The constriction of topological spaces from fuzzy topological spaces

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فى هذالبحث قمنابيناءانواع من الفضاءات التبولوجيه من فضاءات تبولوجيه ضبابية باستخدام تعريف ال(Support) للمجموعه الفضائية كما قمنا ببناء
(Fuzzy cofinite Topology)

Abstract: in this paper, we present the contraction
Of topological space from fuzzy topological space by using the support of a fuzzy set
Also we present the constration fuzzy cofinite topological space

Introduction: in order to exhibit an element $x \in X$

That typically belong to fuzzy set $\widetilde{A}$ we may demand it is membership value to be greater than some threshold $\alpha \in [0,1]$, for a recent collection of papers on fuzzy set theory, $\alpha -$level sets and it is applications one can see the issues published by change c.l , in 1968
“Fuzzy topological spaces” cheng and meng , H, 1985 “Fuzzy topological spaces” etc
Historically, the concept of fuzzy set theory was introduced by I.A Zadeh in 1965 “fuzzy sets” in which Zadeh is original definition to fuzzy sets is defined on a set X of points (objects) with generic element of X denoted by X
Thus a fuzzy set $\widetilde{A}$ is defined on X is characterized by a membership function $\mu_{\widetilde{A}}$ which associate to each $x \in X$ a real number $\mu_{\widetilde{A}}(x)$ in the interval [0,1].

1- fuzzy set

Definition (1.1): let X be non empty set, a fuzzy set $\tilde{A}$ in X is characterized by it is membership function $\mu_{\tilde{A}} : X \to I$, where I is the closed unite interval [0,1] and we write a fuzzy set by the set of points

$$\tilde{A} = \{(x, \mu_{\tilde{A}}(x)) : x \in X, 0 \le \mu_{\tilde{A}} \le 1\}$$

the collection of all fuzzy subsets in X will be denoted by I^x

i.e. $I^x = \{\tilde{A} : \tilde{A}$ is fuzzy subset of $X\}$

Definition (1.2): The support of a fuzzy set $\tilde{A}$ denoted by $S(\tilde{A})$ which is the crisp set of all $x \in X$ such that $\mu_{\tilde{A}}(x) > 0$

i.e. $S(\tilde{A}) = \{x \in X : \mu_{\tilde{A}}(x) > 0\}$

Remark (1.1): we list some concepts related to the Basic operations of fuzzy subset of X.

Let $\tilde{A}$ and $\tilde{B}$ be two fuzzy subsets of X with member ship functions $\mu_{\tilde{A}}(x)$ and $\mu_{\tilde{B}}(x)$ respectively then for all $x \in X$

1- $\tilde{A} \subseteq \tilde{B}$ if and only if $\mu_{\tilde{A}}(x) \le \mu_{\tilde{B}}(x)$

2- $\tilde{A} = \tilde{B}$ if and only if $\mu_{\tilde{A}}(x) = \mu_{\tilde{B}}(x)$

3- $\tilde{A}^C$ is the complement of $\tilde{A}$ with membership function $\mu_{\tilde{A}^C}(x) = 1 - \mu_{\tilde{A}}(x)$

4- $\tilde{A} = \tilde{\phi}$ (empty fuzzy set) if and only if $\mu_{\tilde{A}}(x) = 0$

5- $\tilde{C} = \tilde{A} \cap \tilde{B}$ if and only if $\mu_{\tilde{C}}(x) = \min\{\mu_{\tilde{A}}(x), \mu_{\tilde{B}}(x)\}$

6- $\tilde{D} = \tilde{A} \cup \tilde{B}$ if and only if $\mu_{\tilde{D}}(x) = \max\{\mu_{\tilde{A}}(x), \mu_{\tilde{B}}(x)\}$

Definition (1.3): a fuzzy set $\tilde{A}$ is called finite fuzzy set if and only if $S(\tilde{A})$ is finite set

Remark (1.2): we list some concepts related to the operation of support of $\tilde{A}$

1- $S(\tilde{A} \cap \tilde{B}) = S(\tilde{A}) \cap S(\tilde{B})$

2- $S\left(\bigcup_{i}^{n} \tilde{A}_i\right) = \bigcup_{i}^{n} S(\tilde{A}_i)$

3- $[S(\tilde{A})]^C \subseteq S(\tilde{A}^C)$

2- fuzzy topology

Definition (2.1): fuzzy topology is a family $\tilde{\tau}$ of fuzzy sets on X which satisfying the following condition

1- $\tilde{\phi}, X \in \tilde{\tau}$

2- $\tilde{A}, \tilde{B} \in \tilde{\tau}$, then $\tilde{A} \cap \tilde{B} \in \tilde{\tau}$

3- If $\tilde{A}_i \in \tilde{\tau}$ for each $i \in I$, then $\left(\bigcup_{i} \tilde{A}_i\right) \in \tilde{\tau}$

$\tilde{\tau}$ is called a fuzzy topology for X , and every member of $\tilde{\tau}$ is called a $\tilde{\tau}$ - open fuzzy set

A fuzzy set is called $\tilde{\tau}$ - closed fuzzy set if and only if its complement is $\tilde{\tau}$ - open fuzzy set

Example (2.1): let $X = \{a, b, c\}$

$\tilde{A} = \{(a,0.5),(b,0.6),(c,0.4)\}$

$\tilde{B} = \{(a,1),(b,0.4),(c,0.7)\}$

$\tilde{C} = \tilde{A} \cup \tilde{B} = \{(a,1),(b,0.6),(c,0.7)\}$

$$\tilde{D} = \tilde{A} \cap \tilde{B} = \{(a,0.5),(b,0.4),(c,0.7)\}$$

Then

$$\tilde{\tau}_1 = \{\tilde{\phi}, \tilde{A}, X\}$$
$$\tilde{\tau}_2 = \{\tilde{\phi}, \tilde{B}, \tilde{C}, X\}$$
$$\tilde{\tau}_3 = \{\tilde{\phi}, \tilde{A}, \tilde{B}, \tilde{C}, \tilde{D}, X\}$$

Are fuzzy topology

Example (2.2): let $X = \{a, b, c, d\}$

$$\tilde{A} = \{(a,0.7),(b,0.3),(c,1)\}$$
$$\tilde{\tau}_1 = \{\tilde{\phi}, \tilde{A}, X\}$$

Then

$$\tilde{A}^C = \{(a,0.3),(b,0.7)\}$$
$$\tilde{B}^C = \{(a,1),(b,0.5),(c,0.7)\}$$

Are $\tilde{\tau}$ - close fuzzy set

Theorem (2.1): let X be any infinite set and

$$\tilde{\tau} = \{\tilde{A} : \tilde{A}^C \text{ is fuzzy finite set}\} \cup \{\tilde{\phi}, X\}$$

Then $(X, \tilde{\tau})$ is fuzzy topology and is called fuzzy cofinite topology

proof :

1- if $\tilde{A}, \tilde{B} \in \tilde{\tau}$ then $\tilde{A} \cap \tilde{B} \in \tilde{\tau}$

Since $\tilde{A} \in \tilde{\tau}$ then $\tilde{A}^c$ is fuzzy finite , that is $S(\tilde{A}^c)$ is finite

also $\tilde{B} \in \tilde{\tau}$ then $\tilde{B}^c$ I s fuzzy finite , that is $S(\tilde{B}^c)$ is finite

since $(\tilde{A} \cap \tilde{B})^c = \tilde{A}^c \cup \tilde{B}^c$ and $S(\tilde{A}^c \cup \tilde{B}^c) = S(\tilde{A}^c) \cup S(\tilde{B}^c)$

$\therefore$ $S(\tilde{A}^c) \cup S(\tilde{B}^c)$ is finite

So we get $(\tilde{A} \cap \tilde{B})^c$ is fuzzy finite

$\therefore$ $\tilde{A} \cap \tilde{B} \in \tilde{\tau}$

2- if $\tilde{A}_i \in \tilde{\tau}$ then $\bigcup \tilde{A}_i \in \tilde{\tau}$

Since $\tilde{A}_i \in \tilde{\tau}$ then $\tilde{A}_i^c$ is fuzzy finite , that is $S\left(\tilde{A}_i^c\right)$ is finite

and $\left(\bigcup \tilde{A}_i\right)^c = \bigcap \tilde{A}_i^c$ and $S\left(\bigcap \tilde{A}_i^c\right) = \bigcap S\left(\tilde{A}_i^c\right)$

$\therefore \bigcap S\left(\tilde{A}_i^c\right)$ is finite

So we get $\left(\bigcup \tilde{A}_i\right)^c$ is fuzzy finite

$\therefore \bigcup \tilde{A}_i \in \tilde{\tau}$

Then from 1 and 2 we get $(X, \tilde{\tau})$ is fuzzy topological space

3- **The construction of topology spaces from fuzzy topological spaces**

<u>Theorem (3.1)</u>: let X be any set and $(X, \tilde{\tau})$ be indiscrete fuzzy Topological space then for each $\tilde{A} \in \tilde{\tau}$ we defined $\tau = \{S(\tilde{A}) : \tilde{A} \in \tilde{\tau}\}$ then (X, τ) is indiscrete topological Space

<u>Proof</u>: since $(X, \tilde{\tau})$ is indiscrete fuzzy topological space then $\tilde{\tau} = \{\tilde{\phi}, X\}$ only

then $S(\tilde{\phi}) = \phi$ and $S(X) = X$

$\therefore \tau = \{\phi, X\}$ only

$\therefore (X, \tilde{\tau})$ is indiscrete topological space

<u>Theorem (3.2)</u> : let X be any set and $(X, \tilde{\tau})$ be discrete fuzzy topological space then for each $\tilde{A} \in \tilde{\tau}$ we defined $\tau = \{S(\tilde{A}) : \tilde{A} \in \tilde{\tau}\}$ then (X, τ) is discrete topological Space

Proof: to show that $(X, \tau)$is discrete topological Space we must show that for each $x \in X$ then $\{x\} \in \tau$

Since $(X,\tilde{\tau})$ is discrete fuzzy topological space
then $\exists\tilde{A}\in\tilde{\tau}$ of the form

$$\mu_{\tilde{A}}(y) = 1 \quad \text{if} \quad y = x$$
$$= 0 \quad \text{if} \quad y \neq x$$

Then $S(\tilde{A}) = \{x\}$ only
$\therefore \{x\}\in\tau$

then (X,τ) is discrete topological space

Theorem (3.3): let X be any set and $(X,\tilde{\tau})$ be fuzzy topological space then for each $\tilde{A}\in\tilde{\tau}$ we defined $\tau=\{S(\tilde{A}): \tilde{A}\in\tilde{\tau}\}$ then (X,τ) is topological space

Proof: to show that (X,τ) is topological space

1- $\phi, X\in\tau$
Since $\tilde{\phi}\in\tau$ then $S(\tilde{\phi})=\phi$
$\therefore \phi\in\tau$
And also $X\in\tilde{\tau}$ then $S(X)=X$
$\therefore X\in\tau$

2- Let G,H be two open sets in τ We must show that $G\bigcap H\in\tau$

Since $G\in\tau$ then $\exists\tilde{A}\in\tilde{\tau}$ such that $G=S(\tilde{A})$
and $H\in\tau$ then $\exists\tilde{B}\in\tilde{\tau}$ such that $H=S(\tilde{B})$
Since $\tilde{A},\tilde{B}\in\tilde{\tau}$ then $\tilde{A}\bigcap\tilde{B}\in\tilde{\tau}$
$\therefore S(\tilde{A}\bigcap\tilde{B})\in\tau$
Since $G\bigcap H=S(\tilde{A})\bigcap S(\tilde{B})=S(\tilde{A}\bigcap\tilde{B})\in\tau$
$\therefore G\bigcap H\in\tau$

3– Let G_i be any open set in τ We must show that

$$\bigcup_i G_i \in \tau$$

Since $G_i \in \tau$ then $\exists \tilde{A}_i \in \tilde{\tau}$ such that $G_i = S(\tilde{A}_i)$

Since $\tilde{A}_i \in \tilde{\tau}$ then $\left(\bigcup_i \tilde{A}_i\right) \in \tilde{\tau}$

So we get $S(\tilde{A}_i) \in \tau$

Since $G_i = S(\tilde{A}_i)$

$$\therefore \bigcup_i G_i = \bigcup_i S(\tilde{A}_i) = S\left(\bigcup_i \tilde{A}_i\right)$$

Since $S\left(\bigcup_i \tilde{A}_i\right) \in \tau$

$$\therefore \bigcup_i G_i \in \tau$$

then from 1,2 and 3 we get (X, τ) is topological space

Theorem (3.4): let X be any infinite set and $(X, \tilde{\tau})$ is fuzzy cofinite topological space then for each $\tilde{A} \in \tilde{\tau}$ we defined $\tau = \{S(\tilde{A}) : \tilde{A} \in \tilde{\tau}\}$
then (X, τ) is cofinite topological space

Proof: to show that (X, τ) is cofinite topological

Space we must show that for each $G \in \tau$ emplace that G^C is finite
Since $G \in \tau$ then $\exists \tilde{A} \in \tilde{\tau}$ such that $G = S(\tilde{A})$
Since $\tilde{A} \in \tilde{\tau}$ then $\tilde{A}^C$ is fuzzy finite set
that is $S(\tilde{A})$ is finite set
Since $G = S(\tilde{A})$

$$\therefore G^C = [S(\tilde{A})]^C \subseteq S(\tilde{A}^C)$$

$$\therefore G^C \subseteq S(\tilde{A}^C)$$

$\therefore\ G^C$ is finite

So we get (X,τ) is cofinite topological space

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