Energy loss spectra

Sanar Gasid Hassan

Physics Department, Collage of Sciences, Al-Mustansiriyah University, Baghdad, Iraq.

sanargasid@gmail.com

Abstract

This paper studies the form of the energy spectrum using a statistical methods study the spectra and to fond expressions for energy loss .This paper study of the spectrum of energy loss of abeam of particles penetrate through of small thickness of different atoms Al, W, Au, Ag, Br and Be and study the different spectrum of the energy loss and the loss of these energies.

The energy loss of the spectrum was studied for multi atoms of Al, , W,Au, Ag,Br and Be depending on equation (1) to estimate energy loss for the limited thickness and comparing difference spectrum with difference of atomic numbers .

طيف خسارة الطاقة

سناركاصد حسن

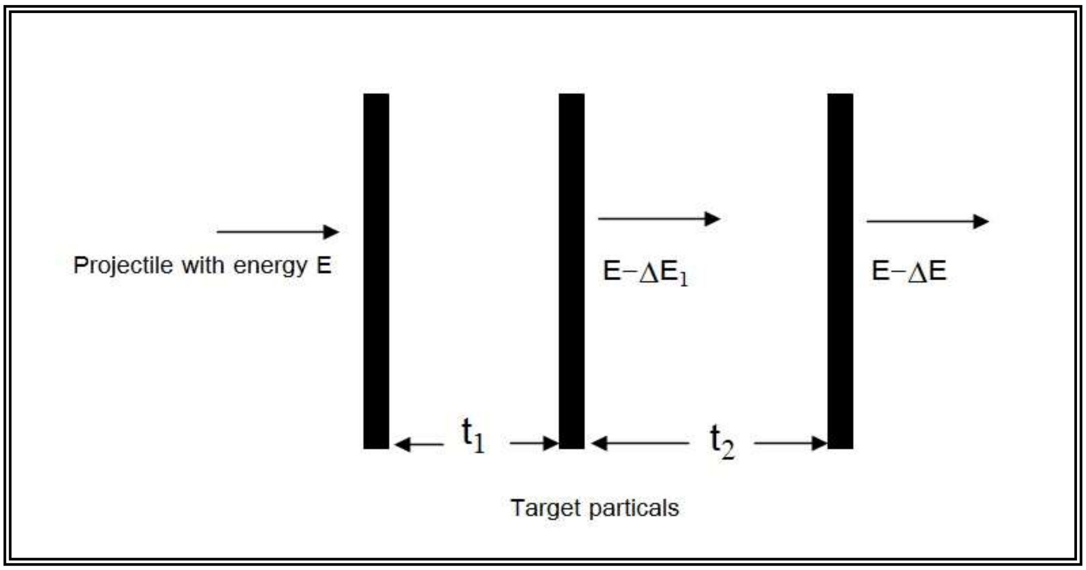
قسم الفيزياء ،كلية العلوم ، الجامعة المستنصرية

الخلاصة

يدرس هذا البحث شكل طيف خسارة الطاقة بأستخدام طرق أحصائية لدراسة الطيف ومن ثم أيجاد صيغة معينة لخسارة الطاقة ، تمت هذه الدراسة لحزمة من الجسيمات النافذة خلال سمك صغير لذرات مختلفة من Al, W, Au, Ag, Br , Be ودراسة أختلاف الطيف الناتج من خسارة الطاقة لطاقات مختلفة . تم الاعتماد على معادلة (١) لايجاد فقدان الطاقة ضمن سمك محدد لذرات متعددة ومقارنة طيف خسارة الطاقة بأختلاف الاعداد الذرية .

Introduction

The collide particles are tabulated accoding to their out come, like the nature of excitation of the atom of the target at the level jth . by using a thin of the beam that interacts with ransoms atoms of the target . and to discuss the energy loss as result of excitation of the target , and to extend the out come to involve other process like multiple scattering and the energy loss accompanied by changing exchange [1].assume $P(\Delta E, t_2)d(\Delta E)$ the probability distribution in energy loss ΔE when it passing through thickness layer t_2 , so the main equation can be found with thickness t_1 and t_2 [1,2].



Fig(1) shows the loss of energy while passing through the thickness of the layer (t_1 and t_2)

Assume te energy loss of beam particles passing through the first layer is ΔE_1. The loss energy will be $\Delta E - \Delta E_1$ in the second layer , when the ΔE is the total energy loss .

As the collision events, which produce two probabilities that define the probability of the joint and Because ΔE_1 is arbitrary and the outcome of the probabilities will be [2],

$$P(\Delta E, t_1 + t_2) = \int_0^{\Delta E} d(\Delta E_1) P(\Delta E_1, t_1) P(\Delta E - \Delta E_1, t_2) \quad \text{---- (1)}$$

The convolution resulting will suggest going application of Fourier space [1]

$$P(\Delta E,t_1)=\frac{1}{2\pi}\int_{\infty}^{-\infty}dk e^{ik\Delta E}P(k,t_2) \qquad \text{---- (2)}$$

And

$$P(k,t_1)=\frac{1}{2\pi}\int_{\infty}^{-\infty}d(\Delta E)e^{-ik\Delta E}P(\Delta E,t_2) \qquad \text{--- (3)}$$

Then equation (1) reduce to

$$P(k,t_1+t_2)=P(k,t_1)P(k,t_2) \qquad \text{--- (4)}$$

by the exponential solution we get

$$P(k,t_2)=e^{t_2C(k)} \qquad \text{--- (5)}$$

Where $C(k)$ is arbitrary function.

This function $C(k)$ is determined by in individual events. Resulting from passing through. These events determined by cross sections σ_j of energy loss T_j. If the thickness of the layer t_2 is very small, the events of the probability can be ignored .the spectrum of the loss of energy passing through the layer t_2 is shown by [2]

$$P(\Delta E,t_2)=\left(1-\sum_j P_j\right)\delta(\Delta E)+\sum_j P_j\delta(\Delta E-T_j) \qquad \text{--- (6)}$$

Due to Lambert & Beer's law

$$pn=\begin{cases}1-Nt\sigma & for \quad n=0\\ Nt\sigma & for \quad n=1\\ 0 & for \quad n\geq 2\end{cases} \qquad \text{--- (7)}$$

The j-event probability is $P_j=Nt_2\sigma_j$

The right side of equation (6) indicates zero energy (probability of nothing happing) and the other side express the collection of collision of energy loss of one collision and energy loss .

By applying Fourier space from of Dirac function we can write equation (6) as ,

$$P(k,t_2)=1-Nt_2\sum_j\sigma_j(1-e^{-ikT_j}) \qquad \text{--- (8)}$$

This equation can by compared with equation (5) for small thickness -t_2 ,

$$P(k,t_2)=1+t_2C(k) \quad \text{--- (9)}$$

So We can identify the function $C(k)$ as

$$C(k)=N\sigma(k) \quad \text{--- (10)}$$

And by transporting the cross section

$$\sigma(k)=\sum_j \sigma_j(1-e^{-ikT_j}) \quad \text{--- (11)}$$

In regard to equations (2),(5) and (10) we can get

$$P(\Delta E,t_1)=\frac{1}{2\pi}\int_{\infty}^{-\infty} dke^{ik\Delta E-Nt_2\sigma\ (k)} \quad \text{--- (12)}$$

It is form of a Bothe-Landau equation that deals with a massed events particle penetration [4]. The continuous single-event spectrum, equation (11) could be written as

$$\sigma(k)=\int d\sigma(T)(1-e^{-ikT}) \quad \text{--- (13)}$$

For the combination of components $l=1,2,3,....$, by replacement

$$N\sigma(k)=\sum_{jl}\sigma_{jl}(1-e^{-ikT_{jl}}) \quad \text{--- (14)}$$

Can be given, Nl denotes of $l-$atoms per volume and

$$\sigma_l(k)=\sum_{jl}\sigma_{jl}(1-e^{-ikT_{jl}}) \quad \text{--- (15)}$$

, the variable jl denotes the states of the atom (type l) .
By Multiplication of equation (14) by the power of the energy loss and then to integrated ΔE given [5]

$$\left\langle (\Delta E)^n \right\rangle=\frac{1}{2\pi}\int d(\Delta E)\int dke^{-Nt_2\sigma(k)}(\Delta E)e^{ik\Delta E} \quad \text{--- (16)}$$

the order of integrations can be rearranged and using of

$$\frac{\partial}{\partial k}e^{ik\Delta E}=i\Delta Ee^{ik\Delta E} \quad \text{--- (16)}$$

And to replace by ΔE^n $i(\frac{\partial}{\partial k})^n$ and by doing partial integration this gives [5,6] :

$$\int_{-\infty}^{\infty} d(\Delta E)(\Delta E)^n P(\Delta E,t_2) = i(\frac{\partial}{\partial k})^n e^{-Nt_2\sigma(k)}\Big|_{k=0} \qquad \text{--- (17)}$$

By butting n = 0 the right-hand side of equation (13). This expresses minimize to 1

That means the probability $P(\Delta E,t_2)$ is normalized to 1 for all t_2. It is clear that, the particles number should be conserved for penetration of particles[7].

By butting n=1 the equation (13) after the differentiation and doing the limit when $k=0$, putting (Ω^2) as the mean-square fluctuation to the (ΔE) referring to equation (17), so the mean loss in energy

$$\Omega^2 = N\Delta x \sum_j T_j^2 \sigma_j \qquad \text{--- (18)}$$

At the same time taking n = 2, the equation (18) the energy loss straggling will be recovered for the mean-square fluctuation (Ω^2).higher probability distribution can be derive (Symon, 1948) and the equation (12) can be reformed,

$$P(\Delta E,t_1) = \frac{1}{2\pi}\int_{\infty}^{-\infty} dk e^{ik(\Delta E - \langle \Delta E \rangle) - Nt_2\sigma\ (k)} \qquad \text{--- (19)}$$

Where

$$\sigma_1(k) = \int d\sigma(T)(1 - ikT - e^{-ikT}\ ) \qquad \text{--- (20)}$$

Approximation by Diffusion

The relationship between Gaussian approximations and the loss of the energy and Bothe-Landau formula (12) is illustrate by Equation (18), which expect that increase the energy loss when the thickness of the target increase .so the Fourier integral of the same values k received amount of fraction thats lead to [9]

$$P(\Delta E,t_1) = \frac{1}{\sqrt{2\pi NxW}} \exp\left(- \frac{(\Delta E - NxS)^2}{2NxW} \right) \qquad \text{--- (21)}$$

With standard deviation $\sqrt{NxW}$, Gaussian located on the mean loss of energy NxS . the method sketch is named diffusion of approximation . The accuracy can be expected in higher−order limit . the finite term show by Lindhard an Nielsen when the value above ν = 2,this leads to the probality density $P(k,t_1)$ will be negative value for certain duration of ΔE in respect to cross section [10]

$$\sigma(k) = ikS + \frac{1}{2}k^2W + \sigma 2(k) \qquad \text{--- (22)}$$

With

$$\sigma 2(k) = \int d\sigma(T)(1 - ikT - \frac{1}{2}k^2T^2 - e^{-ikT}) \qquad \text{--- (23)}$$

Where $\sigma 2(k)$ correction term that leads to suitable approximation.

The spectrum of energy loss integral

By analytical solution Lindhard and Nielsen showed [7]

$$d\sigma(T) = \frac{C}{T^{3/2}} e^{-\alpha T} d(T) \quad \text{for} \quad 0 \langle T \langle \infty \qquad \text{--- (24)}$$

α and C are constants. By replacement T^{-2} by $T^{3/2}$ and T_{max} was replaced by an exponential eeeeeeeeeee and the parameter is chosen to be T_{max}, both eqn (13) and (14) leads to transport of the cross section [12]

$$\sigma(k) = 2\sqrt{\pi}C(\sqrt{\alpha + ik} - \sqrt{\alpha}) \qquad \text{--- (25)}$$

the integration of gamma function, this integration gives to [11,12]

$$P(\Delta E, t_1) = \frac{NC_t}{\Delta E^{3/2}} \exp\left[-\frac{\alpha}{\Delta E}(\Delta E - N_t S)^2\right] \qquad \text{--- (26)}$$

S is stopping cross section

$$S = \int_0^\infty T d\sigma(T) = C\sqrt{\frac{\pi}{\alpha}} \qquad \text{--- (27)}$$

Replacing to (26) and (1) [12]

$$\Omega^2 = \left\langle \left(\Delta E - \langle \Delta E \rangle\right)^2 \right\rangle = \frac{1}{2\alpha} \langle \Delta E \rangle \qquad \text{--- (28)}$$

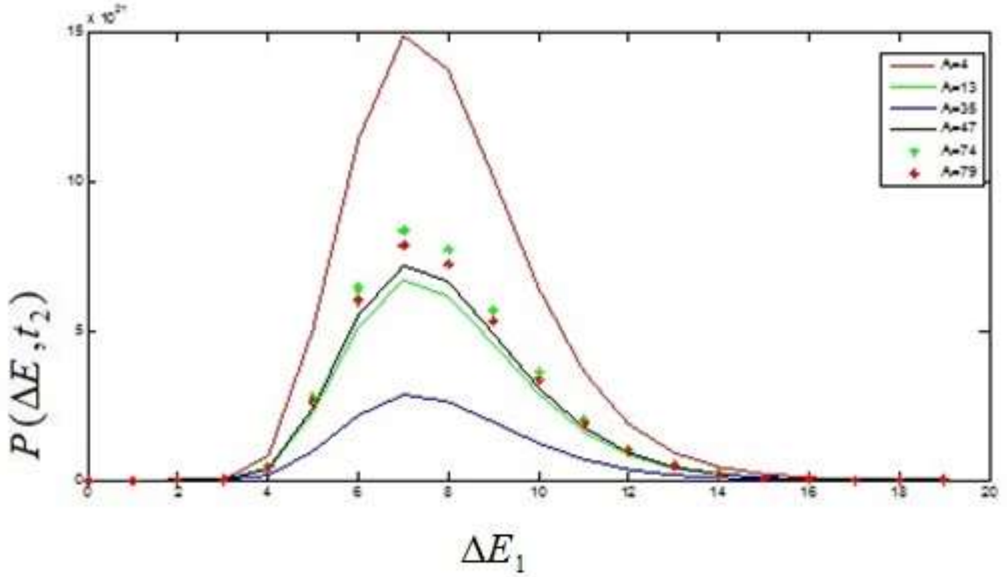
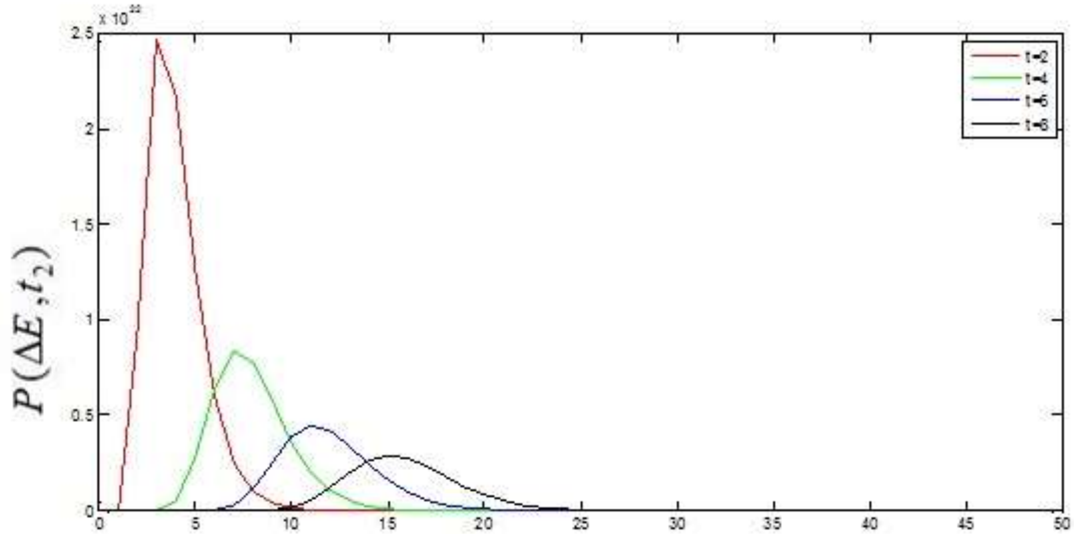
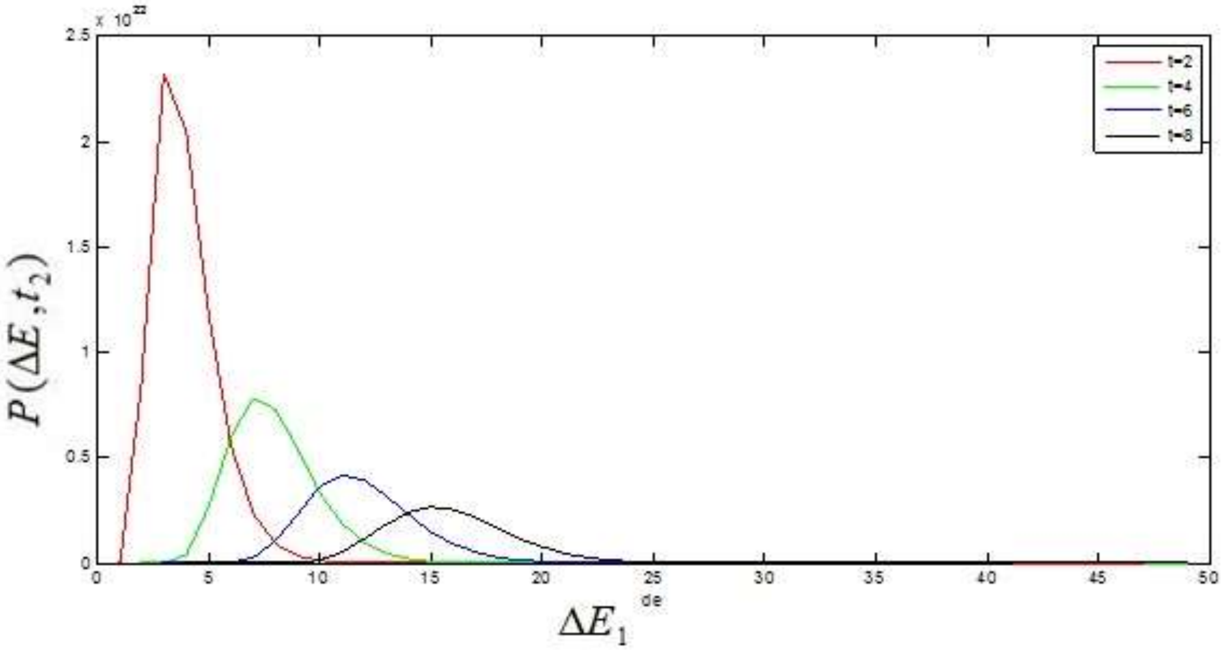


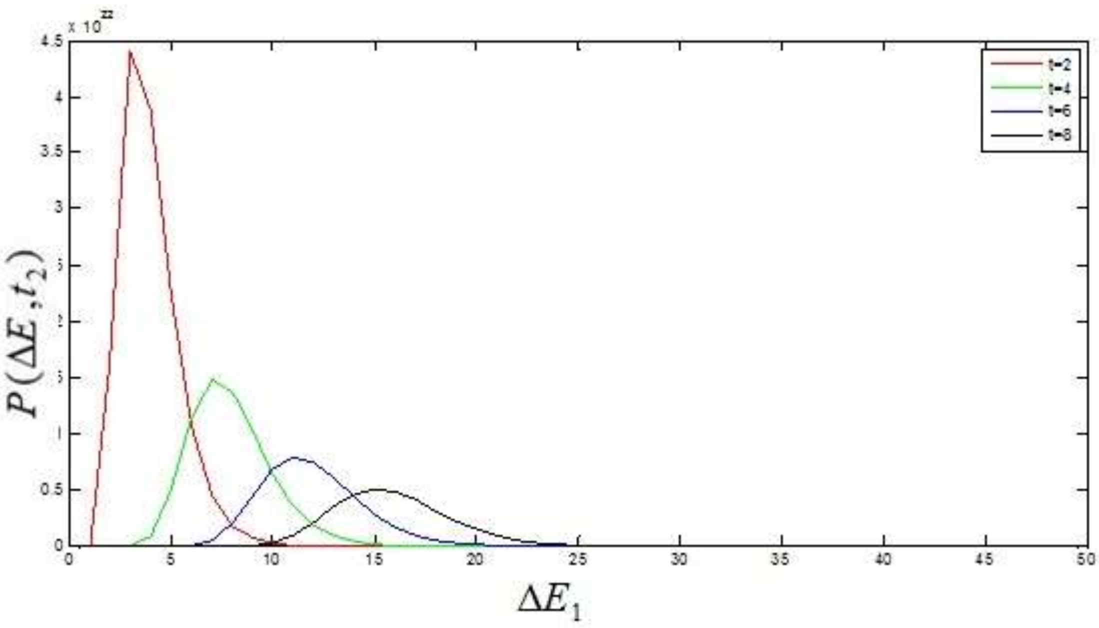
Figure (1) shows the relation between probability and energy loss for particles for different atomic numbers at thickness $t_2 = 4$ with energy range $0.1 \rangle E \rangle 20$



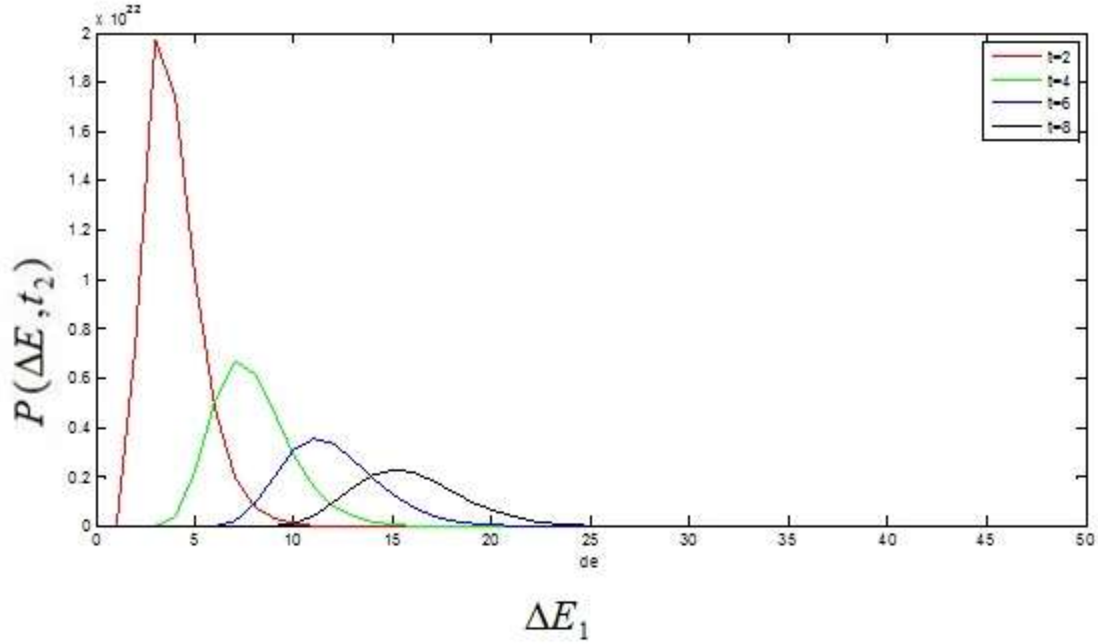
(a)


t=2
t=4
t=6
t=8
$P(\Delta E, t_2)$
ΔE_1
de

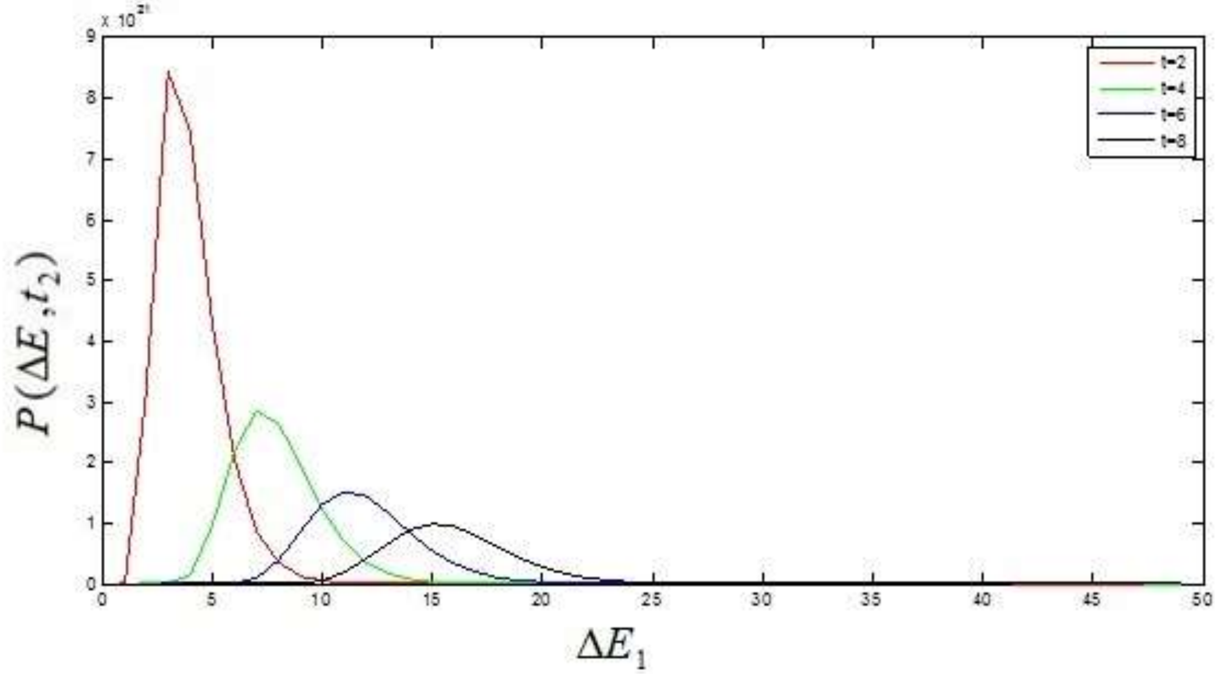
(b)


t=2
t=4
t=6
t=8
$P(\Delta E, t_2)$
ΔE_1

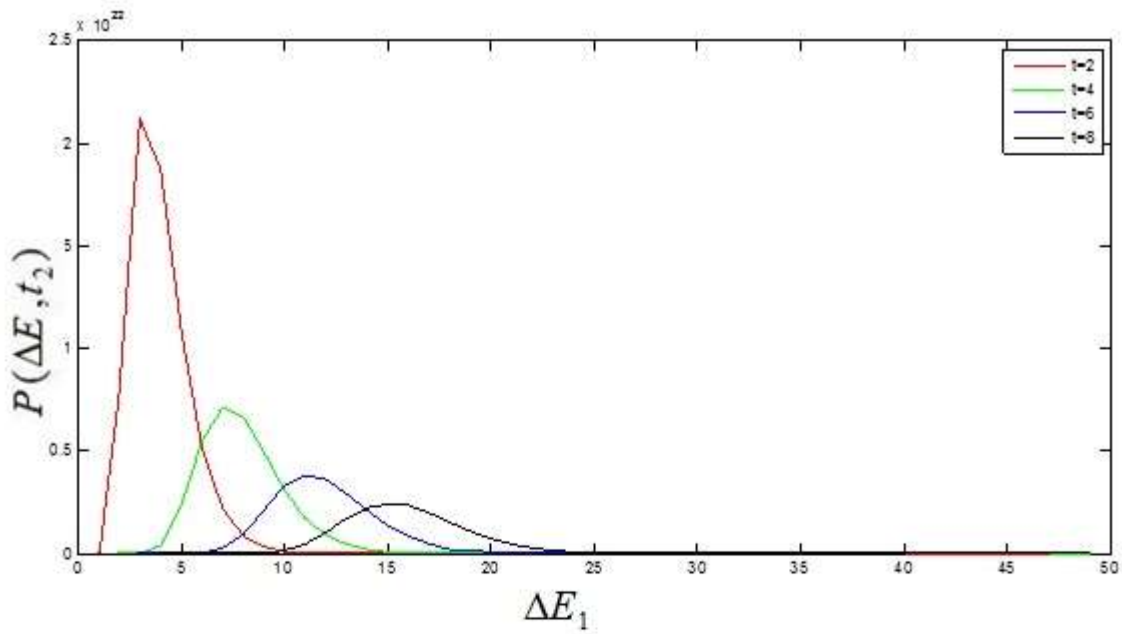
(c)


x 10^22
t=2
t=4
t=6
t=8
de
$P(\Delta E, t_2)$
ΔE_1

(d)


x 10^21
t=2
t=4
t=6
t=8
$P(\Delta E, t_2)$
ΔE_1

(e)



(f)

Figure (2) shows the relation between probability and energy loss for particles fo atomic numbers (a) A=74, (b) A=79 (c) A= 4 (d) A=13 , (e) A=35 , (f) A=47 at thickness $t_2 = 2, 4, 6, 8$ **with energy range** $0.1 \rangle E \rangle 100$**.**

The energy loss spectra in equation (1) depends on thickness of the layer , in the figure (2) $\langle \alpha \Delta E \rangle \geq 1$ for the moderate layer thickness of the target , and this energy loss is similar to the label of pertinent spectrum on the abscissa of the graph .

Discussion

The Convolution equation (1) extends to involve the loss of energy spectrum, which depends totally on the energy. Increase in the layer thickness causes decrees in the speed of projectile to the degree that assumption is not maintained. Because the analytical expressions of the energy loss spectrum cannot be easily obtained so the numerical expressions can be describe in equation (1).the whole energy loss spectra must be to small to meet justification of the neglect of changing collision cross section of energy beam . i.e. the equation (12) is restricted for the thin and moderate of layer thickness of the targets .

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