(\omega,.): \partial A \to A$ for all $\in \Omega$, it follows that $G(\omega, y) \in A$. Since $G(\omega, x_\circ) = x_\circ$ and G is a h −nonexpansive random operator , thus

$$\|G(\omega, y) - x_\circ\|_p = \|G(\omega, y) - G(\omega, x_\circ)\|_p \le \|h(\omega, y) - h(\omega, x_\circ)\|_p$$

As $h(\omega, x_\circ) = x_\circ$ and $h(\omega, y) \in M$,

$\|G(\omega, y) - x_\circ\|_p \leq \|h(\omega, y) - x_\circ\|_p = d_p(x_\circ, A)$

Since $G(\omega, y) \in A$ we have $(\omega, y) \in M$. Thus $G: \Omega \times P_A(x_\circ) \to P_A(x_\circ)$.

Hence the result follows by Theorem (2.2) ,(2.4) and (2.6) respectively .■

As a consequence of theorem(3.2) we get

Corollary (3.3):

Let X be a p-normed space. Let $h, G: \Omega \times X \to X$ be two random operators and A a nonempty subset of X such that $G(\omega, .): \partial A \to A$ and $x_\circ = h(\omega, x_\circ) = G(\omega, x_\circ)$ for all $\omega \in \Omega$. Let $P_A(x_\circ)$ be a nonempty q-starshaped with $h(\omega, q) = q$ and $h(\omega, P_A(x_\circ)) = P_A(x_\circ)$ for all $\omega \in \Omega$ and let G is h -nonexpansive random operator and continuous on $P_A(x_\circ) \cup \{x_\circ\}$ where h has property (a_2) , continuous on $P_A(x_\circ)$ and $h(\omega, G(\omega, x)) = G(\omega, h(\omega, x))$ for all $x \in P_A(x_\circ)$ and all $\omega \in \Omega$ then there exists a measurable map $\delta: \Omega \to P_A(x_\circ)$ such that $\delta(\omega) = h(\omega, \delta(\omega)) = G(\omega, \delta(\omega))$ for each $\omega \in \Omega$ if one of the following conditions is satisfied :

i. $P_A(x_\circ)$ is compact and G is continuous on $P_A(x_\circ) \cup \{x_\circ\}$;

ii. $P_A(x_\circ)$ is separable weakly compact subset of a complete p-normed space X , h weakly continuous random operator , $(h - G)(\omega, .)$ is demiclosed at zero for all $\omega \in \Omega$;

iii. $P_A(x_\circ)$ is separable weakly compact subset of a complete p- normed space X, h weakly continuous random operator and X is Opial space.

Remark (3.4):

All Theorems and Corollaries in this paper are true for one random operator by taking $h = I$ (is the identity random operator) .

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