

Back bending phenomena evaluation and energy band crossing of some deformed nuclei

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ABSTRACT

In the present work, the moment of inertia models(VMI,VAVM,GVMI)are used to study the nuclear structure of even-even nuclei (${}^{56}_{26}\text{Fe}_{30}$, ${}^{74}_{34}\text{Se}_{40}$, ${}^{88}_{40}\text{Zr}_{48}$, ${}^{106}_{46}\text{Pd}_{60}$, ${}^{156}_{64}\text{Gd}_{92}$, ${}^{180}_{72}\text{Hf}_{108}$, ${}^{186}_{74}\text{W}_{112}$). These nuclei are belong to a dynamical symmetry O(6), SU(5)-O(6)-SU(3), SU(5), O(6)-SU(5), SU(3)-SU(5), SU(3), SU(3)-O(6) respectively, throw the study energy band crossing (energy level as a function of angular momentum), and back bending phenomena (moment of inertia $(2\mathcal{I}/\hbar^2)$ as a function of rotational energy squared $(\hbar\omega)^2$).

This study shows that the three moment of inertia models that are used here have no effect on the nuclear structure of the nucleus with the dynamical symmetry SU(3), but the nuclear structure of the other dynamical symmetries have been effected, because the back bending does appear in them .

حساب ظاهرة الانحناء الخلفي وظاهرة تقاطع حزم الطاقة لبعض الانوية المشوهة

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المستخلص

تم في هذا البحث استخدام ثلاثة نماذج من نماذج عزم القصور الذاتي هي

(VMI,VAVM ,GVMI) لغرض دراسة التركيب النووي لبعض الانوية الزوجية-الزوجية

الديناميكية O(6),SU(5)-O(6)-SU(3),SU(5),O(6)-SU(5),SU(3)-SU(5), SU(3), التي تنتمي الى التناظرات

${}^{186}_{74}\text{W}_{112}$ ${}^{180}_{72}\text{Hf}_{108}$ ${}^{156}_{64}\text{Gd}_{92}$ ${}^{106}_{46}\text{Pd}_{60}$ ${}^{88}_{40}\text{Zr}_{48}$ ${}^{74}_{34}\text{Se}_{40}$ ${}^{56}_{26}\text{Fe}_{30}$

(6) $SU(3) - O$ على التوالي من خلال دراسة ظاهرة تقاطع حزم الطاقة (مستويات الطاقة كدالة للزخم الزاوي) وظاهرة الانحناء الخلفي (عزم القصور الذاتي $(2\mathcal{I}/\hbar^2)$) ومربع الطاقة الدورانية $(\hbar\omega)^2$. اظهرت الدراسة الحالية للنماذج الثلاثة المستخدمة انه لا تأثير لعزم القصور الذاتي على شكل الانوية الدورانية عند التناظر الديناميكي $SU(3)$ لعدم ظهور الانحناء الخلفي في الحزم التي تنتمي الى هذا التناظر. أما بالنسبة للتناظرات الديناميكية الاخرى فقد تبين وجود تأثير لعزم القصور الذاتي على تركيبها النووي بسبب ظهور ظاهرة الانحناء الخلفي.

INTRODUCTION

Moment of inertia is an important subjects that are used to explain the energy band crossing, and back bending phenomena.

The nucleus extension due to the centrifugal force effect, which increases the value of moment of inertia, happened only for rotational nuclei if it is supposed that the nucleus is fluid, where the theoretical moment of inertia value (θ_{fluid}) is very small compared with that experimental. But this extension does not happen if we suppose that the nucleus is a rigid body and moment of inertia in this case (θ_{rigid}) is large [1].

The Back Bending Phenomena

This phenomenon has been discovered by Johnson A., et al. (1971) [2]. They found that moment of inertia at certain angular momentum significantly increases accompanied with a decrease of rotational energy value of some nuclear. It also occurs because rotational energy is greater than the stopping power of a pair of nucleons to couple [1]. Where the single nucleons occupy different orbits which results in changing of the moment of inertia value for the nucleus.

The back bending is one feature of moment of inertia of deformed nuclei, in case of non appearance of back bending; this means that there is no effect of moment of inertia on the deformation of the nuclei. This can be attributed to:

(i): Energy Band Crossing

The band crossing means that the certain energy band like (β or γ) with high moment of inertia (\mathcal{I}_2) to replace or cross with another energy band as (g- band) state with lower moment of inertia (\mathcal{I}_1) (i.e. these two bands having same spin, and different in energy and moment of inertia will intersect with themselves) at a particular moment of inertia called (\mathcal{I}_c) where $\mathcal{I}_c = \mathcal{I}_{cross}$.

This phenomenon is considered an important feature of moment of inertia of deformed nuclei [3].

(ii): Corielis force effect

Corielis force effect increases with rotational motion at high spins for some nuclei; this increase in corielis force leads to depairing between numbers of nucleon pairs. The depairing of the first pair leads to the appearance of new band namely "two quasi particles" which probably intersects the ground state band at a certain angular momentum, while ground band remains completely paired. Where as the depairing of the second pair leads to the appearance of "four-quasi particles" band .This causes nuclear deformations and deviations at high angular momentum of some nucleus which causes back bending phenomena [4].

Habeeb N.A., et al. (1976)[5] explained the back bending phenomenon for $^{162}_{68}\text{Er}$ and $^{154}_{64}\text{Gd}$ nuclei depending on Corielis force effect.

Yu-Xin-Lin (1998) [6] have been calculated moment of inertia for high deformed nuclei using (IBM-1).

Hirsch, Jorge G., et al. (2005)[7] showed that the Pseudo SU(3) model and abnormal parity states in heavy deformed nuclei is based on the Pseudo spin symmetry, and is able to correctly predict the collective spectra and their transition amplitudes, and is also able to give nice description of the back bending phenomena.

THEORETICAL PART

1. Rotational and Vibration Energy Levels

The simple formula of the rotational or vibrational energy levels in even-even nuclei is determined by [8]:

$$E = \frac{1}{2} \mathcal{I} \omega^2 = \frac{L^2}{2\mathcal{I}} \dots \dots \dots (1)$$

Where ω is a rotational or vibrational frequency. The above equation is quantized to [9]:

$$E(L) = \frac{\hbar^2}{2\mathcal{I}} L(L+1) \dots \dots \dots (2)$$

Mariscotti M., et al., (1969)[10] proposed another formula from this equation as:

$$E(L) = \frac{1}{2} \left[\frac{L(L+1)}{\mathcal{I}(L)} + C(\mathcal{I}(L) - \mathcal{I}_0)^2 \right] \dots \dots \dots (3)$$

Where

$\mathcal{I}(L)$ is a moment of inertia as a function of angular momentum

C, \mathcal{I}_0 are parameters which are suitable with fitted to the experimental data.

The physical meanings of these parameters are [11]:

\mathcal{I}_0 is a moment of inertia for the ground state or (rotated nucleus).

C: is a meaning of “elasticity.

The derivative of equation (3) is

$$\frac{\partial E(L)}{\partial \mathcal{G}(L)} = -\frac{1}{2} \frac{L(L+1)}{\mathcal{G}^2(L)} + C(\mathcal{G}(L) - \mathcal{G}_o) \dots \dots \dots (4)$$

Moment of inertia $\mathcal{G}(L)$ can be determined from the equilibrium condition [10, 11]:

$$\frac{\partial E(L)}{\partial \mathcal{G}(L)} = 0 \dots \dots \dots (5)$$

$$\mathcal{G}^3(L) - \mathcal{G}^2(L)\mathcal{G}_o - \frac{L(L+1)}{2C} = 0 \dots \dots \dots (6)$$

The solution of above equation is [12, 13]:

$$\mathcal{G}(L) = (x + y)^{1/3} + (x - y)^{1/3} + \frac{1}{3}\mathcal{G}_o \dots \dots \dots (7)$$

Where

$$x = \frac{L(L+1)}{4C} + \frac{\mathcal{G}_o^3}{27}, y = \left[\frac{(L(L+1))^2}{16C^2} + \frac{\mathcal{G}_o^3 L(L+1)}{54C} \right]^{1/2} \dots \dots \dots (8)$$

The nuclear softness coefficient (σ) can be written as [10]:

$$\sigma = \frac{1}{\mathcal{G}(L)} \left(\frac{d\mathcal{G}}{dL} \right)_{\mathcal{G}=0} \dots \dots \dots (9)$$

σ can be written according to variable moment of inertia model as [10,12]

$$\sigma = \frac{1}{2C\mathcal{G}_o^3} \dots \dots \dots (10)$$

2. The back bending phenomena

Rotational energy square ($\hbar\omega$)² and moment of inertia can be written as[14]

$$(\hbar\omega)^2 = \left[\frac{E(L)}{\sqrt{L(L+1)}} \right]^2 \dots \dots \dots (11)$$

$$\frac{2\mathcal{G}}{\hbar^2} = \frac{L(L+1)}{E(L)} \dots \dots \dots (12)$$

Equation (11) can be written for g-band and β -band as following [12, 13]:

$$(\hbar\omega)^2 = \left[\frac{E(L \rightarrow L-2)}{\sqrt{L(L+1)} - \sqrt{(L-2)(L-2+1)}} \right]^2 \dots\dots\dots(13)$$

$$(\hbar\omega)^2 = \left[\frac{E_\gamma}{\sqrt{L(L+1)} - \sqrt{(L-2)(L-1)}} \right]^2 \dots\dots\dots(14)$$

and for γ -band is

$$(\hbar\omega)^2 = \left[\frac{E(L \rightarrow L-1)}{\sqrt{L(L+1)} - \sqrt{L(L-1)}} \right]^2 \dots\dots\dots(15)$$

Also from equation (12) can be evaluating moment of inertia for g-band and β -band as [15]:

$$\frac{2\mathcal{G}}{\hbar^2} = \frac{L(L+1) - (L-2)(L-2+1)}{E(L \rightarrow L-2)} = \frac{4L-2}{E(L_i) - E(L_f)} \dots\dots\dots(16)$$

and for γ -band is:

$$\frac{2\mathcal{G}}{\hbar^2} = \frac{L(L+1) - L(L-1)}{E(L \rightarrow L-1)} = \frac{2L}{E_\gamma} \dots\dots\dots(17)$$

Where $E(L \rightarrow L-2)$ is the energy difference between any two states which have angular momentum L and $(L-2)$ for g-band and β -band while $E(L \rightarrow L-1)$ is the energy difference between any two states having angular momentum L and $(L-1)$ for γ -band

RESULTS AND DISCUSSION

In the present study we can evaluate the following calculations by programming (VMI, VAVM, and GVMI) model using FORTRAN 77 language:

1. Theoretical energy levels E_{cal} . Using equation (3).
2. Moment of inertia as a function of angular momentum using equation (7).
3. Rotational energy square $(\hbar\omega)^2$ and $(2\mathcal{G}/\hbar^2)$ as a function of angular momentum.
4. Chi-squared which can be calculated from the following equation [16]:

$$\chi^2 = \frac{(E_{cal} - E_{exp})^2}{E_{cal}^2} \dots\dots\dots(18)$$

5. Nuclear softness parameters (σ) from equation (10).

6. Standard deviation (Root Mean Square Deviation) from the following equation [11, 16]:

$$deviation(\Delta) = \left[\frac{1}{N} \sum_{i=1}^N (E_{cal} - E_{exp})^2 \right]^{1/2} \dots\dots\dots(19)$$

Table (1) shows the parameters (\mathcal{G}_0/\hbar^2 , C, Y, E_k) which are used in (VMI, VAVM,GVMI) program to calculate the above results.

1. The Energy Band crossing

The increasing of rotational energy between any two excited states due to reduce moment of inertia at large angular momentum. Here, Corielis force effect will increase with the increase of rotational energy between these bands. This effect leads to reducing the energy of two nucleons, which occupy these states because, the depairing of the nucleon pairs. In this case, the direction of angular momentum of these nucleons is in same direction with the rotational axis with low rotational energy and maximum angular momentum. The unstable of these two excited states leads to crossing band with any other band at certain angular momentum called " L_{cross} ".

Figure (1) indicates the energy band crossing as a function of angular momentum for all chosen nuclei using (VMI) model.

It is noticed in this figure that the $(g, \beta_2), (g, \beta_3), (\beta_1, \beta_2, \gamma_2, \gamma_3)$ of the $^{56}_{26}Fe_{30}$ nucleus will intersect at angular momentum $L_c=(14,7,5)$ respectively, while the band crossing would happened at $L_c = (6,7,11,14,22)$ for $^{74}_{34}Se_{40}$ nuclei. This figure also shows that the bands of $^{88}_{40}Zr_{48}$ and $^{156}_{64}Gd_{92}$ are not crossing between them, while, all bands of $^{106}_{46}Pd_{60}$ are crossing at $L_c=(5,7,11)$, while, it happened at $L_c=13$ in $^{180}_{72}Hf_{108}$ and at $L_c=(5,7)$ of $^{186}_{74}W_{112}$.

Figure(2) shows the energy band crossing for the same nuclei using (VAVM).This figure shows the bands $(g, \beta_3), (g, \gamma_1), (\beta_1, \beta_2, \gamma_2, \gamma_3)$ of $^{56}_{26}Fe_{30}$ intersect at angular momentum $L_c=(15,7,5)$ respectively , while it happened at $L_c = (4,8,17)$ in $^{74}_{34}Se_{40}$, $L_c= (4,5,7,12,17,21)$ in $^{106}_{46}Pd_{60}$, $L_c=14$ in $^{180}_{72}Hf_{108}$ and $L_c = (10,20,22)$ in $^{186}_{74}W_{112}$, while in the bands of $^{156}_{64}Gd_{92}$ there are no crossing between them .

Figure(3) shows the band crossing using (GVMI) model for all selected nuclei.

In this figure we noticed that the $(g, \beta_3, \gamma_1), (\beta_1, \beta_2, \gamma_2, \gamma_3)$ bands crossing at $L_c = (14,6)$ in $^{56}_{26}Fe_{30}$ respectively, $L_c=(4,7,9,12)$ in $^{74}_{34}Se_{40}$. This figure also

shows that the band crossing happened between (β_1, β_2) bands at $L_c=18$, $L_c=(5,6,8,11)$ in $^{106}_{46}\text{Pd}_{60}$, $L_c=14$ in $^{180}_{72}\text{Hf}_{108}$, and $L_c=(4,6,10)$ in $^{186}_{74}\text{W}_{112}$, while, there is no band crossing in $^{156}_{64}\text{Gd}_{92}$.

2. The Back Bending Phenomenon

In order to study the back bending phenomenon, we must calculate and draw the values of moments of inertia ($2\mathcal{I}/\hbar^2$) and rotational energy squared $(\hbar\omega)^2$ as a function of angular momentum (L) by using (VMI, VAVM, and GVMI) model using the parameters (\mathcal{I}_0 , C, E_K , Y) in table(1) for the ground state, and the excited state bands.

Figure (4) indicates that the back bending (moment of inertia ($2\mathcal{I}/\hbar^2$)) as a function of rotational energy squared $(\hbar\omega)^2$ using VMI model for chosen nuclei. This figure shows that the back bending occurs in γ_1 -band of $^{56}_{26}\text{Fe}_{30}$ and $^{74}_{34}\text{Se}_{40}$ at angular momentum $L_c=9,12$ respectively, and no effect of moment of inertia on nuclear structure of other nuclei, due to the back bending not occurs.

Figure (5) shows the same plot but for (VAVM) model. It is noticed in this figure that the back bending occurs in γ_1 -band of $^{106}_{46}\text{Pd}_{60}$ at angular momentum $L_c=9$, while it is not occurs in the other band.

Figure (6) also shows the back bending using (GVMI) model. This figure noticed that the back bending occurs in (g, β_1, β_3) , (β_3, β_4) , (g, β_1, β_2) , $(\beta_1, \beta_3, \beta_4)$, (β_1) bands of the nuclei $^{56}_{26}\text{Fe}_{30}$, $^{74}_{34}\text{Se}_{40}$, $^{88}_{40}\text{Zr}_{48}$, $^{106}_{46}\text{Pd}_{60}$, $^{186}_{74}\text{W}_{112}$ at angular momentum $L_c=(8,8,18)$, $(16,12)$, $(10,10,6)$, $(8,8,10)$, (12) respectively, while it is not happened in $^{156}_{64}\text{Gd}_{92}$, $^{180}_{72}\text{Hf}_{108}$ nuclei.

We noticed from these figures in which the back bending appearance, the moment of inertia ($2\mathcal{I}/\hbar^2$) increases with rotational energy squared $(\hbar\omega)^2$, but at certain angular momentum (L_c) the rotational energy squared $(\hbar\omega)^2$ decreases, while the moment of inertia ($2\mathcal{I}/\hbar^2$) increases, causing the back bending phenomena. This reason belonging to “*Coriolis antipiring effect*”. In this case the moment of inertia will effect on nuclear structure of these nuclei.

The figures in which no back bending appearance; this means that there is no effect of moment of inertia on nuclear structure of these nuclei. Such, that these nuclei are belonging to dynamical symmetry SU(3) as in $^{180}_{72}\text{Hf}_{108}$ nuclei.

Table (1): The corresponding parameters of (VMI, GVMI, VAVM) model for even-even nuclei.

Nuclei	band	parameter	\mathcal{A}_0/\hbar^2 (MeV) ⁻¹	C (MeV) ³	E_k (MeV)	Y (MeV)	O'	Standard deviation	χ^2
		Model							
⁵⁴ Fe ₃₀ O(6)	g	VMI	1.8100	0.0470	0.0000	—	1.7941	0.0709	0.0068
		GVMI	0.0280	0.044	0.0000	0.4680	51.78	0.0763	0.0075
		VAVM	24.1800	0.01000	0.0000	0.4280	-0.0141	0.0603	0.0054
	β_1	VMI	6.0000	5.0000	2.9417	—	0.0005	0.3956	0.1605
		GVMI	9.1200	7.8800	2.9417	1.0000	-0.0002	0.1666	0.0278
		VAVM	7.9000	18.0000	2.9417	0.0280	-0.0002	0.0464	0.0019
	β_2	VMI	13.0000	8.8000	3.5992	—	0.0003	0.1305	0.0132
		GVMI	54.0000	0.0020	3.5992	5.2000	-0.0298	0.0198	0.0003
		VAVM	10.1600	12.0000	3.5992	0.0016	-0.0001	0.0004	0.0000
	β_3	VMI	27.9000	0.08000	4.3002	—	0.0003	0.0529	0.0019
		GVMI	93.4000	0.0110	4.3002	3.9900	-0.0008	0.0010	0.000001
		VAVM	23.2000	11.0000	0.0096	4.3002	-0.00001	0.0007	0.00000
	γ_1	VMI	12.4000	2.0160	2.3800	—	0.00001	0.1960	0.0632
		GVMI	17.9000	3.9800	2.6575	0.8400	-0.00003	0.1879	0.05950
		VAVM	0.2800	0.0008	2.6460	0.00588	-0.00004	0.6688	0.6776

Table (1): To be continued (2/8).

Nuclei	band	parameter	\mathcal{A}_0/\hbar^2 (MeV) ⁻¹	C (MeV) ³	E_k (MeV)	Y (MeV)	O'	Standard deviation	χ^2
		Model							
⁵⁶ Fe ₃₀ O(6)	γ_2	VMI	6.4000	0.6920	3.0000	—	0.0028	0.1626	0.0233
		GVMI	83.0200	0.9900	3.3697	7.9960	-0.00003	0.1579	0.0198
		VAVM	8.2000	6.6000	3.2600	0.0020	-0.0005	0.1278	0.0137
	γ_3	VMI	8.8600	0.6900	3.5400	—	0.0010	0.0956	0.0070
		GVMI	99.8800	1.0000	3.7480	7.6000	-0.00001	0.0944	0.0066
		VAVM	11.0000	8.8800	3.6900	0.0010	-0.0002	0.0741	0.0042
	γ_4	VMI	8.8800	4.0900	3.6800	—	0.00002	0.0550	0.0022
		GVMI	85.0400	1.0000	3.8303	7.4800	-0.00002	0.0517	0.0019
		VAVM	11.0000	4.8800	3.7200	0.0360	-0.0003	0.0392	0.0011
⁷⁴ Se ₄₀ SU(5) - O(6) - SU(3)	g	VMI	1.0000	0.0090	0.0000	—	55.5556	0.0561	0.0089
		GVMI	3.1642	1.0600	0.0000	0.0710	0.0255	0.0186	0.0011
		VAVM	72.9200	0.4990	0.0000	0.3162	-0.00001	0.0071	0.0002
	β_1	VMI	6.5500	0.0452	0.8538	—	0.0394	0.0053	0.00006
		GVMI	4.6000	1.1880	0.8538	0.2220	0.0048	0.0171	0.0006
		VAVM	23.0000	0.2990	0.8538	0.2200	-0.0005	0.0194	0.0009
	β_2	VMI	7.5000	0.0110	2.1300	—	0.1077	0.1459	0.0272
		GVMI	5.7000	5.6000	2.1300	0.1600	0.0006	0.1358	0.0235
		VAVM	28.3200	0.4990	2.1300	0.1526	-0.0002	0.1232	0.0208

Table (1): To be continued (3/8).

Nuclei	band	parameter Model	a_0/h^2 (MeV) ⁻¹	C (MeV) ³	E_k (MeV)	Y (MeV)	O'	Standard deviation	χ^2
⁷⁴ Se ₃₀ SU(5) - O(6) - SU(3)	β_3	VMI	6.4400	0.0087	2.7180	-----	0.2152	0.1470	0.0211
		GVMi	12.7000	0.8000	2.7180	1.0000	-0.0006	0.0340	0.0012
		VAVM	10.5580	0.4990	2.7180	0.0505	-0.0034	0.0012	0.000001
	β_4	VMI	6.6000	0.0053	2.9180	-----	0.3281	0.0794	0.0058
		GVMi	4.4600	0.7000	2.9180	0.00006	0.0161	0.0152	0.0002
		VAVM	98.0000	0.0480	2.9160	0.2000	-0.00004	0.0395	0.0014
	γ_1	VMI	9.0000	0.0200	1.6600	-----	0.0343	0.3310	0.2229
		GVMi	20.0000	1.0000	1.7400	1.3700	-0.0002	0.1900	0.0683
		AVMV	74.0000	0.4400	0.8300	0.4250	-0.00001	0.1732	0.0714
⁸⁸ Zr ₄₈ SU(5)	g	VMI	1.0000	0.01000	0.0000	-----	50.0000	0.4674	0.9331
		GVMi	1.0000	0.2860	0.0000	0.0290	3.2934	0.2740	0.1656
		VAVM	99.8000	0.3288	0.0000	0.3720	-0.000006	0.4316	0.5778

Table (1): To be continued (4/8).

Nuclei	band	parameter Model	a_0/h^2 (MeV) ⁻¹	C (MeV) ³	E_k (MeV)	Y (MeV)	O'	Standard deviation	χ^2
⁸⁸ Zr ₄₈ SU(5)	β_1	VMI	9.2800	0.8900	1.5214	-----	0.0007	0.0170	0.0004
		GVMi	2.8000	0.4000	1.5214	0.0180	0.1098	0.3597	0.1828
		VAVM	48.0000	0.0088	1.5214	0.2200	-0.0020	0.1339	0.0257
	β_2	VMI	7.4400	0.9990	2.2250	-----	0.0012	0.0339	0.0013
		GVMi	3.0000	0.0820	2.2250	0.0320	0.4227	0.4908	0.2423
		VAVM	0.7000	0.00038	2.2250	0.1900	3836	0.0891	0.0088
¹⁰⁶ Pd ₅₀ O(6) - SU(5)	g	VMI	2.0000	0.0077	0.0000	-----	8.1169	0.7749	0.8603
		GVMi	2.2800	0.1000	0.0000	0.08200	0.7053	0.1395	0.0793
		VAVM	99.6000	0.1660	0.0000	0.2610	-0.00001	0.2968	0.2094
	β_1	VMI	6.2000	0.0090	1.1338	-----	0.2331	0.0419	0.0033
		GVMi	4.6500	1.0000	1.1338	0.0610	0.0087	0.0020	0.000007
		VAVM	91.0000	0.00238	1.1338	0.2138	-0.0011	0.0005	0.000000

Table (1): To be continued (5/8).

Nuclei	band	parameter Model	φ_0/\hbar^2 (MeV) ⁻¹	C (MeV) ³	E_k (MeV)	Y (MeV)	σ	Standard deviation	χ^2
¹⁰⁶ Pd ₅₀ O(6)-SU(5)	B ₂	VMI	4.2000	0.0020	1.7064	-----	3.3744	0.1636	0.0374
		GVMi	2.6000	0.0135	1.7064	0.0000	4.2145	0.0100	0.0134
		VAVM	99.6000	0.1860	1.7064	0.1590	-0.00001	0.1496	0.0316
	β ₃	VMI	5.6000	0.00019	2.0015	-----	14.9848	0.0882	0.0108
		GVMi	7.6000	0.0070	2.0015	0.0260	0.3085	0.0594	0.0046
		VAVM	99.6000	0.1860	2.0015	0.0900	-0.00001	0.0855	0.0098
	β ₄	VMI	5.0000	0.0002	2.2781	-----	19.9005	0.0013	0.000002
		GVMi	12.1700	0.0500	2.2781	0.0790	0.0093	0.0010	0.000001
		VAVM	99.6000	0.1860	2.2781	0.0742	-0.00001	0.0081	0.00008
	γ ₁	VMI	7.8000	1.9800	0.7700	-----	0.0005	0.0709	0.0094
		GVMi	21.5000	2.0000	1.0600	2.0560	-0.0001	0.0566	0.0074
		AVMV	29.0000	0.2000	0.4000	0.3500	-0.0004	0.0859	0.0137

Table (1): To be continued (6/8).

Nuclei	band	parameter Model	φ_0/\hbar^2 (MeV) ⁻¹	C (MeV) ³	E_k (MeV)	Y (MeV)	σ	Standard deviation	χ^2
¹⁵⁶ Gd ₇₂ SU(3)-SU(5)	m	VMI	32.7580	0.0025	0.0000	-----	0.0057	0.0079	0.0003
		GVMi	21.7600	0.0057	0.0000	0.3307	0.0057	0.0079	0.0003
		VAVM	94.4000	0.0040	0.0000	0.0534	-0.0006	0.0399	0.0098
	β ₁	VMI	40.8000	0.00223	1.0495	-----	0.0033	0.0133	0.0006
		GVMi	23.6000	1.0600	1.0495	0.2250	0.00004	0.0074	0.0002
		VAVM	98.9900	0.0640	1.0495	0.0384	-0.00003	0.0132	0.0007
	β ₂	VMI	30.6000	0.0016	1.1682	-----	0.0109	0.0213	0.0016
		GVMi	16.8800	0.01400	1.1682	0.2020	0.0088	0.0158	0.0010
		VAVM	99.2000	0.0740	1.1682	0.0583	-0.00003	0.0223	0.0017
	β ₃	VMI	46.0000	0.09988	1.7152	-----	0.0001	0.0234	0.0011
		GVMi	31.9000	0.0380	1.7152	0.3640	0.0002	0.0219	0.0009
		VAVM	68.0000	0.0740	1.7152	0.0200	-0.00008	0.0131	0.0004
	γ ₁	VMI	30.6000	0.0016	1.0660	-----	0.0109	0.0117	0.0007
		GVMi	52.2000	0.000356	1.1300	1.1080	-0.0240	0.0133	0.0007
		AVMV	81.0000	0.0040	1.0700	0.0460	-0.0009	0.0182	0.0014
	γ ₂	VMI	20.0000	0.0016	1.6000	-----	0.0391	0.1198	0.0337
		GVMi	47.0000	0.8000	1.7660	1.0000	-0.00001	0.0676	0.0102
		VAVM	62.0000	0.0080	1.6900	0.0520	-0.0010	0.0730	0.0119

Table (1): To be continued (7/8).

Nuclei	band	parameter	β_0, \hbar^2 (MeV) ⁻¹	C (MeV) ³	E _k (MeV)	Y (MeV)	σ	Standard deviation	χ^2
		Model							
¹⁸⁰ ₇₂ Er SU(3)	π	VMI	32.0540	0.01370	0.0000	—	0.0011	0.0002	0.00000
		GVMII	21.5800	0.0280	0.0000	0.3400	0.0011	0.0009	0.00001
		VAVM	70.4800	3.8800	0.0000	0.0490	-0.000001	0.0087	0.0005
	β_1	VMI	44.6000	0.8800	1.1073	—	0.0000	0.0228	0.0015
		GVMII	30.8000	1.8820	1.1073	0.3540	0.000005	0.0221	0.0014
		VAVM	73.6000	1.8800	1.1073	0.0256	-0.000003	0.0192	0.0011
	β_2	VMI	0.0600	0.0020	1.3157	—	1162	0.1643	0.0443
		GVMII	4.4000	0.4800	1.3157	0.0000	0.0244	0.1386	0.0301
		VAVM	99.8800	0.0800	1.3157	0.2000	-0.000002	0.1620	0.0426
	γ_1	VMI	0.9990	0.0003	1.0600	—	1671	0.05147	0.0095
		GVMII	42.2000	1.2800	1.1998	0.6800	-0.000004	0.0357	0.0049
		VAVM	99.0000	0.1801	1.1000	0.0640	-0.000001	0.0325	0.0039

Table (1): To be continued (8/8).

Nuclei	band	parameter	β_0, \hbar^2 (MeV) ⁻¹	C (MeV) ³	E _k (MeV)	Y (MeV)	σ	Standard deviation	χ^2
		Model							
¹⁸⁰ ₇₄ Yb SU(3)- O(6)	π	VMI	12.0000	0.0010	0.0000	—	0.2894	0.1126	0.0739
		GVMII	13.2400	9.0000	0.0000	0.2000	0.00003	0.0242	0.0081
		VAVM	57.3440	0.0096	0.0000	0.06420	-0.0011	0.0059	0.0004
	β_1	VMI	0.0400	0.0008	0.8820	—	9803	0.0768	0.0141
		GVMII	7.4000	0.4900	0.8820	0.2600	0.0024	0.1471	0.0535
		VAVM	98.0000	0.0640	0.8820	0.1420	-0.00003	0.0806	0.0156
	β_2	VMI	10.0000	0.0006	1.1530	—	0.8333	0.1086	0.0265
		GVMII	3.0000	0.0040	1.1530	0.0300	8.7037	0.5383	0.0057
		VAVM	99.6000	0.0740	1.1530	0.1158	-0.00003	0.0935	0.0179
	γ_1	VMI	20.0000	0.8800	0.6600	—	0.00007	0.0527	0.0096
		GVMII	30.4000	0.0080	0.7100	1.4500	-0.0084	0.0493	0.0077
		VAVM	70.0000	0.0001	0.4400	0.1600	-0.0583	0.0346	0.0038

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Table (1): The corresponding parameters of (VMI), GVMI, VAVM) model for even-even nuclei.

Nuclei	band	parameter Model	g_0/h^2 (MeV) ⁻¹	C (MeV) ³	E _k (MeV)	Y (MeV)	O'	Standard deviation	χ^2
⁵⁶ Fe ₃₀ O(6)	g	VMI	1.8100	0.0470	0.0000	—	1.7941	0.0709	0.0068
		GVMI	0.0280	0.044	0.0000	0.4680	51.78	0.0763	0.0075
		VAVM	24.1800	0.01000	0.0000	0.4280	-0.0141	0.0603	0.0054
	β_1	VMI	6.0000	5.0000	2.9417	—	0.0005	0.3956	0.1605
		GVMI	9.1200	7.8800	2.9417	1.0000	-0.0002	0.1666	0.0278
		VAVM	7.9000	18.0000	2.9417	0.0280	-0.0002	0.0464	0.0019
	β_2	VMI	13.0000	8.8000	3.5992	—	0.0003	0.1305	0.0132
		GVMI	54.0000	0.0020	3.5992	5.2000	-0.0298	0.0198	0.0003
		VAVM	10.1600	12.0000	3.5992	0.0016	-0.0001	0.0004	0.0000
	β_3	VMI	27.9000	0.08000	4.3002	—	0.0003	0.0529	0.0019
		GVMI	93.4000	0.0110	4.3002	3.9900	-0.0008	0.0010	0.000001
		VAVM	23.2000	11.0000	0.0096	4.3002	-0.00001	0.0007	0.00000
	γ_1	VMI	12.4000	2.0160	2.3800	—	0.00001	0.1960	0.0632
		GVMI	17.9000	3.9800	2.6575	0.8400	-0.00003	0.1879	0.05950
		VAVM	0.2800	0.0008	2.6460	0.00588	-0.00004	0.6688	0.6776

Table (1): To be continued (2/8).

Nuclei	band	parameter Model	\mathcal{G}_0/\hbar^2 (MeV) ⁻¹	C (MeV) ³	E _k (MeV)	Y (MeV)	O'	Standard deviation	χ^2	
⁵⁶ ₂₆ Fe ₃₀	γ_2	VMI	6.4000	0.6920	3.0000	—	0.0028	0.1626	0.0233	
		GVM	83.0200	0.9900	3.3697	7.9960	-0.00003	0.1579	0.0198	
		VAVM	8.2000	6.6000	3.2600	0.0020	-0.0005	0.1278	0.0137	
	O(6)	γ_3	VMI	8.8600	0.6900	3.5400	—	0.0010	0.0956	0.0070
			GVM	99.8800	1.0000	3.7480	7.6000	-0.00001	0.0944	0.0066
			VAVM	11.0000	8.8800	3.6900	0.0010	-0.0002	0.0741	0.0042
		γ_4	VMI	8.8800	4.0900	3.6800	—	0.00002	0.0550	0.0022
			GVM	85.0400	1.0000	3.8303	7.4800	-0.00002	0.0517	0.0019
			VAVM	11.0000	4.8800	3.7200	0.0360	-0.0003	0.0392	0.0011
⁷⁴ ₃₄ Se ₄₀	g	VMI	1.0000	0.0090	0.0000	—	55.5556	0.0561	0.0089	
		GVM	3.1642	1.0600	0.0000	0.0710	0.0255	0.0186	0.0011	
		VAVM	72.9200	0.4990	0.0000	0.3162	-0.00001	0.0071	0.0002	
	SU(5) - O(6) - SU(3)	β_1	VMI	6.5500	0.0452	0.8538	—	0.0394	0.0053	0.00006
			GVM	4.6000	1.1880	0.8538	0.2220	0.0048	0.0171	0.0006
			VAVM	23.0000	0.2990	0.8538	0.2200	-0.0005	0.0194	0.0009
		β_2	VMI	7.5000	0.0110	2.1300	—	0.1077	0.1459	0.0272
			GVM	5.7000	5.6000	2.1300	0.1600	0.0006	0.1358	0.0235
			VAVM	28.3200	0.4990	2.1300	0.1526	-0.0002	0.1232	0.0208

Table (1): To be continued (3/8).

Nuclei	band	parameter Model	ρ_0/h^2 (MeV) ⁻¹	C (MeV) ³	E _k (MeV)	Y (MeV)	O	Standard deviation	χ^2
⁷⁴ ₃₄ Se ₄₀	β_3	VMI	6.4400	0.0087	2.7180	—	0.2152	0.1470	0.0211
		GVMi	12.7000	0.8000	2.7180	1.0000	-0.0006	0.0340	0.0012
		VAVM	10.5580	0.4990	2.7180	0.0505	-0.0034	0.0012	0.000001
	β_4	VMI	6.6000	0.0053	2.9180	—	0.3281	0.0794	0.0058
		GVMi	4.4600	0.7000	2.9180	0.00006	0.0161	0.0152	0.0002
		VAVM	98.0000	0.0480	2.9160	0.2000	-0.00004	0.0395	0.0014
	γ_1	VMI	9.0000	0.0200	1.6600	—	0.0343	0.3310	0.2229
		GVMi	20.0000	1.0000	1.7400	1.3700	-0.0002	0.1900	0.0683
		AVMV	74.0000	0.4400	0.8300	0.4250	-0.00001	0.1732	0.0714
⁸⁸ ₄₀ Zr ₄₈	g	VMI	1.0000	0.01000	0.0000	—	50.0000	0.4674	0.9331
		GVMi	1.0000	0.2860	0.0000	0.0290	3.2934	0.2740	0.1656
		VAVM	99.8000	0.3288	0.0000	0.3720	-0.000006	0.4316	0.5778

Table (1): To be continued (4/8).

Nuclei	band	parameter Model	ρ_0/h^2 (MeV) ⁻¹	C (MeV) ³	E _k (MeV)	Y (MeV)	O'	Standard deviation	χ^2	
⁸⁸ Zr ₄₈	β_1	VMI	9.2800	0.8900	1.5214	—	0.0007	0.0170	0.0004	
		GVMi	2.8000	0.4000	1.5214	0.0180	0.1098	0.3597	0.1828	
		VAVM	48.0000	0.0088	1.5214	0.2200	-0.0020	0.1339	0.0257	
	SU(5)	β_2	VMI	7.4400	0.9990	2.2250	—	0.0012	0.0339	0.0013
			GVMi	3.0000	0.0820	2.2250	0.0320	0.4227	0.4908	0.2423
			VAVM	0.7000	0.00038	2.2250	0.1900	3836	0.0891	0.0088
¹⁰⁶ Pd ₆₀	g	VMI	2.0000	0.0077	0.0000	—	8.1169	0.7749	0.8603	
		GVMi	2.2800	0.1000	0.0000	0.08200	0.7053	0.1395	0.0793	
		VAVM	99.6000	0.1660	0.0000	0.2610	-0.00001	0.2968	0.2094	
	β_1	VMI	6.2000	0.0090	1.1338	—	0.2331	0.0419	0.0033	
		GVMi	4.6500	1.0000	1.1338	0.0610	0.0087	0.0020	0.000007	
		VAVM	91.0000	0.00238	1.1338	0.2138	-0.0011	0.0005	0.000000	

Table (1): To be continued (5/8).

Nucl ei	band	parameter Model	g_0/\hbar^2 (MeV) ⁻¹	C (MeV) ³	E _k (MeV)	Y (MeV)	O'	Standard deviation	χ^2	
¹⁰⁶ ₄₆ Pd ₆₀	B ₂	VMI	4.2000	0.0020	1.7064	—	3.3744	0.1636	0.0374	
		GVMi	2.6000	0.0135	1.7064	0.0000	4.2145	0.0100	0.0134	
		VAVM	99.6000	0.1860	1.7064	0.1590	-0.00001	0.1496	0.0316	
	β ₃	VMI	5.6000	0.00019	2.0015	—	14.9848	0.0882	0.0108	
		GVMi	7.6000	0.0070	2.0015	0.0260	0.3085	0.0594	0.0046	
		VAVM	99.6000	0.1860	2.0015	0.0900	-0.00001	0.0855	0.0098	
	O(6)-SU(5))	β ₄	VMI	5.0000	0.0002	2.2781	—	19.9005	0.0013	0.000002
			GVMi	12.1700	0.0500	2.2781	0.0790	0.0093	0.0010	0.000001
			VAVM	99.6000	0.1860	2.2781	0.0742	-0.00001	0.0081	0.00008
	γ ₁	VMI	7.8000	1.9800	0.7700	—	0.0005	0.0709	0.0094	
		GVMi	21.5000	2.0000	1.0600	2.0560	-0.0001	0.0566	0.0074	
		AVMV	29.0000	0.2000	0.4000	0.3500	-0.0004	0.0859	0.0137	

Nuclei	band	parameter Model	g_0/h^2 (MeV) ⁻¹	C (MeV) ³	E _k (MeV)	Y (MeV)	O	Standard deviation	χ^2
¹⁵⁶ ₆₄ Gd ₉₂ SU(3)- SU(5)	σ	VMI	32.7580	0.0025	0.0000	—	0.0057	0.0079	0.0003
		GVMi	21.7600	0.0057	0.0000	0.3307	0.0057	0.0079	0.0003
		VAVM	94.4000	0.0040	0.0000	0.0534	-0.0006	0.0399	0.0098
	β_1	VMI	40.8000	0.00223	1.0495	—	0.0033	0.0133	0.0006
		GVMi	23.6000	1.0600	1.0495	0.2250	0.00004	0.0074	0.0002
		VAVM	98.9900	0.0640	1.0495	0.0384	-0.00003	0.0132	0.0007
	β_2	VMI	30.6000	0.0016	1.1682	—	0.0109	0.0213	0.0016
		GVMi	16.8800	0.01400	1.1682	0.2020	0.0088	0.0158	0.0010
		VAVM	99.2000	0.0740	1.1682	0.0583	-0.00003	0.0223	0.0017
	β_3	VMI	46.0000	0.09988	1.7152	—	0.0001	0.0234	0.0011
		GVMi	31.9000	0.0380	1.7152	0.3640	0.0002	0.0219	0.0009
		VAVM	68.0000	0.0740	1.7152	0.0200	-0.00008	0.0131	0.0004
	γ_1	VMI	30.6000	0.0016	1.0660	—	0.0109	0.0117	0.0007
		GVMi	52.2000	0.000356	1.1300	1.1080	-0.0240	0.0133	0.0007
		AVMV	81.0000	0.0040	1.0700	0.0460	-0.0009	0.0182	0.0014
	γ_2	VMI	20.0000	0.0016	1.6000	—	0.0391	0.1198	0.0337
		GVMi	47.0000	0.8000	1.7660	1.0000	-0.00001	0.0676	0.0102
		VAVM	62.0000	0.0080	1.6900	0.0520	-0.0010	0.0730	0.0119

Table (1): To be continued (7/8).

Nucl ei	band	paramet er Model	g_0/h^2 (MeV ⁻¹)	C (MeV ³)	E_k (MeV)	Y (MeV)	O'	Standa rd deviati on	χ^2
¹⁸⁰ ₇₂ Hf ₁₀₈ SU(3)	g	VMI	32.0540	0.0137 0	0.0000	—	0.0011	0.0002	0.00000
		GVMi	21.5800	0.0280	0.0000	0.3400	0.0011	0.0009	0.00001
		VAVM	70.4800	3.8800	0.0000	0.0490	-0.000001	0.0087	0.0005
	β_1	VMI	44.6000	0.8800	1.1073	—	0.0000	0.0228	0.0015
		GVMi	30.8000	1.8820	1.1073	0.3540	0.000005	0.0221	0.0014
		VAVM	73.6000	1.8800	1.1073	0.0256	-0.000003	0.0192	0.0011
	β_2	VMI	0.0600	0.0020	1.3157	—	1162	0.1643	0.0443
		GVMi	4.4000	0.4800	1.3157	0.0000	0.0244	0.1386	0.0301
		VAVM	99.8800	0.0800	1.3157	0.2000	-0.00002	0.1620	0.0426
	γ_1	VMI	0.9990	0.0003	1.0600	—	1671	0.0514 7	0.0095
		GVMi	42.2000	1.2800	1.1998	0.6800	-0.000004	0.0357	0.0049
		VAVM	99.0000	0.1801	1.1000	0.0640	-0.00001	0.0325	0.0039

Table (1): To be continued (8/8).

Nuclei	band	parameter	g_0/\hbar^2 (MeV) ⁻¹	C (MeV) ³	E _k (MeV)	Y (MeV)	σ	Standard deviation	χ ²
		Model							
¹⁸⁶ ₇₄ W ₁₁₂	g	VMI	12.0000	0.0010	0.0000	—	0.2894	0.1126	0.0739
		GVMi	13.2400	9.0000	0.0000	0.2000	0.00003	0.0242	0.0081
		VAVM	57.3440	0.0096	0.0000	0.06420	-0.0011	0.0059	0.0004
	β ₁	VMI	0.0400	0.0008	0.8820	—	9803	0.0768	0.0141
		GVMi	7.4000	0.4900	0.8820	0.2600	0.0024	0.1471	0.0535
		VAVM	98.0000	0.0640	0.8820	0.1420	-0.00003	0.0806	0.0156
	β ₂	VMI	10.0000	0.0006	1.1530	—	0.8333	0.1086	0.0265
		GVMi	3.0000	0.0040	1.1530	0.0300	8.7037	0.5383	0.0057
		VAVM	99.6000	0.0740	1.1530	0.1158	-0.00003	0.0935	0.0179
	γ ₁	VMI	20.0000	0.8800	0.6600	—	0.00007	0.0527	0.0096
		GVMi	30.4000	0.0080	0.7100	1.4500	-0.0084	0.0493	0.0077
		VAVM	70.0000	0.0001	0.4400	0.1600	-0.0583	0.0346	0.0038

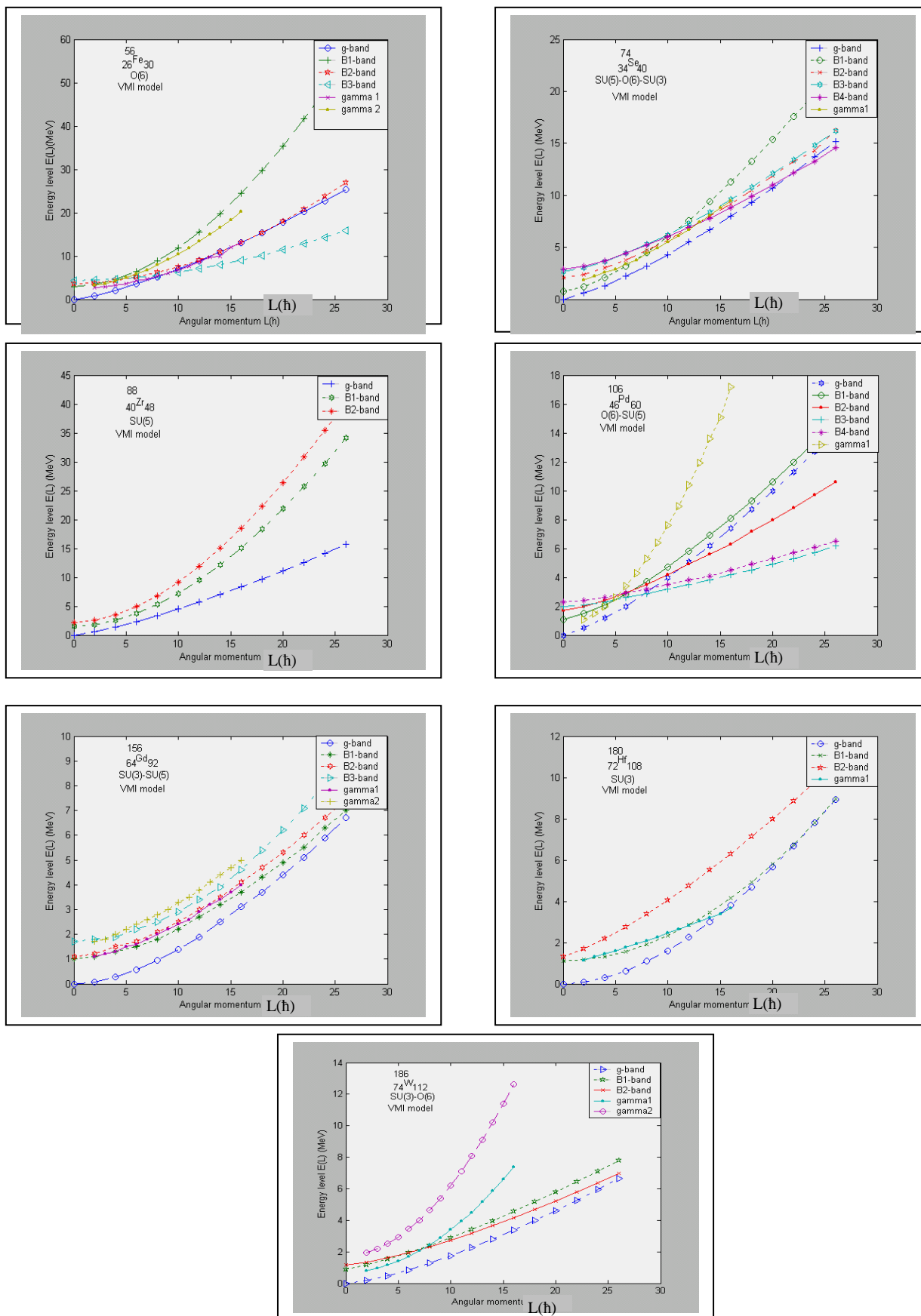


Figure (1): The energy band crossing $E(L)$ as a function of angular momentum L for chosen nuclei using (VMI) model. The parameters used for calculations are in table (1).

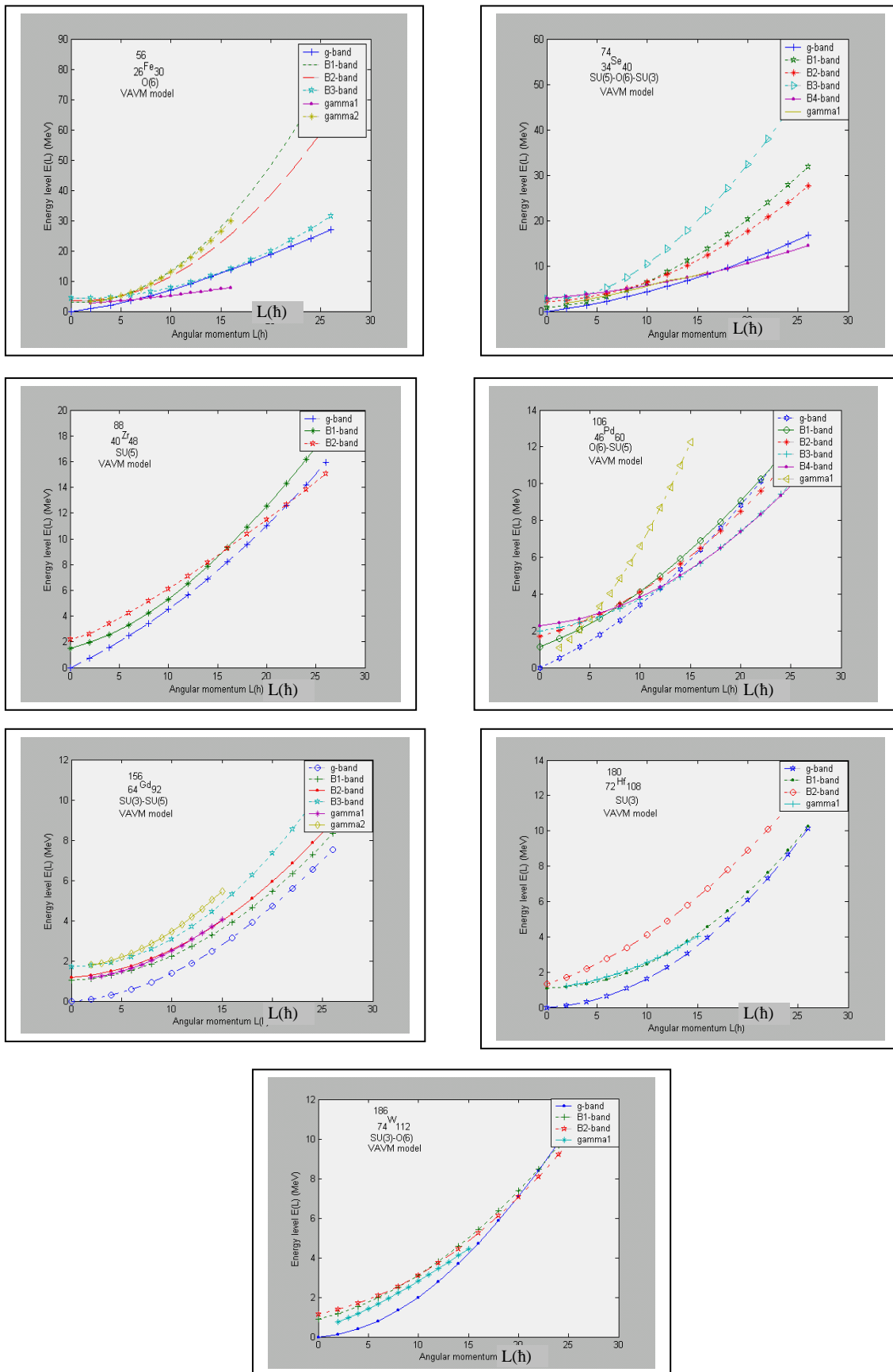
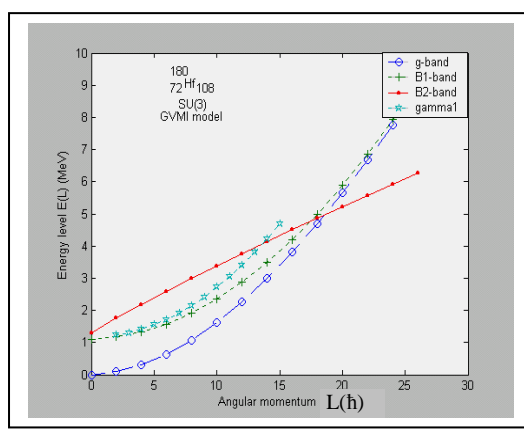
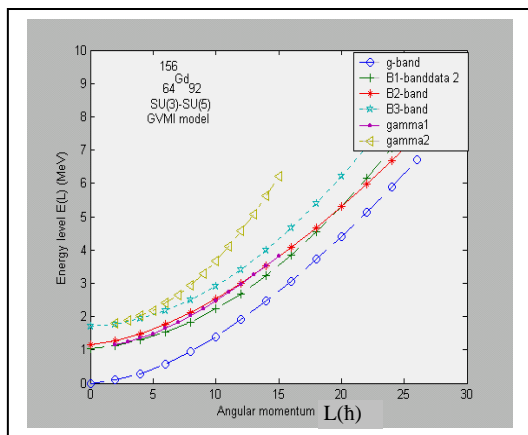
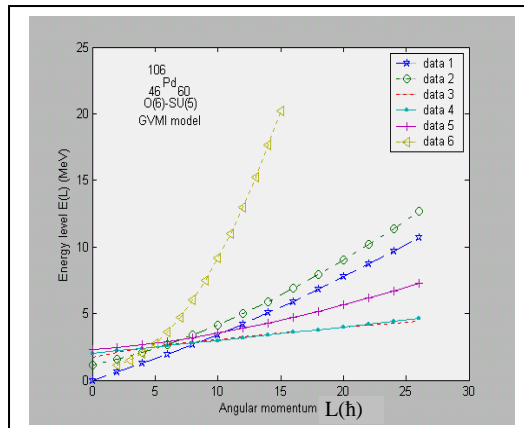
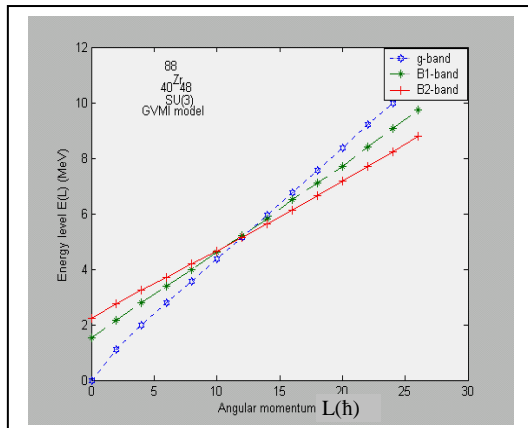
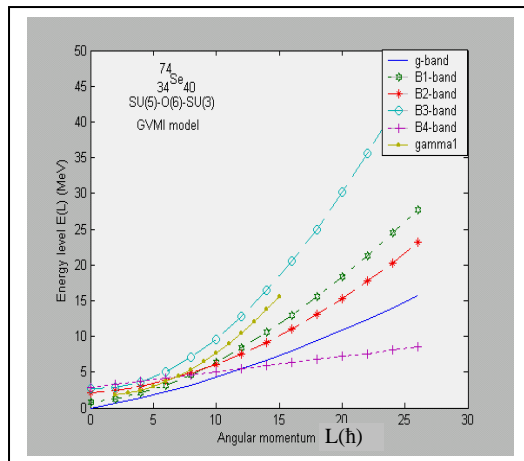
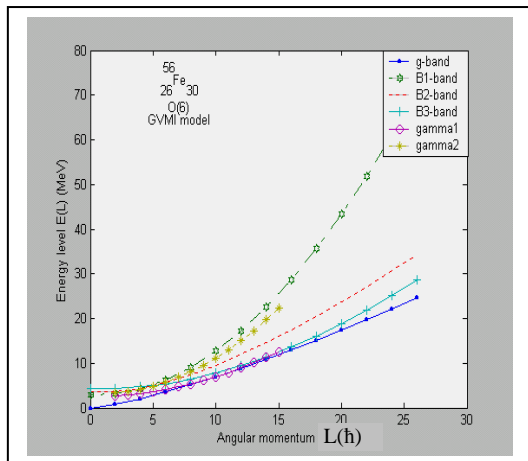


Figure (2): The energy band crossing $E(L)$ as a function of angular momentum L for chosen nuclei using (VAVM) model. The parameters used for calculations are in table (1).



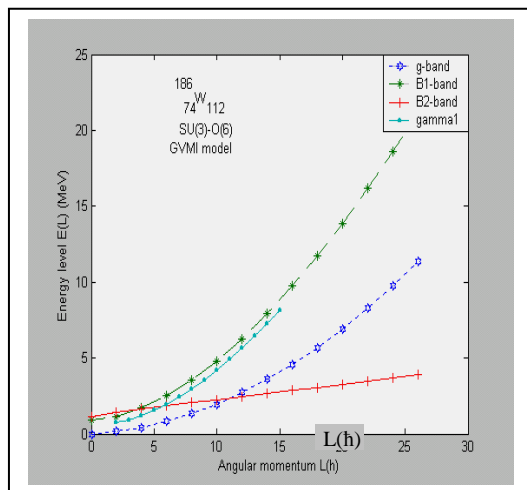


Figure (3): The energy band crossing $E(L)$ as a function of angular momentum L for chosen nuclei using (GVMI) model .The parameters used for calculations are in table (1) .

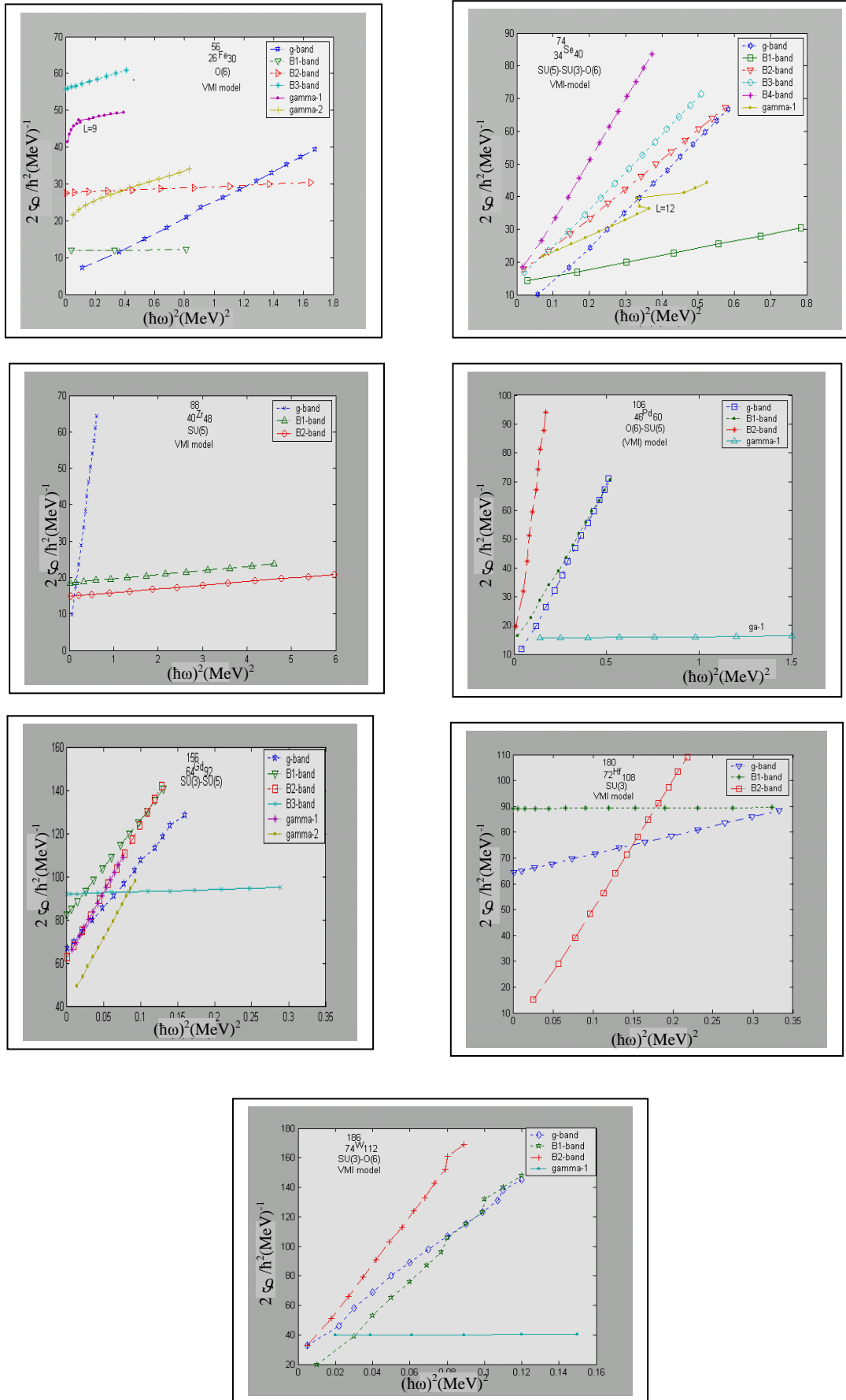


Figure (4): The back bending ($2\epsilon_0/\hbar^2$) versus $(\hbar\omega)^2$ plot for chosen nuclei using (VMI) model . The parameters used for calculations are in table (1).

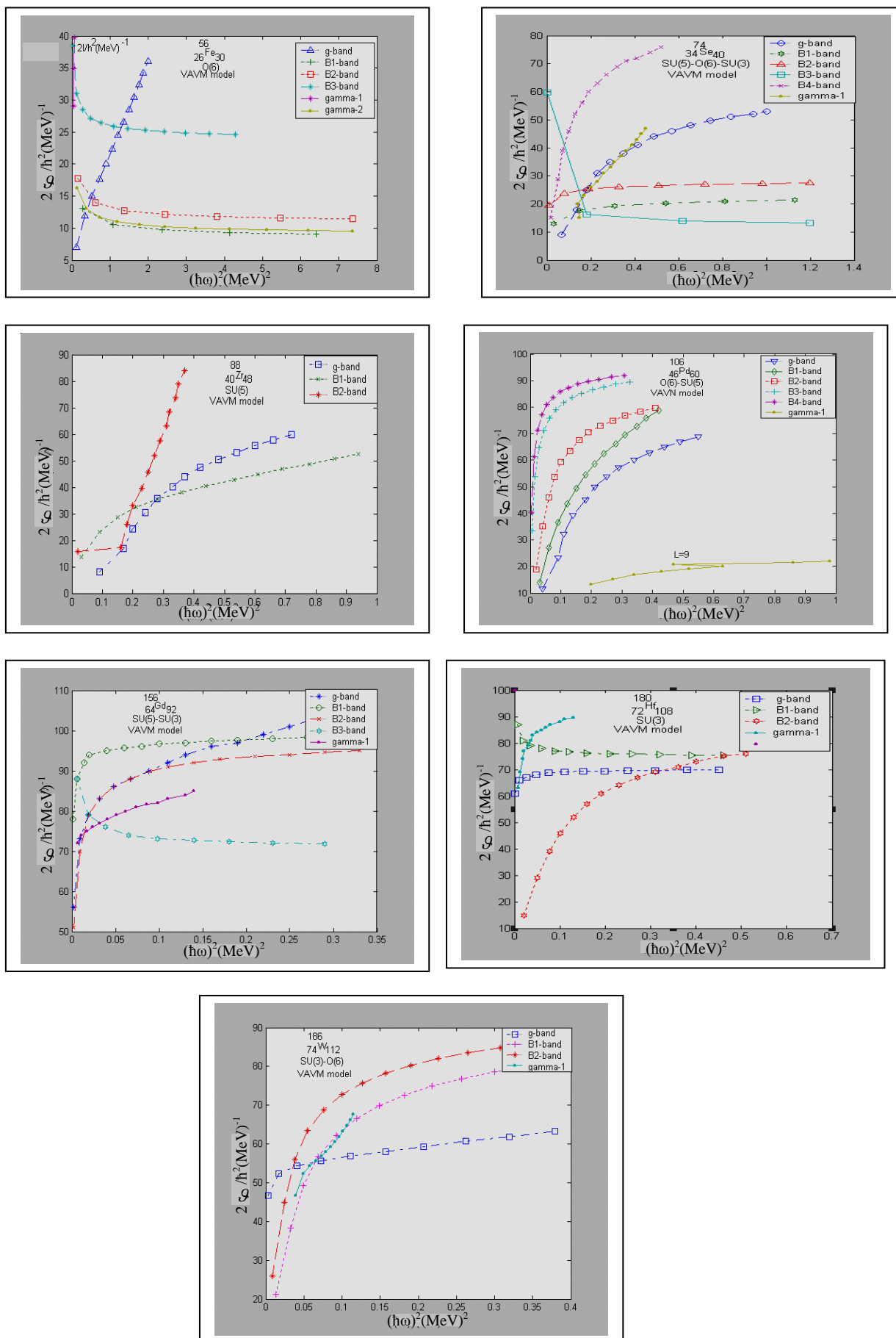


Figure (5): The back bending ($2\epsilon_0/\hbar^2$) versus $(\hbar\omega)^2$ plot for chosen nuclei using (VAVM) model. The parameters used for calculations are in table (1).

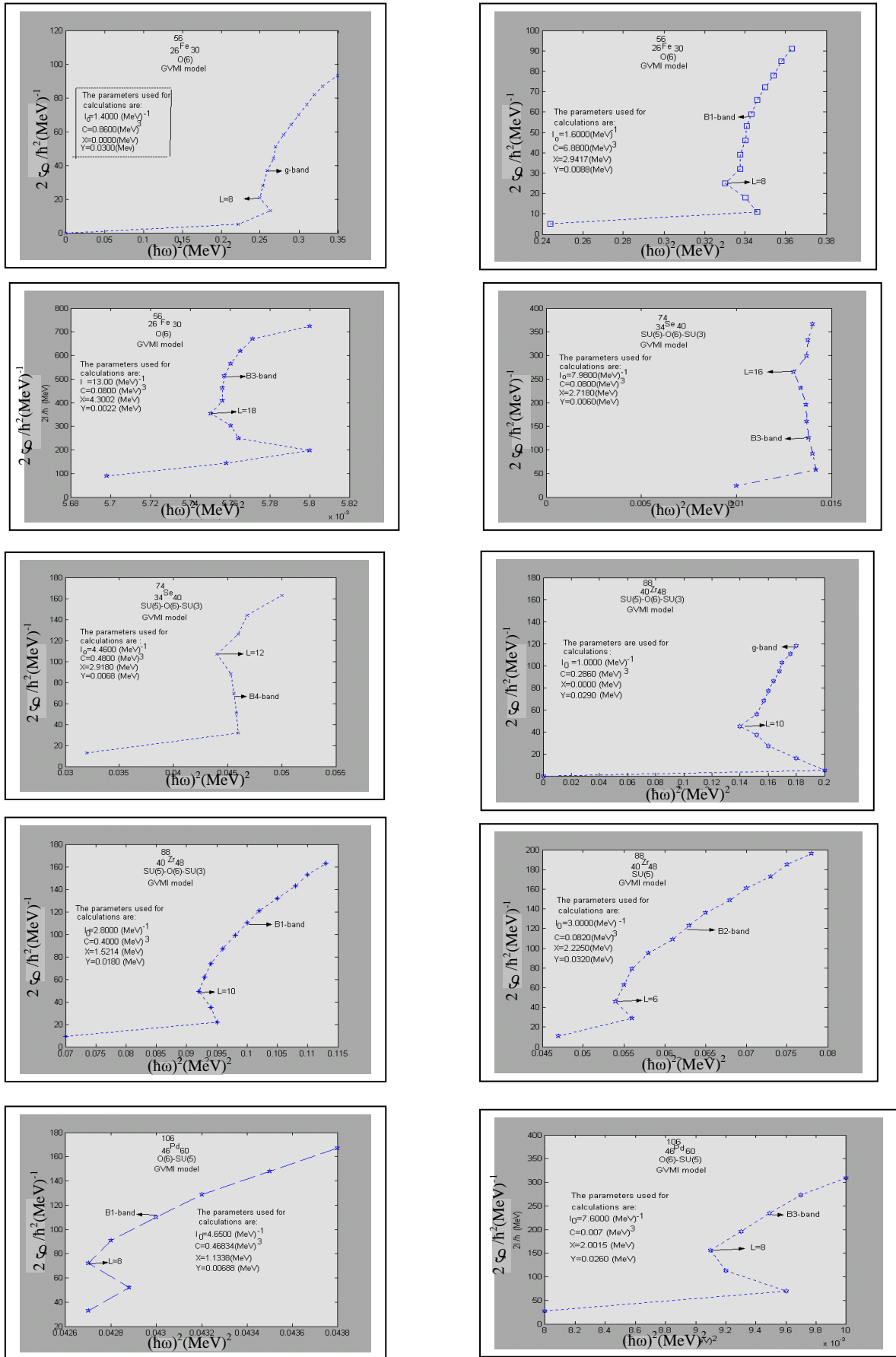


Figure (6): The back bending ($2\psi/h^2$) versus $(\hbar\omega)^2$ plot for chosen nuclei using (GVMI)model.

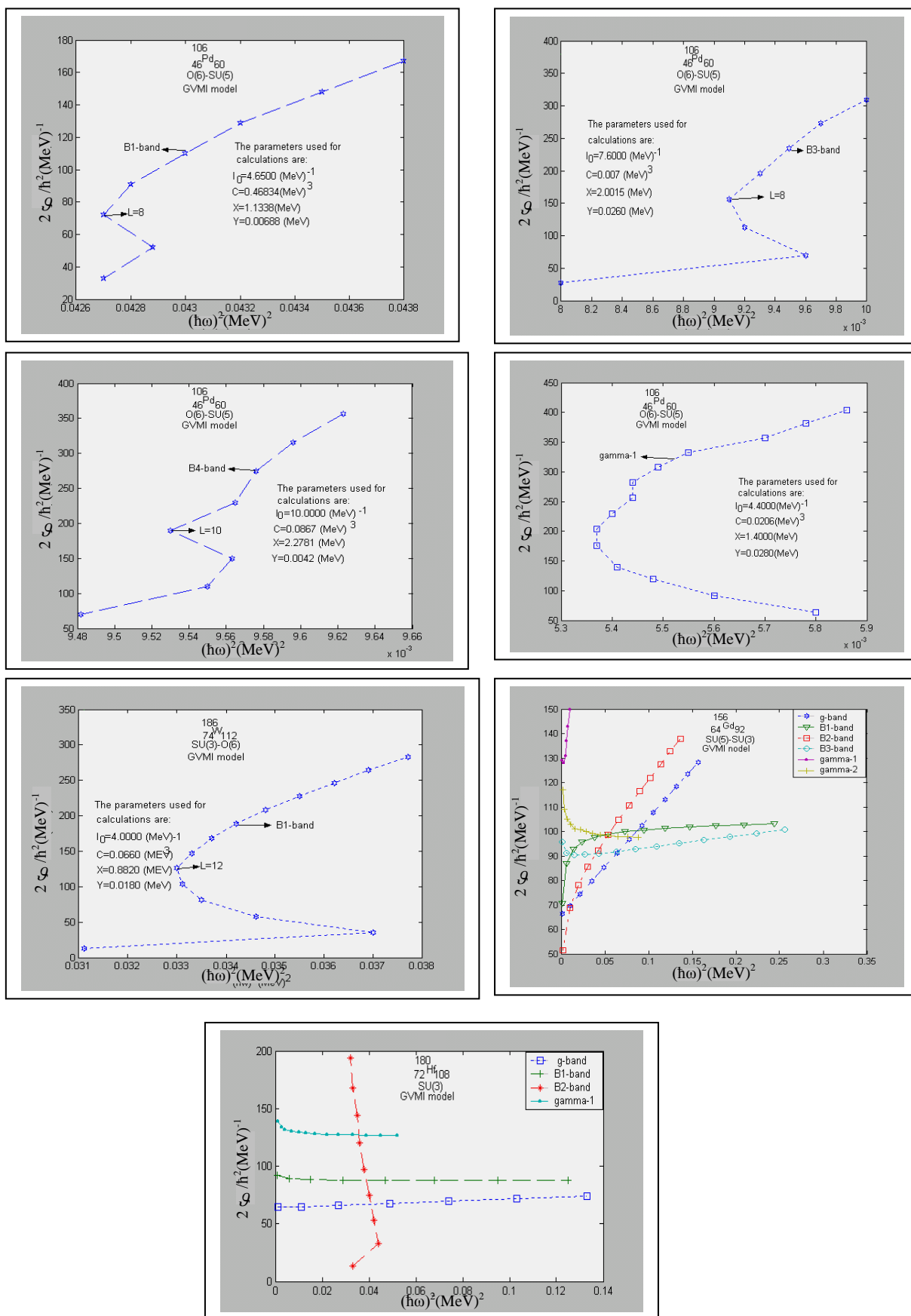


Figure (6): To be continued (2/2). The parameters used for calculations ^{156}Gd and ^{180}Hf nuclei are in table (1).