Estimation of Right Truncated Laplace Distribution, Simulation Study

م.م احمد جاسم خميس
وزارة التربية / المديرية العامة لتربية بغداد الرصافة /٣
a.jasem92@yahoo.com

ABSTRACT

this papers aim to parameter right truncated Laplace distribution estimate by some estimation methods (MOM, MM, PER and MLE), Simulation experiments was used for comparison between each of the estimators methods to obtain the best method to estimate the parameter. The simulation is to generate random data follow the distribution of Right truncated Laplace Distribution on three models of the real values of the parameter $A_1(\beta = 1)$, $A_2(\beta = 0.5)$, $A_3(\beta = 3.4)$, and with samples size (n=10, 25, 50, 75, 100) and sample iteration (N=1000). Comparisons have been made between the obtained results by using two creteriar, which they are estimation (MSE) and (MAPE), the results are given in tables for comparison purpose. The results indicate that:-In all the samples sizes, the (mom) method is the best of the other methods.

Key Words: Right Truncated Laplace Distribution, Estimation Methods

المستخلص

يهدف البحث إلى تقدير معلمة توزيع لابلاس الايمن المبتور من خلال استخدام بعض طرائق التقدير (MOM,MM,PER and MLE) واستخدم اسلوب المحاكاة للمقارنة بين كل طرق المقدرات للحصول على افضل طريقة لتقدير معلمة التوزيع وتمت المحاكاة بتوليد بيانات عشوائية تتبع توزيع لابلاس الايمن المبتور، بإخذ ثلاث حالات للقيم الحقيقية لمعلمة التوزيع $A_1(\beta = 1)$, $A_2(\beta = 0.5)$, $A_3(\beta = 3.4)$ ، وبحجوم عينة مختلفة (n=10, 25, 50, 75, 100) ، وبتكرار حجم العينة (N=1000)، تمت المقارنة بين النتائج المستحصلة من طرائق التقدير باستخدام معيارين متوسط مربعات الخطأ (MSE) ومتوسط مطلق الخطأ (MAPE) وقد وضعت النتائج في جداول اعدت لذلك، وقد وجدنا ان افضل تقدير لمعلمة التوزيع هي: (mom)

1 .Introduction

The literal meaning of truncation is to “shorten” or “cut-off” something. Extending this definition to our world of situations, we can define the truncation of a distribution as a process, which results in certain values being “cut-off”, there by resulting in a “shorted” distribution[9]. Statistical inference can be subdivided into two groups, first of them is estimation and the other is hypothesis testing one of the most important estimation is the point estimation of unknown parameter, The process of estimation is one of corners tones in the process of statistical inference, which is known as “process” which is made of preliminary conclusions about community on the basis of the results extracted from the sample drawn for the point estimation used in this thesis. The important branches of estimation, which is known as the “finding the numerical value” and used to estimate the parameter of the community of the samples community is almost the characteristics of the original community, in which the samples are drawn[2].

Let X be a random variable of the laplace distribution, the probability density function :[11]

$$f(x\backslash\mu,\beta) = \frac{1}{2\beta}\, e^{-\left|\frac{x-\mu}{\beta}\right|} \qquad \ldots(1)$$

Where:

x : is a value of random variable X, and $-\infty < x < \infty$

μ : is the location parameter, and $-\infty < \mu < \infty$

β : is the scale parameter, and $\beta > 0$.

and cumulative distribution function for the laplace distribution:

$$F(x\backslash\mu,\beta) = \begin{cases} 1 - \frac{1}{2}\, e^{-\left(\frac{\mu-x}{\beta}\right)} & for \quad x \geq \mu \\ \frac{1}{2}\, e^{-\left(\frac{x-\mu}{\beta}\right)} & for \quad x < \mu \end{cases} \qquad \ldots(2)$$

2. Right-Truncated Laplace Distribution

Let g be a random variable of the right truncated Laplace distribution, the probability density function (p.d.f) for general truncated distribution as the following formula[12]:

$$\boldsymbol{g(x)} = \frac{\boldsymbol{f(x)}}{\boldsymbol{F(b)-F(a)}} \qquad \ldots(3)$$

Where:

$\boldsymbol{x}$: value of the random variable , $a \leq \boldsymbol{x} < b$.

$\boldsymbol{f(x)}$: is the (p.d.f) of the distribution.

F(a): is the (c.d.f) value of the lower bound of truncation interval (a).

F(b): is the (c.d.f) value of the upper bound of truncation interval (b).

If a = $-\infty$ and b =0, then the equation (3) become:

$$\boldsymbol{g(x)} = \frac{\boldsymbol{f(x)}}{\boldsymbol{F(0)-F(-\infty)}} \qquad \ldots(4)$$

From equation (2) for Laplace distribution with location parameter (μ) and scale parameter (β), we get:

$$\mathrm{F}(-\infty) = \frac{1}{2}\, e^{-\frac{(\mu+\infty)}{\beta}}$$
$$= 0$$
$$\mathrm{F}(0) = \frac{1}{2}\, e^{-\frac{(\mu-0)}{\beta}}$$
$$= \frac{\mathbf{1}}{\mathbf{2}}\, \boldsymbol{e}^{-\frac{\boldsymbol{\mu}}{\boldsymbol{\beta}}}$$

Then:

$$\mathrm{F}(0) - \mathrm{F}(-\infty) = \frac{1}{2}\, e^{-\frac{\mu}{\beta}} \qquad \ldots(5)$$

We take one sided from equation (1) which is left Laplace distribution we get:

$$f(x) = \frac{e^{-\left(\frac{\mu-x}{\beta}\right)}}{2\beta} \qquad \ldots(6)$$

By substituting equations (5) and (6) in equation (3), we obtain the (p.d.f) for right truncated Laplace Distribution:

$$g(x) = \frac{\frac{e^{-\left(\frac{\mu-x}{\beta}\right)}}{2\beta}}{\frac{1}{2}\, e^{-\frac{\mu}{\beta}}}$$
$$= \frac{\mathrm{e}^{\frac{\mathrm{x}}{\beta}}}{\beta} \qquad \ldots(7)$$

Where:

$\boldsymbol{x}$: is a value of random variable , and $-\infty \leq \boldsymbol{x} < 0$.

β : is the scale parameter, and $\beta > 0$.

Now $\int_{-\infty}^{0} g(x)dx = 1$, and from equation (7), we get:

$$\int_{-\infty}^{0} \frac{1}{\beta}\, e^{\frac{x}{\beta}}\, dx = e^{\frac{x}{\beta}}\Big|_{-\infty}^{0} = 1$$

The (c.d.f) for general truncated distribution as the following formula[10]:

$$G(x) = \frac{\int_a^x f(u)\, du}{F(b)-F(a)} \qquad \ldots(8)$$

Where

$f(u)$ is (p.d.f) for distribution, to find the cumulative distribution function (c.d.f) for the right truncate Laplace Distribution, as following:

$$G(x) = \frac{\int_a^x f(u)\,du}{F(0)-F(-\infty)}$$

By substituting equations (5) and (6) in the general formula of equation (8), we get:

$$G(x) = \frac{\int_a^x \frac{e^{-\left(\frac{\mu-u}{\beta}\right)}}{2\beta}\,du}{\frac{1}{2}\,e^{-\frac{\mu}{\beta}}}$$

$$= \frac{e^{-\left(\frac{\mu-u}{\beta}\right)}\Big|_{-\infty}^{x}}{e^{-\frac{\mu}{\beta}}}$$

$$= \frac{e^{-\left(\frac{\mu-x}{\beta}\right)} - e^{-\left(\frac{\mu+\infty}{\beta}\right)}}{e^{-\frac{\mu}{\beta}}}$$

$$G(x) = e^{\frac{x}{\beta}} \qquad \ldots(9)$$

3.Some Properties of the right truncated Laplace Distribution

3.1 Median:

The median of a random variable G of the (p.d.f) for the right- truncated Laplace Distribution can be obtained as follows[8]:

$$G(x) = \frac{1}{2}$$

$$e^{\frac{x}{\beta}} = \frac{1}{2}$$

$$x = -\beta \ln 2 \qquad \ldots(10)$$

3.2 Some Moments of right truncated Laplace Distribution:

The general formula for moments[10]:

$$E(x^n) = \int_x x^n\, g(x)\,dx \qquad \ldots(11)$$

We can find the first moment by substituting (n = 1) and equation (7) is equation (11), we get:

$$E(x) = \int_{-\infty}^{0} x\,\frac{1}{\beta}\,e^{\frac{x}{\beta}}\,dx$$

Let $\begin{cases} u = x & , \quad \boldsymbol{du = dx} \\ dv = \frac{1}{\beta}\, e^{\frac{x}{\beta}}\; dx & , \quad v = e^{\frac{x}{\beta}} \end{cases}$

$$E(x) = x\, e^{\frac{x}{\beta}}\Big|_{-\infty}^{0} - \int_{-\infty}^{0} e^{\frac{x}{\beta}}\; dx$$

$$= -\,\beta \qquad \ldots(12)$$

We can find the second moment by substituting (n = 2) and equation (7) in equation (11), we get:

$$E(x^2) = \int_{-\infty}^{0} x^2\; \frac{1}{\beta}\, e^{\frac{x}{\beta}}\; dx$$

Let $\begin{cases} u = x^2 & , \quad du = 2x\, dx \\ dv = \frac{1}{\beta}\, e^{\frac{x}{\beta}}\; dx & , \quad v = e^{\frac{x}{\beta}} \end{cases}$

$$E(x^2) = x^2\, e^{\frac{x}{\beta}}\Big|_{-\infty}^{0} - 2\int_{-\infty}^{0} x\, e^{\frac{x}{\beta}}\; dx$$

Let $\begin{cases} w = x & , \quad dw = dx \\ dz = e^{\frac{x}{\beta}}\; dx & , \quad z = \beta\, e^{\frac{x}{\beta}} \end{cases}$

$$= -2\left(x\,\beta\, e^{\frac{x}{\beta}}\Big|_{-\infty}^{0} - \int_{-\infty}^{0} \beta\, e^{\frac{x}{\beta}}\; dx \right)$$

$$E(x^2) = 2\beta^2 \qquad \ldots(13)$$

4.Estimation Methods

In this section, we shall discuss some methods to estimate the unknown parameters.

4.1.Moment Method (MOM)

This method is one of the simplest techniques commonly used in the field of parameter estimation[5] In a wide variety of problems the parameter to be estimated is some known function of given finite number of moments about zero.

The moment method leads to get consistent and unbiased estimators, in this case the estimators are asymptotically normally distribution[1] .

The r^{th} moments of sample and population for parameter truncated distribution are given, respectively:

$$\mu_r = E\left(x^r\right) = \frac{\sum_{i=1}^{n} x_{(i)}^r}{n} \qquad \ldots(14)$$

$$\mu_r' = -\beta \qquad \ldots(15)$$

From equation (14) and (15) the first moment, where (r=1), are given from follows:

$$\mu_1 = \mu_1'$$

$$\bar{x} = E(x) = \frac{\sum_{i=1}^{n} x_{(i)}}{n} = -\beta$$

Then:

$$\hat{\beta}_{mom} = -\bar{x} \qquad \ldots(16)$$

4.2 Modification Moment Method (MM)

In year (1982) the two researchers Whitten and Cohen suggest new modify by used equation[6]:

$$E\left(\hat{G}(x_{(i)})\right) = G(x_{(i)}) \qquad \ldots(17)$$

i represents the views rank after arranged in ascending order, $\widehat{G}(x_{(i)})$ is estimated unbiased for function distribution $G(x_{(i)})$ and by replacement $G(x_{(i)})$ by plotting position formula[3]:

$$G(x_{(i)}) = \frac{i}{n+1} \quad , \quad i = 1,2,\ldots,n$$

Then

$$G(x_{(1)}) = \frac{1}{n+1} \quad \text{, when } i = 1 \qquad \ldots(18)$$

From two formulas(9) and (18), we get:

$$e^{\frac{x_{(1)}}{\beta}} = \frac{1}{n+1} \qquad \ldots(19)$$

By taking the natural logarithm for equation (19), we get:

$$\frac{x_{(1)}}{\beta} = \ln\left(\frac{1}{n+1}\right)$$

$$\frac{x_{(1)}}{\beta} = -\ln(n+1)$$

$$\hat{\beta}_{mm} = \frac{-x_{(1)}}{\ln(n+1)} \qquad \ldots(20)$$

4.3 Percentile Method (PER):

This method depends on parameter estimation of any distribution of inverse distribution function. as depends on the estimation of the parameter of the right truncated Laplace distribution and on minimizes the value represented by the equation (9), we get[8] :

$$x = \beta \ln(G) \qquad \dots(21)$$

From the equation (21), through equated with zero, then squared and take its sum, we get the following:

$$\sum_{i=1}^{n}(x - \beta \ln(G))^2 = 0 \qquad \dots(22)$$

The replacement $G(x)$ by formula plotting position, $P_i = \frac{i}{n+1}$, $i = 1,2,\dots,n$

Now, the equation (22) becomes as follows:

$$\sum_{i=1}^{n}(x - \beta \ln(P_i))^2 = 0 \qquad \dots(23)$$

And to estimate β the scale parameter, we take the partial derivative for equation (23), with respect to β and solving this equation:

$$\sum_{i=1}^{n} 2(x - \beta \ln(P_i)) \ [-\ln(P_i)] = 0$$

$$\sum_{i=1}^{n}(x \ln(P_i) - \beta(\ln(P_i))^2) = 0$$

$$\sum_{i=1}^{n} x \ln(P_i) - \sum_{i=1}^{n} \beta(\ln(P_i))^2 = 0$$

$$\sum_{i=1}^{n} x \ln(P_i) = \beta \sum_{i=1}^{n}(\ln(P_i))^2$$

$$\hat{\beta} = \frac{\sum_{i=1}^{n} x \ln(P_i)}{\sum_{i=1}^{n}(\ln(P_i))^2} \qquad \dots(24)$$

4.4 Maximum Likelihood Estimation Method (MLE):

It is one of the most popular and reliable methods to obtaine a point estimator of parameters in any distribution[4].It copes with all types of samples, whether uncensored or censored in one way or another and whether the data are grouped or not[7] .

Maximum likelihood method is preferred over all estimation methods because the MLE has many excellent statistical properties.

Let $x_1, x_2, \dots, x_n$ be order random sample of size (n) from a distribution with (p.d.f) $g(x, \beta)$, such that β the parameter, then the likelihood function is the joint (p.d.f) of the random samples is[8]:

$$L(n,\beta) = g(x_1,\beta) \, . \, g(x_2,\beta) \, . \dots . \, g(x_n,\beta)$$

$$= \prod_{i=1}^{n} g(x_i,\beta)$$

The likelihood function for equation (7) is:

$$L(x_1, x_2, \dots, x_n, \beta) = \prod_{i=1}^{n} g(x_i,\beta)$$

$$= \beta^{-n} e^{\frac{\sum_{i=1}^{n} x_i}{\beta}} \qquad \ldots(25)$$

Taking the natural logarithm for the equation (25), so we get the function:

$$\ln L = -n \ln \beta + \frac{\sum_{i=1}^{n} x_i}{\beta} \qquad \ldots(26)$$

The partial derivative for the equation (26) with respect to the unknown parameters , we get:

$$\frac{d \ln L}{d\beta} = \frac{-n}{\beta} - \frac{\sum_{i=1}^{n} x_i}{\beta^2} \qquad \ldots(27)$$

We place equation (27) to zero as follows:

$$0 = \frac{-n}{\hat{\beta}} - \frac{\sum_{i=1}^{n} x_i}{\hat{\beta}^2}$$

$$\frac{-n}{\hat{\beta}} = \frac{\sum_{i=1}^{n} x_i}{\hat{\beta}^2} \qquad \ldots(28)$$

Now, from equation (29) we can obtain formula for β as follows:

$$\hat{\beta} = -\frac{\sum_{i=1}^{n} x_i}{n} \qquad \ldots(29)$$

5. Simulation experiment and results

5.1 Introduction

With the aim of comparing experimentally between estimation methods which have been adopted and studied in the theoretical part of parameter the right truncated Laplace Distribution. We will use the simulation technique, this part includes some of the general concepts of simulation as well as a description of the experience of simulation in this research in terms of the volumes of generated samples. Also we present the results of test simulation that have been obtained as well as a description of the program which has been built.

5.2 Simulation

"Simulation" is defined as the process of representation or imitation of reality using real-specific models. We often find operations in reality that are complicated in understanding and analysis, so it is best that we describe these processes that are similar to the pictures real specific examples. Understanding the model will be achieved when we have a good understanding of the process of the original or true reality through the simulation model. It is natural that the degree of any similar experience between simulation and reality-based real extent of identical or similar simulation model of the real system .Its basic program is to develop a simulation represents the behavior of the calculation a little like real reality as much as possible. And even building a simulation program, it is the generation of random data using a sampling taken as a sample of the community which is supposed to represent the phenomenon in real, rather than to be taken from the real community and then applied statistical methods and sports on these random variables to reach the desired results and then conduct the process analysis and comparison to take the appropriate decision [1].

As a result of the rapid development in the use of electronic computers, colorful simulation methods are the effective scientific method, which can secure the researcher base experimental. It is a guide with a theoretical base for the selection of the appropriate method for analysis and study of experimental data and phenomena under study by comparing the characteristics that have been applied by the simulation.

5.3 Stages of Simulation Experiment Building

building simulation experiments contain the following five important stages:

First stage (stage set default values):

This is the most important stage because other stages depend on it and at this stage it has been set the default parameters and values as follows:

1. Choose the sample size n: (n= 10, 25, 50, 75, 100).
2. Choose different values of the scale parameter β scale and conduct three, $\beta = (1, 0.5, 3.4)$
3. Choose the number of sample replicated size (N): (N = 1000).

The Second stage (data generation):

At this stage, random data is generated by the inverse function for the right-truncated laplace

Distribution of the equation (9), assuming U = G where U represents a continuous random variable structured and defines the period (0,1), and the equation (9) becomes as follows:

$$x = \beta \ \ln G \qquad \ldots(30)$$

Equation (30) represents random variables required for the one parameters of the right- truncated Laplace distribution.

The third stage (find estimators):

At this stage of parameter the discretion of each (β) of the right- truncated Laplace distribution through estimation methods is presented with the theoretical side of this thesis. Equations (16) the Moments Method (MOM). Equations (20) the Modification Moment Method (MM). Equations (24) the Percentile Method (PER). (29) are for the Maximum likelihood Method (MLE).

The fourth stage (stage comparison):

At this stage of comparison between different estimation methods is done by using the standard deviations of the comparison between the estimated methods:

1. Mean square error (MSE)[6]:

$$MSE(\hat{\beta}) = \frac{\sum_{i=1}^{N}(\hat{\beta}-\beta)^2}{N}$$

2.Mean absolute percentage error (MAPE)[6] :

$$MAPE(\hat{\beta}) = \frac{\sum_{i=1}^{N}\left|\frac{\hat{\beta}-\beta}{\beta}\right|}{N}$$

Where

$\hat{\beta}$ is the estimated parameter for the parameter β.

N is the number of replicated.

6. The simulation Results

In this section will analyze the results of the simulation methods for estimating the parameter and for right- truncated Laplace distribution , in order to reache to the best estimation method.

6.1 Estimation results of the Parameters:

6.1.1 Estimation results of the scale parameter (β):

Results of the values for empirical estimates shows in tables (1) , (2) and (3):

Table (1) Show the (Mean) of parameter ($\beta = 1$).

N	Method			
	Mom	Mle	mm	per
10	1.0018	1.0018	1.2615	1.1649
25	1.0009	1.0009	1.1559	1.0849
50	1.0021	1.0021	1.1536	1.0564
75	0.9975	0.9975	1.1454	1.0514
100	0.9985	0.9985	1.1046	1.0401

Table (2) Show the (Mean) of parameter ($\beta = 0.5$).

N	Method			
	Mom	Mle	mm	per
10	0.496	0.5009	0.6107	0.5784
25	0.5003	0.5037	0.5837	0.5485
50	0.5023	0.4977	0.5706	0.5291
75	0.4991	0.4995	0.56	0.5201
100	0.4982	0.4993	0.566	0.5167

Table (3) Show the (Mean) of parameter ($\beta = 3.4$).

N	Method			
	Mom	Mle	mm	per
10	3.3908	3.4439	4.1346	3.9734
25	3.3862	3.3988	3.9461	3.6995
50	3.397	3.3977	3.94	3.5908
75	3.3866	3.3918	3.823	3.5558
100	3.3946	3.3897	3.855	3.5208

6.1.2 Results of the values for (MSE) estimates:

Results are shows in Table (4), (5), and (6), the results will be analyzed by sequence tables, as follows:

Table (4) Show the (MSE) of parameter ($\beta = 1$).

n	Method				
	Mom	mle	Mm	Per	BEST
10	0.0991	0.0991	0.3558	0.1774	Mom- mle
25	0.0402	0.0402	0.1679	0.0606	Mom- mle
50	0.0195	0.0195	0.1336	0.0298	Mom- mle
75	0.0136	0.0136	0.103	0.0204	Mom- mle
100	0.0105	0.0105	0.0814	0.0149	Mom- mle

Table (5) Show the (MSE) of parameter ($\beta = 0.5$).

n	Method				
	Mom	mle	Mm	Per	BEST
10	0.0249	0.0248	0.0819	0.0426	mle
25	0.0091	0.01	0.0478	0.0166	Mom
50	0.0048	0.005	0.0341	0.0068	Mom
75	0.0032	0.0034	0.0235	0.0047	Mom
100	0.0024	0.0025	0.0242	0.0034	Mom

Table (6) Show the (MSE) of parameter ($\beta = 3.4$).

n	Method				
	Mom	mle	mm	per	BEST
10	1.1863	1.1228	3.6032	1.9931	mle
25	0.4893	0.4378	2.0814	0.7242	mle
50	0.2336	0.2405	1.7725	0.3237	Mom
75	0.1402	0.1381	1.0925	0.23	mle
100	0.1101	0.1139	1.1676	0.1623	Mom

Through the study and analysis of the results obtained from the tables (4), (5), (6) we found that the orders of preference estimation methods for the parameter (β) for most of the values in the three cases are as follows:

Order	**1**	**2**	**3**	***4***
Method	**MOM**	**MLE**	**PER**	***MM***

6.1.3 Results of the values for (MAPE) estimates:

Results are shows in Table (7), (8), (9), the results will be analyzed by sequence tables, as follows:

Table (7) Show the (MAPE) of parameter ($\beta = 1$).

n	Method				
	Mom	mle	Mm	per	BEST
10	0.2447	0.2447	0.4282	0.3155	Mom- mle
25	0.1572	0.1572	0.3044	0.1906	Mom- mle
50	0.1115	0.1115	0.2595	0.134	Mom- mle
75	0.093	0.093	0.2306	0.1103	Mom- mle
100	0.0816	0.0816	0.205	0.095	Mom- mle

Table (8) Show the (MAPE) of parameter ($\beta = 0.5$).

n	Method				
	Mom	mle	Mm	per	BEST
10	0.2535	0.2447	0.4059	0.3064	mle
25	0.1521	0.1595	0.3074	0.2034	Mom
50	0.1119	0.1136	0.2629	0.1301	mle
75	0.0912	0.0918	0.2223	0.1074	Mom
100	0.0777	0.0796	0.2232	0.0933	Mom

Table (9) Show the (MAPE) of parameter ($\beta = 3.4$).

n	Method				
	Mom	mle	Mm	per	BEST
10	0.2545	0.2452	0.4041	0.3141	Mom
25	0.1623	0.1554	0.3049	0.1956	mle
50	0.1123	0.1157	0.2695	0.132	Mom
75	0.0873	0.088	0.2253	0.1114	Mom
100	0.0779	0.0784	0.2288	0.0928	Mom

Through the study and analysis of the results obtained from the tables (7), (8), (9) we found that the orders of preference estimation methods for the parameter (β) for most of the values in the three cases are as follows:

Order	**1**	**2**	**3**	***4***
Method	**MOM**	**MLE**	**PER**	***MM***

7. Conclusions

Through the simulation, we compare the estimation methods of parameters for right- truncated Laplace Distribution through which we obtain the conclusions of this study, which are: In all the samples sizes, the (mom) method is the best of the other methods.

References

المصادر العربية

Arabic References

١- الدراجي، الحان نهاد جمعه (٢٠٠٤)، " استخدام المحاكاة للمقارنة بين بعض مقدرات معلمة القياس ودالة معولية النظام المتسلسل لتوزيع ويبل" ، رسالة ماجستير في الرياضيات ، كلية العلوم ، الجامعة المستنصرية .

٢- الجميلي، صبا صباح أحمد (٢٠٠٧)، "مقارنة بعض طرائق تقدير المعلمة والمعولية لأنموذج رَيلي للفشل بإستخدام المحاكاة"، رسالة ماجستير، كلية الإدارة والاقتصاد، جامعة بغداد.

٣- الطواش، ميسون سالم مجيد (١٩٩٠)، "دراسة إحصائية حول توزيع ويبل وتوزيع القيمة المتطرفة مع تطبيقات عملية"، رسالة ماجستير، كلية الإدارة والاقتصاد، جامعة بغداد.

٤- العطواني، شيماء عباس جاسم (٢٠١٠)، " تقدير دالة البقاء لمرض سرطان الدماغ باستخدام توزيع ويبل ذي الثلاث معلمات"، رسالة ماجستير ، كلية التربية- ابن الهيثم ، جامعة بغداد.

٥- الهلالي، فراس صدام عبد (٢٠٠٤)، " مقارنة طرائق تقدير معالم انموذج ويبل للفشل بثلاث معالم

٦- صالح، حكيم حسين حمد (٢٠١٣) ، "مقارنة بعض طرائق تقدير دالة معولية توزيع ويبل العام ذي المعلمتين"، رسالة ماجستير علوم في الرياضيات ، كلية التربية ، الجامعة المستنصرية.

٧- مجيد، ضفاف فاضل (٢٠١٣)، "تقدير المعلمات توزيع ويبل المعدل بالاعتماد على عينات المراقبة من نوع التام مع تطبيق عملي"، رسالة ماجستير علوم في الرياضيات، كلية العلوم للبنات، جامعة بغداد.

المصادر الأجنبية

Foreign References

8-AL-Yasseri,Ahmed Yassin Taqi (2014),"using simulation to estimate two parameters & reliability function for logistic distribution" master thesis, college of education, Al-Mustansirah University.

9-Balakrishnan,N. and Aggarwala, Rita (1996),"Relationships for moments of order statistics from the right-truncated generalized half logistic distribution", Department of Mathematics and Statistics, Mcmaster University, Vol.48, No.3, pp.519-534.

10- Evans, M ; Hastings , N.A.J. and Peacock , J.B. (2000), "Statistical distribution", Third Edition , John Wiley and Sons, Inc., New York.

11- Krishnanorthy, K.(2006),"Handbook of statistical distribution with applicat-ion", University of Louisiana at Lafayette, USA.

12- Savalees Nadavajah and Samuel Kotz (2006), "Programs for computing truncated distributions", Journal of Statistical Software, Vol.16, Code Snippet 2.