Study The Deformation Parameters (β_2, δ) For Even-Even Nuclei (A=38-48 & A=74-92)

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Abstract

The aim of the present work is to calculate the deformation parameters (β_2, δ)and its relation with neutrons magic number.

The relationship between the deformation parameters (β_2, δ) which is obtained by two different methods, first: nucleus quadrupole deformation parameter β_2 from reduced transition probability $B(E2)\uparrow$ for $0^+\rightarrow2^+$ transitions, second: nucleus quadrupole deformation parameter δ from quadrupole moment Q_o ,with neutrons magic number have been studied. These parameters have been calculated for even-even nuclei $_{20}$Ca $40\leq A\leq48$and $_{36}$Kr $74\leq A\leq92$ using deformed shell model equation.

From this study we shows that the deformation parameters decreased when we are closer to the neutrons magic number.

Key Words: transition probability $B(E2;0^+\rightarrow2^+)\uparrow$, electric quadrupole moment Q_o , mean-squared charge distribution radius $\langle r^2\rangle$, deformation parameters(β_2, δ).

دراسة معاملات التشوه (β_2, δ) للنوى زوجية- زوجية (A=38-48 & A=74-92)

الخلاصة: الهدف من هذه الدراسة هو حساب معاملات التشوه (β_2, δ) وعلاقتها مع الاعداد السحرية للنيوترونات.

حيث تم دراسة العلاقة بين معاملات التشوه (β_2, δ) والمحسوبة بطريقتين مختلفتين، الاولى: معامل تشوه رباعي القطب النووي β_2 بالاعتماد على احتمالية الانتقال $B(E2;0^+\rightarrow2^+)\uparrow$ والثانية: معامل تشوه رباعي القطب النووي δ بالاعتماد على عزم رباعي القطب، مع الاعداد السحرية للنيوترونات .

هذه المعاملات تم حسابها للنوى زوجية - زوجية $_{20}$Ca $40\leq A\leq48$ و$_{36}$Kr $74\leq A\leq92$ باستخدام نموذج القشرة المشوهة.

حيث تبين أن معاملات التشوه هذه تنخفض عندما تقترب عدد النيوترونات من العدد السحري للنيوترونات .

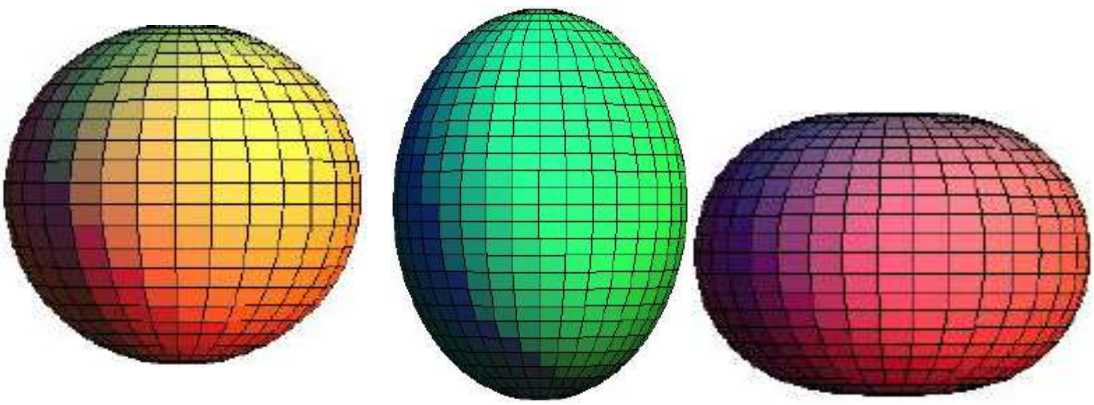
الكلمات المفتاحية: احتمالية الانتقال $B(E2;0^+\rightarrow 2^+)\uparrow$، عزم رباعي القطب الكهربائي Q_o ، معدل مربع توزيع الشحنة لنصف القطر $<r^2>$، معاملات التشوه (β_2, δ).

Introduction

The nucleus which has a spherical shape, lose its spherical shape and become deformed when the number of nucleons are not equal to magic number (2, 8, 20, 28, 50, 82, and 126), in other word, the nuclei which have magic numbers of nucleons (neutrons N and protons Z) it will be more stable [1].

The deformation in the nucleus originates because of the way valence nucleons order themselves in an unfilled shell, that's mean the deformation occurs only when both proton (P) and neutron (N) shells are partly filled [2].

The simplest deformations are called quadrupole deformation, in this type of deformation the nucleus can take either a prolate shape or oblate shape, as illustrated in fig. (1) [3].



Spherical **Prolate** **Oblate**

Figure (1): Diagram showing spherical, prolate and oblate shapes [4]

Theory

Nuclei are expected to be soft and flexible according to the liquid-drop model, i.e. the shape of a nucleus may be strongly deviate from a spherical shape. Experimentalists found many nuclei with a remarkably deformed charge distribution in large regions of neutron number (N) and proton number (P) far from the magic numbers [5].

The assumption of independent motion of nucleons in an average potential well is the theoretical basis of the shell model. For a particular nucleon, this potential well represents the nucleon's interaction with all other nucleons in the nucleus [5].

Quadrupole deformation parameters can be obtaining by two ways:

1- Quadrupole deformation parameter β_2 from reduced transition probability $B(E2)\uparrow$ for $0^+ \rightarrow 2^+$ transition.

The reduced electric quadrupole transition probability *B(E2)* between the 0^+ ground state and first-excited 2^+ state , represents basic nuclear information complementary to our knowledge of the energies of low–lying levels in the nuclides[6].

Generally the *B(E2)* values emphasize the widespread occurrence of quadruple distortion in nuclides which calculated from Global Beast Fit equation [6]:

$$B(E2)\uparrow = 2.6\, E_{\gamma_0}^{-1} Z^2 A^{-2/3} \quad \text{------------------ (1)}$$

Where:

E_{γ_0}: energy of the gamma ray transitions in KeV units.

Z: atomic number.

A: mass number of a nucleus.

From this equation we can shows that *B(E2)* values are basic on experimental quantities ($E\gamma_0$, Z, A) that do not depend on nuclear models. A quantity that, model dependent, is quite useful because of its easy visualization is the deformation parameter β_2. Assuming a nuclide with uniform charge distribution out to the distance $R(\theta, \emptyset)$ (the two polar angle) and zero charge beyond, β_2 *is* related to $B(E2)$ by the formula[6]:

$$\beta_2 = \frac{4\pi}{3ZR_o^2}\left[B(E2)\uparrow \frac{e^2\, b^2}{e^2}\right]^{1/2} \quad \text{-------------------- (2)}$$

Where:

R_o:is the average radius nuclear which can be obtained from the following equation:

$$R_o^2 = 0.0144\, A^{2/3}\, barn \quad \text{----------------------- (3)}$$

2- Quadrupole deformation parameter δ from nucleus quadrupole moment

The electric quadrupole moment Q_o, measures the deviation of the charge distribution from spherical symmetry. Therefore its measurement gives good information about the shape and deformation of the nucleus [7]. From quadrupole moment Q_o (degree of nucleus shape difference from spherical shape) we can be obtained quadrupole deformation parameter δ [8]:

$$\delta = 0.75 Q_o / (< r^2 > Z) \quad \text{---------------------- (4)}$$

Where:

$< r^2 >$: mean squared charge distribution radius which calculated by the following equations:

$$< r^2 >= 0.63\left[1.2A^{1/3}\right]^2 \quad , A\leq 100 \text{ ----------------------- } (5)$$

$$< r^2 >= \frac{\left[0.63\, R_O^2\, (1+\frac{10}{3}(\frac{\pi\, a_O}{R_O})^2\right]}{\left[1+(\frac{\pi a_O}{R_O})^2\right]} \quad , A>100 \text{------- } (6)$$

Where:

R_o: Parameters of radial Woods Saxon potential

$R_o=1.07A^{1/3}$ *fm* $a_o=0.55$ *fm*

Where: a_o is obtained from the data on fast electrons scattering.

The intrinsic quadrupole moment Q_o calculated by equation [6]:

$$Q_o = \left[\left(\frac{16\,\pi}{5}\right) \frac{B(E2)e^2 b^2}{e^2}\right]^{1/2} \text{-------------------- } (7)$$

Calculations and Results

In the present study, we have been calculated deformation parameters which are obtained by two different methods, nucleus quadrupole deformation parameter β_2 from reduced transition probability $B(E2)\uparrow$ for $0^+\rightarrow 2^+_1$ transitions and nucleus quadrupole deformation parameter δ from quadrupole moment Q_0 . These parameters have been calculated for even-even nuclei $_{20}$Ca 40≤A≤48 and $_{36}$Kr 74≤A≤92 using deformed shell model.

In addition, to these calculations we have been studied the relationship between two deformation parameters (β_2, δ) and comparison between them.

The calculations of deformation parameters (β_2, δ) were calculated as follows:

1-Transition Probabilities *B* (*E2*) ↑

The transition probability for a number of γ-transition from the energy levels of $_{20}$Ca and $_{36}$Kr nuclei have been calculated, using eq. (1), the results are presented in table (1) for $_{20}$Ca isotopes and in table (3) for $_{36}$Kr isotopes.

2-Quadrupole Moments Q_o

The intrinsic quadrupole moments Q_o have been calculated using eq. (7) for nuclides under the study. The results are presented in tables (2 & 4).

3-deformation parameters

3-1 Nucleus quadrupole deformation parameter β_2 from reduced transition probability *B(E2)*↑ for $0^+\rightarrow 2^+_1$ transitions

1- Mass number A and neutron number N are presented in columns 1 and 2 in tables (1, 2, 3 & 4).

2- Quadrupole deformation parameter β_2 for each element was obtained by using eq. (2). To applied this eq. we need to calculate the average radius of nucleus (R_o^2) using eq. (3).The result of β_2 have been presented in coulomb (7) in tables (1and 2) for ($_{20}$Ca and $_{36}$Kr) nuclei respectively.

3-2 Nucleus Quadrupole deformation parameters δ from quadrupole moment Q_o

Calculation of quadrupole deformation parameters δ were applied as follows:

1- Mean-squared charge distribution radius $<r^2>$ values which are used in eq.(4), and calculated from eq.(5) and(6), where eq.(5)used for calculated $<r^2>$ for nuclei with atomic number($A \leq 100$), while the eq.(6) used for nuclei with atomic number($A > 100$) .

2-The nucleus quadrupole deformation parameter δ has been calculated by using eq. (4),The results of quadrupole deformation parameter δ are presented in tables(2 & 4) ($_{20}$Ca and $_{36}$Kr) nuclide respectively.

4- Comparison between quadrupole deformation parameters (β_2, δ) obtained from two methods

Nucleus quadrupole deformation parameters *(β_2,δ)* obtained using two different methods which are described above, quadrupole deformation parameter β_2 from reduced transition probability *B(E2)*↑ and quadrupole deformation parameters δ from quadrupole moment Q_o ,were compared systematically. This comparison are explained in figures (1&2) for nuclides $_{20}$Ca $40 \leq A \leq 48$ and $_{36}$Kr $74 \leq A \leq 92$ respectively.

5- The relationship between two methods

To fined the relationship between quadrupole deformation parameter β_2 and quadrupole deformation parameters δ we calculated the ratio of mean values of δ to mean values of β_2 for each element as shown in table (5).

Table (1): Mass number A, neutron number N, transition gamma energy E_{γ_0} , reduced transition probabilities *B(E2)* $e^2b^2\uparrow$ and deformation parameters β_2 for $_{20}$Ca isotopes.

A	N	E_{γ_0} (KeV)[9]	Theoretical values [6]		Present work	
			B(E2)↑ e^2b^2 Adopted values	β_2	*B(E2)*↑ e^2b^2	β_2
38	18	2206	0.0096	0.125	0.0417	0.2628
40	20	3904.17	0.0099	0.123	0.0228	0.1878
42	22	1524.70	0.0420	0.247	0.0565	0.2861
44	24	1157.031	0.0470	0.253	0.0721	0.3133
46	26	1346	0.0182	0.153	0.0602	0.2780
48	28	3832.2	0.0095	0.106	0.0205	0.1577

Table (2): Mass number A, neutron number N, transition gamma energy E_{γ_0} , reduced transition probabilities *B(E2)* $e^2b^2\uparrow$ and deformation parameter β_2 for $_{36}$Kr isotopes

A	N	E_{γ_0} (KeV)[9]	Theoretical values [6]		Present work	
			B(E2)↑ e^2b^2 Adopted values	β_2	*B(E2)*↑ e^2b^2	β_2
74	38	455.8	0.840	0.419	0.419	0.296
76	40	424.0	0.824	0.409	0.442	0.299
78	42	454.97	0.633	0.352	0.405	0.281
80	44	616.6	0.370	0.265	0.294	0.236
82	46	776.517	0.223	0.2021	0.229	0.205
84	48	881.610	0.125	0.1489	0.199	0.188
86	50	1564.92	0.122	0.145	0.110	0.137
88	52	775.28	-	-	0.219	0.191
90	54	707.05	-	-	0.237	0.196
92	56	769	-	-	0.215	0.183

Table (3): Mass number A, neutron number N , transition gamma Energy E_{γ_0} , root mean square radii $<r^2>^{1/2}$, mean square radii $<r^2>$, quadrupole moment Q_o and deformation parameters δ for $_{20}$Ca isotopes .

A	N	E_{γ_0}(keV)[9]	Theoretical values		Present work			
			$<r^2>^{1/2}$ *fm*[10]	Q_o(b)[6]	$<r^2>$*fm*^{2}	$<r^2>^{1/2}$ *fm*	Q_o (b)	δ
38	18	2206	-	0.309	10.2540	3.2022	0.647	0.2368
40	20	3904.17	3.4776	0.314	10.6107	3.2574	0.478	0.1691
42	22	1524.70	3.4781	0.649	10.9615	3.3108	0.753	0.2576
44	24	1157.031	3.5179	0.687	11.3068	3.3626	0.851	0.2822
46	26	1346	3.4953	0.427	11.6468	3.4127	0.777	0.2502
48	28	3832.2	3.4771	0.30	11.9820	3.4615	0.454	0.1421

Table (4): Mass number A , neutron number N, transition gamma Energy E_{γ_0}, root mean square radii $<r^2>^{1/2}$, mean square radii $<r^2>$, quadrupole moment Q_o and deformation parameters δ for $_{36}$Kr isotopes.

A	N	E_{γ_0}(keV)[9]	Theoretical values		Present work			
			$<r^2>^{1/2}$ *fm*[10]	Q_o(b)[6]	$<r^2>$*fm*^{2}	$<r^2>^{1/2}$ *fm*	Q_o (b)	δ
74	38	455.8	4.1870	2.90	15.9903	3.9988	2.053	0.267
76	40	424.0	4.2020	2.878	16.2772	4.0345	2.110	0.270
78	42	454.97	4.2038	2.52	16.5615	4.0696	2.019	0.254
80	44	616.6	4.1970	1.93	16.8434	4.1041	1.720	0.212
82	46	776.517	4.1919	1.497	17.1230	4.1380	1.520	0.185
84	48	881.610	4.1884	1.121	17.4003	4.174	1.415	0.169
86	50	1564.92	4.1835	1.107	17.6754	4.2042	1.054	0.124
88	52	775.28	4.2171	-	17.9484	4.2366	1.486	0.172
90	54	707.05	4.2423	-	18.2193	4.2684	1.544	0.176
92	56	769	4.2724	-	18.4882	4.2998	1.470	0.165

Table (5): relationship between nucleus quadrupole deformation parameter β_2 from reduced transition probability *B(E2)*↑ and nucleus quadrupole deformation parameters δ from quadrupole moment Q_o.

Elements	The ratio between δ and β_2
$^{38-48}_{20}Ca$	0.9006
$^{72-92}_{36}Kr$	0.9011
The average value of (δ / β_2) =0.9009	

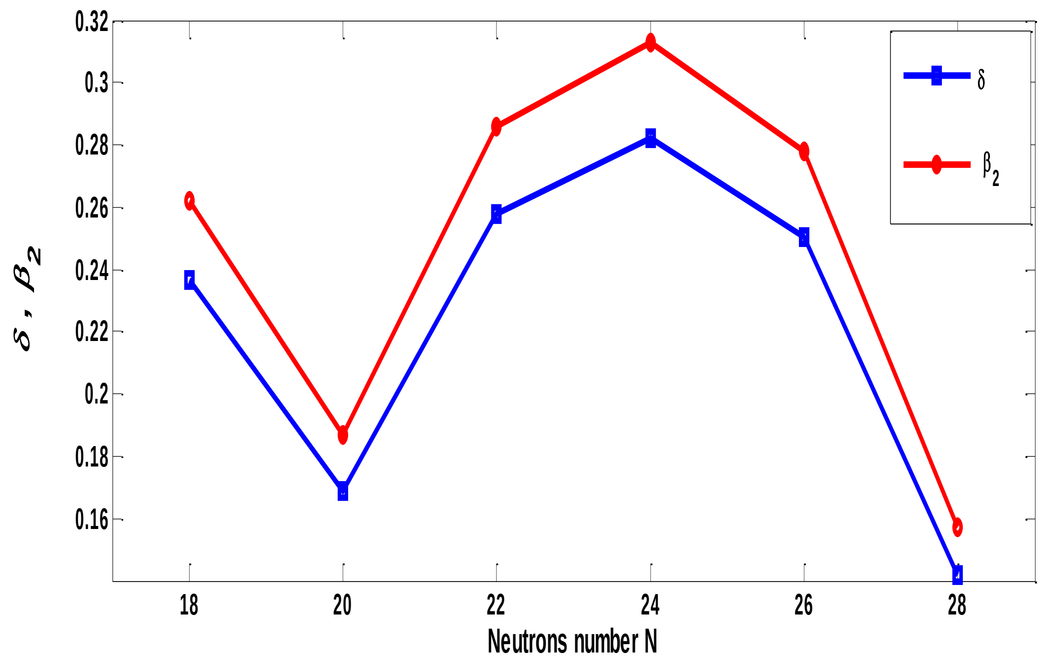


Figure (1): comparison between β_2 and δ values as function to neutron numbers for $_{20}$Ca isotopes.

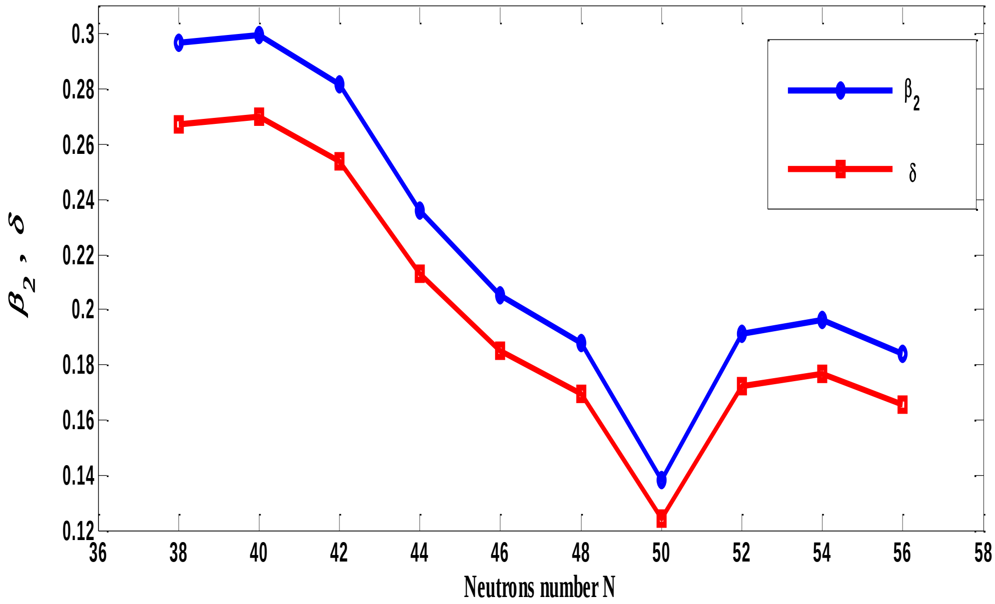


Figure (2): comparison between β_2 and δ values as function to neutron numbers for $_{36}$Kr isotopes.

Discussion

This study focused on the calculation of deformation parameters for even- even ($_{20}$Ca, $_{36}$Kr) isotopes nuclei because some of these nucleus have neutrons magic number (N=20, 28 for $_{20}$Ca nuclides and N=50 for $_{36}$Kr nuclides) .

The calculation of deformation parameters in this study have been calculated two different methods. The dependence of the first method of quadrupole deformation parameter β_2 dependence is on the reduced transition probability $B(E2)\uparrow$ while the second method of quadrupole deformation parameters δ dependence on the quadrupole moment Q_o.

For comparison purpose we plotted deformation parameters β_2 and δ as a function of neutron numbers which are explained in figures (1 and 2) for nuclei ($_{20}$Ca and $_{36}$Kr)respectively. From these figures we shows that the deformation parameters decreases when the neutrons number be closer to the neutrons magic number, on the other word, nuclei with neutron number (N) far from a magic number are generally deformed. It means that , nuclei with magic numbers of neutrons have a "closed shell" that encourages a spherical shape.

From tables (1and2) the results show clear different between theoretical values for $B(E2)\uparrow$ and β_2 [6] and that calculated in the present work, the reason of this difference comes from using Global Best Fit equation in the present work to calculate of the $B(E2)\uparrow$ which is using for calculating the deformation parameter β_2,while the reference data[6] represents adopted values of $B(E2)\uparrow$.

Figures (1&2) show the comparison between β_2 and δ for $_{20}$Ca, $_{36}$Kr isotopes respectively. It is very clear that the values of β_2 are larger than δ values for all isotopes, which means that the deformation which comes from transition probability $B(E2)\uparrow$ is larger than the deformation which comes from Q_o.

The calculation for root mean square radii $<r^2>^{1/2}$ and quadrupole moment Q_o have been checked by comparison with theoretical values of $<r^2>^{1/2}$ [10] and Q_o [6] for $_{20}$Ca and $_{36}$Kr nuclides.
From this comparison we can shows clearly a good agreement for $<r^2>^{1/2}$ values for $_{20}$Ca and $_{36}$Kr nuclides as shown in tables(3 and 4) respectively.

While, for Q_o values we can notice less agreement between calculated values and theoretical values for $_{20}$Ca nuclide ,and better agreement for calculated values and theoretical values for $_{36}$Kr isotopes.

Finally, from table (5) we can shows that two deformation parameters are connected by the relation (δ= 0.9009 β_2) which is shows very small difference between these two parameters.

For more explained we while discus each nuclei separately.

Isotopes of Calcium (Ca)

The results of deformation parameters β_2 and δ which is listed in the table (1and 3) respectively shows that the values of β_2 and δ in $^{38}_{20}Ca$, $^{42}_{20}Ca$,$^{44}_{20}Ca$ and $^{46}_{20}Ca$ isotopes be greater than these values in $^{40}_{20}Ca$ and $^{48}_{20}Ca$ isotopes this is due to the neutrons magic number (N=20,28) (which produce more stability for nuclide), because of this neutrons magic number the value of reduced transition probability *B(E2)*↑ and quadrupole moment Q_o will be less than other isotopes. These results have been explained in figure (1) which is shows the relationship between deformation parameters (β_2,δ) and neutrons number . Finally it can be noticed that there are two neutrons magic number of this nucleus which are 20 and 28.

Isotopes of krypton (Kr)

From tables (2 and 4), it can be shows that the deformation parameters (β_2,δ) decrease slightly with neutrons number increase until neutrons number equal to magic number (N=50) which mean closed shell, at this point the deformation parameters (β_2,δ) have minimum value (β_2=0.137, δ=0.124) and this indicates that low deformation can be occur in the neutrons magic number. At N> 50 deformation parameters (β_2,δ) increased with neutrons number increase, the maximum value of deformation parameters (β_2,δ) at (N=40) which means more deformation in shape .

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