Solving fuzzy fractional linear programming problems by ranking function methods

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Abstract

In this paper we present solving fuzzy fractional linear programming problems by ranking function methods in which both the right- hand side and objective function coefficients are fuzzy numbers , numerical examples are solved to illustrate the proposed methods .

الخلاصة :

في هذا البحث سنقدم حل مسائل البرمجة الكسرية الخطية الضبابية بواسطة طرق دالة تخفيض الرتبة التي تكون فيها كل من الجهة اليمنى ومعاملات دالة الهدف من الأعداد الضبابية ، حلت الأمثلة العددية لتوضيح الطرق المقترحة.

Keyword : fuzzy set , fuzzy number ,trapezoidal fuzzy number , ranking function , fuzzy linear programming , fuzzy fractional linear programming.

1-Introduction

Fuzzy set theory has been applied to many fields of operation research . Bellman and Zadeh [١٢] introduced the concept of a fuzzy decision in environment was first proposed . and their are others studied the fuzzy environment , Zimmermann [٨] was presented early works of fuzzy programming , A.Kumar and P.Singh [1] introduced a new method for solving full fuzzy linear programming problems . And T.Beaula and S.Rajalakshmi[16] proposed a new method to solve fuzzy linear programming problems using Duality , P.A.Thakra , D.S.Shelar and S.P.Thakre [11] solved fuzzy linear programming problem as multi objective linear programming problem . There are many applications of fractional programming in fuzzy as T.Peric , Z . Babic and S.Resic [17] proposed a goal programming procedure for solving fuzzy multi objective fractional linear programming problems . B.Stanojevie and M.Stanojevie [7] prove an approach for solving method for linear fractional optimization problem with fuzzy coefficients in the objective function.

Ranking fuzzy number is important in decision to making data analysis economic systems and operations research . so A.Nchammai and P.Thangaraj [3] solving intuitionistic fuzzy linear programming by using metric distance ranking . Also H.M.Nehi and H.Hajmohamadi [9] introduced a ranking function method for solving fuzzy multi – objective linear programming problem . And A.N.Gani and V.N.Mohammed [6] give a way to find the solution of a fuzzy assignment problem by using a new ranking method .

In this paper present solving fuzzy fractional linear programming problems by ranking function methods in which both the right- hand side and objective function coefficients are fuzzy numbers, And then the complementary development method is used to convert fuzzy fractional linear programming problems .

2-Basic concepts

We need the following definitions of the basic arithmetic operators and partial ordering relations on fuzzy trapezoidal numbers based on the function principle which can be found in [5 , 14 , 15] .

Definition 2.1 : A fuzzy set A is defined by $\tilde{A} = \{ (x , \mu_A(x)) : x \in A , \mu_A(x) \in [0,1] \}$. In the pair $(x, \mu_A(x))$ the first element x belong to the classical set A , the second element $\mu_A(x)$, belong to the interval [0,1] , called Membership function .

Definition 2.2: The support of a fuzzy set $\tilde{A}$ on X is the crisp set of all $x \in X$ such that $\mu_A(x) > 0$.

Definition 2.3 : The α - cut of a fuzzy set $\tilde{A}$ is the subset of X defined as : $\tilde{A}_\alpha=\{ x \in X , \mu_A(x) > \alpha \}$.

Definition 2.4 : A fuzzy set $\tilde{A}$, defined on the universal set of real numbers R , is said to be a fuzzy number if its membership function has the following characteristics:

(i) $\tilde{A}$ is convex, i.e. $\forall x, y \in R, \forall \lambda \in [0,1]$, $\mu_A (\lambda x + (1- \lambda) y) \geq \min \{ \mu_A(x) , \mu_A (y)\}$,

(ii) $\tilde{A}$ is normal, i.e., $\exists \bar{x} \in R ; \mu_A(\bar{x}) = 1 ;$

iii) $\mu_A(x)$ is piecewise continues.

Definition 2.5 : We can define trapezoidal fuzzy number $\tilde{A}$ as $\tilde{A} = (a_1 , a_2 , a_3 , a_4)$

the membership function of this fuzzy number will be interpreted as

$$\mu_{\tilde{A}}(x) = \begin{cases} 0, & x < a_1, \\ \dfrac{x-a_1}{a_2-a_1}, & a_1 \le x \le a_2, \\ 1, & a_2 \le x \le a_3, \\ \dfrac{a_4-x}{a_4-a_3}, & a_3 \le x \le a_4, \\ 0, & x > a_4 \end{cases}$$

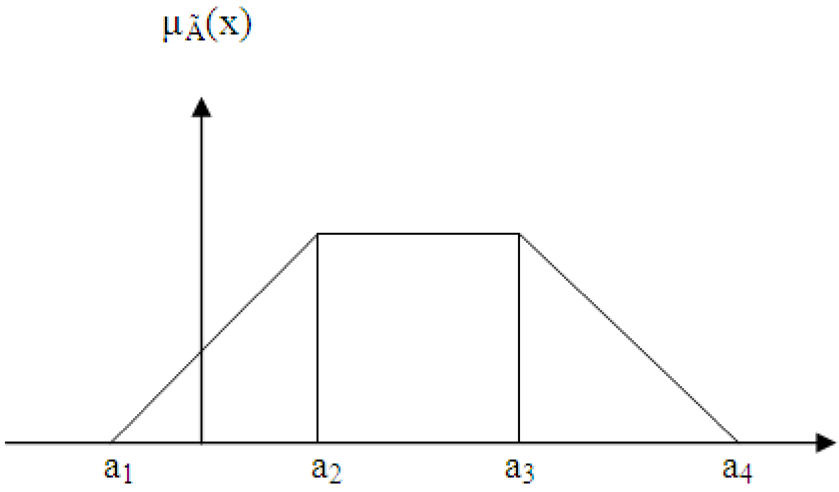


Fig. 1

Trapezoidal fuzzy number $\tilde{A} = (a_1 , a_2 , a_3 , a_4)$

The Operations of Trapezoidal Fuzzy Number

Let $\tilde{A} = (a_1 , a_2 , a_3 , a_4)$ and $\tilde{B} = (b_1 , b_2 , b_3 , b_4)$ be two trapezoidal fuzzy numbers.

Addition : $\tilde{A}\ (+)\ \tilde{B} = (a_1 , a_2 , a_3 , a_4)\ (+)\ (b_1 , b_2 , b_3 , b_4)$
$= (a_1 + b_1 , a_2 + b_2 , a_3 + b_3, a_4 + b_4)$

Subtraction: $\tilde{A}\ (-)\tilde{B} = (a_1 , a_2 , a_3 , a_4)\ (-)\ (b_1 , b_2 , b_3 , b_4)$
$= (a_1 - b_4 , a_2 - b_3 , a_3 - b_2, a_4 - b_1)$

Scalar Multiplication : $x > 0$, $x\tilde{A} = (xa_1 , xa_2 , xa_3 , xa_4)$
$x < 0$, $x\tilde{A} = (xa_4 , xa_3 , xa_2 , xa_1)$.

2.1-Ranking functions [14]

A convenient method for comparing of the fuzzy numbers is by use of ranking functions . So we define a ranking function $\Re: F(R)\rightarrow R$, which maps for each fuzzy number into the real line. Now, suppose that $\tilde{a}$ and $\tilde{b}$ be two trapezoidal fuzzy numbers. Then , we define orders on

$F\,(R\,)$ as following:

$\tilde{a} \underset{R}{\geq} \widetilde{b}$ if and only if R $(\tilde{a}) \geq$ R $(\tilde{b})$

$\tilde{a} \underset{R}{>} \widetilde{b}$ if and only if R $(\tilde{a}) >$ R $(\tilde{b})$

$\tilde{a} \underset{R}{=} \widetilde{b}$ if and only if R $(\tilde{a}) =$ R $(\tilde{b})$

where $\tilde{a}$ and $\tilde{b}$ are in F(R) . Also we write $\tilde{a} \underset{R}{\leq} \widetilde{b}$ if and only if $\widetilde{b} \underset{R}{\geq} \widetilde{a}$.

Lemma 2.1 : [1,10]

Let $\Re$ be any linear ranking function. Then

i) $\tilde{a} \underset{R}{\geq} \widetilde{b}$ if and only if $\tilde{a} - \tilde{b} \underset{R}{\geq} 0$ if and only if $-\tilde{b} \underset{R}{\geq} -\tilde{a}$

ii) if $\tilde{a} \underset{R}{\geq} \widetilde{b}$ if and only if $\tilde{c} \underset{R}{\geq} \widetilde{d}$, then $\widetilde{a + \tilde{c}} \underset{R}{\geq} \widetilde{b} + \widetilde{d}$.

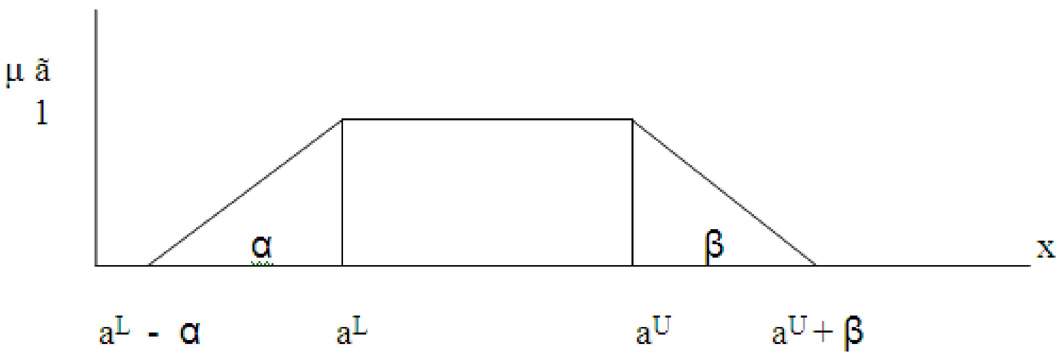
These are many numbers ranking function for comparing fuzzy numbers. Here, we use from linear ranking functions , that is, a ranking function $\Re$ such that

$\Re(\, k\tilde{a} + \tilde{b}) = k\,\Re(\tilde{a}) + \Re(\tilde{b})$.

Maleki ranking function suggestion f or a linear ranking function as following:

$\Re(\tilde{a}) = \frac{L}{a} + \frac{U}{a} + \frac{1}{2}\ (\beta - \alpha)$

Where $\tilde{a} = (\frac{L}{a}, \frac{U}{a}, \alpha, \beta) \in F\,(R\,)$.



Yager ranking function suggestion f or a linear ranking function as following:

$\Re(\tilde{a}) = \frac{1}{2}(\,\frac{L}{a} + \frac{U}{a} - \frac{4}{5}\,\alpha + \frac{2}{3}\ \beta\,)$

Where $\tilde{a} = (\frac{L}{a}, \frac{U}{a}, \alpha, \beta) \in F\,(R\,)$.

2.2-Fuzzy linear programming [13]

A linear programming problem with fuzzy right – hand side constants (FLPP) is defined as

Maximize $Z = \sum_{j=1}^{n} c_j \; x_j$

Subject to

$$\sum_{j=1}^{n} a_j \; x_j \; \leq \; b_i \quad , \quad 1 \leq i \leq \mathrm{m}$$

$$x_j \geq 0 \quad ; \quad 1 \leq j \leq \mathrm{n}$$

Where at least one $x_j \geq 0$ and $\tilde{b}$ is a fuzzy number and $\mathrm{x} \in \mathrm{R}$.

2.3-Fuzzy linear fractional programming [2]

Fuzzy linear fractional programming problem is defined as

Maximize $(\tilde{c} \;\; \tilde{x} + \alpha) / (\tilde{d} \;\; \tilde{x} + \beta)$

Subject to the constraints

$$\tilde{A}\,\tilde{X} \; = \; \tilde{b}$$

$$\tilde{X} \; \geq 0$$

2.4-Fractional linear programming problems with fuzzy numbers [4]

A fuzzy number fractional programming linear programming (FNFLP) problems is defined as following :

Maximize $\tilde{z} \; = \; \dfrac{\tilde{c} \;\; \tilde{x} + \tilde{p}}{\tilde{d} \;\; \tilde{x} + \tilde{q}}$

Subject to the constraints

$$\mathrm{AX} \; \leq \; \mathrm{B}$$

$$\mathrm{X} \geq 0$$

Where $b□R^{m}$, $x□R^{n}$, $x□ \; X$, $A□R^{n*m}$, $\tilde{c}\,,\tilde{d}\,,\tilde{p}\,,\tilde{q} \in (\,\mathrm{F(R)}\,)$ and R is the linear ranking function .

2.5-**Complementry development method [4]**

Use the Complementary development method to covert fuzzy fractional programming problem to fuzzy linear programming problem.

3- Solving fuzzy fractional linear programming problems by ranking function methods

We have used algorithm to convert fuzzy fractional programming problem to fuzzy linear programming problem and solving fuzzy fractional linear programming problems by ranking function .

3.1-Algorithm to solve fuzzy fractional linear programming problems by ranking function methods

Step(1) : dividing objective function into two linear functions , the first function is represent the numerator function and the second is denominator function and the value of objective function to be maximum at possible it is must be the numerator function to be maximum ($\max z_1(x)$) and the denominator function be minimum ($\min z_2(x)$).

Step(2) : reclamation a function max ($z^*(x)$) from subtracting the denominator function from nominator function and this function is putting in mathematical module made up of original restriction of problem in addition to nonnegative conditions and to solve this linear system we going to step(3) .

Step(3) : we make the objective function coefficients as trapezoidal fuzzy numbers which is as follows :

$$Max\ z = \sum_{j=1}^{n} \tilde{c}_j\, x_j$$

s.to

$$\sum_{j=1}^{n} a_{ij}\, x_j \le b_i \quad , \ i=1,2,\ldots,m$$

$$x_j \ge 0$$

Where c_j : fuzzy coefficients of objective function $a_{ij} \in R^{n\times m}$, $b_i \in R^m$. then we solve the fuzzy linear programming by using two ranking function:

(i) the linear programming problem when using Maleki ranking function:

$$Max\ z = \sum_{j=1}^{n} \left[c_j^l + c_j^u + \frac{1}{2}(\beta - \alpha) \right] x_j$$

s.to

$$\sum_{j=1}^{n} a_{ij}\, x_j \le b_i \quad , \ i=1,2,\ldots,m$$

$$x_j \ge 0$$

(ii) the linear programming problem when using Yager ranking function:
$Max\ z = \sum_{j=1}^{n} \frac{1}{2} [\, c_j^l + c_j^u - \frac{4}{5}\alpha + \frac{2}{3}\beta \,]\, x_j$

s.to

$\sum_{j=1}^{n} a_{ij}\, x_j \le b_i$, $i=1,2,\ldots,m$

$x_j \ge 0$

Step(4) : we make the right-hand side coefficients as trapezoidal fuzzy numbers which is as follows:

$Max\ z = \sum_{j=1}^{n} c_j\, x_j$

s.to

$\sum_{j=1}^{n} a_{ij}\, x_j \le \tilde{b}_i$, $i=1,2,\ldots,m$

$x_j \ge 0$

Where b_i : fuzzy right-hand side coefficient , $c_j \in R^n$, $a_{ij} \in R^{n\times m}$.

Then we solve the fuzzy linear programming by using two ranking function:

(i) the linear programming problem when using Maleki ranking function:

$Max\ z = \sum_{j=1}^{n} c_j\, x_j$

s.to

$\sum_{j=1}^{n} a_{ij}\, x_j \le [b_j^l + b_j^u + \frac{1}{2}(\beta - \alpha)\,]$, $i=1,2,\ldots,m$

$x_j \ge 0$

(ii) the linear programming problem when using Yager ranking function:

$Max\ z = \sum_{j=1}^{n} c_j\, x_j$

s.to

$\sum_{j=1}^{n} a_{ij}\, x_j \le \frac{1}{2} [\, b_i^l + b_i^u - \frac{4}{5}\alpha + \frac{2}{3}\beta \,]$, $i=1,2,\ldots,m$

$x_j \ge 0$

Step(5) : we make both the objective function coefficients and right-hand side coefficients as trapezoidal fuzzy numbers which is as follows:

$Max\ z = \sum_{j=1}^{n} \tilde{c}_j\, x_j$

s.to

$\sum_{j=1}^{n} a_{ij}\ x_j\ \leq\ \widetilde{b}_i$, *i=1,2, . . . , m*

$x_j\ \geq\ 0$

Where b_i *and* c_j are trapezoidal fuzzy numbers , $a_{ij} \in R^{n\times m}$ .then we solve the fuzzy linear programming by using two ranking function:

(i) the linear programming problem when using Maleki ranking function:

$Max\ z = \sum_{j=1}^{n}\ [\ c_j^l +\ c_j^u\ + \frac{1}{2}(\beta - \alpha)]\ x_j$

s.to

$\sum_{j=1}^{n} a_{ij}\ x_j\ \leq [b_j^l +\ b_j^u + \frac{1}{2}(\beta - \alpha)\ ]$, *i=1,2,…,m*

$x_j\ \geq\ 0$

(ii) the linear programming problem when using Yager ranking function:

$Max\ z = \sum_{j=1}^{n}\ \frac{1}{2}\ [\ c_j^l +\ c_j^u\ - \frac{4}{5}\ \alpha + \frac{2}{3}\ \beta\ ]\ x_j$

s.to

$\sum_{j=1}^{n} a_{ij}\ x_j\ \leq\ \frac{1}{2}\ [\ b_i^l +\ b_i^u - \ \frac{4}{5}\ \alpha + \frac{2}{3}\ \beta\ ]$, *i=1,2,…,m*

$x_j\ \geq\ 0$

4 -Numerical examples:

Now we illustrate the proposed method using the following numerical examples .

Example 1:

$$\text{Max } z = \frac{(7,4,3,3)\ x_1 + (10,9,6,6)\ x_2 + (14,13,9,9)}{(1,5,2,2)\ x_1 + (3,6,4,4)\ x_2 + (5,9,6,6)}$$

s.to

$3x_1+6x_2\ \leq\ 13$

ٍ$x_1+6x_2\ \leq\ 12$

$x_1\ , x_2\ \geq\ 0$

By algorithm (4.1) we can write it as:

Max $z_1 = (7,4,3,3)\ x_1 + (10,9,6,6)\ x_2 + (14,13,9,9)$

s.to

$3x_1+6x_2 \leq 13$

٤$x_1+6x_2 \leq 12$

$x_1, x_2 \geq 0$

And

Min $z_2 = (1,5,2,2)\ x_1 + (3,6,4,4)\ x_2 + (5,9,6,6)$

s.to

$3x_1+6x_2 \leq 13$

٤$x_1+6x_2 \leq 12$

$x_1, x_2 \geq 0$

Now , we construct Max z^* from step (2) in algorithm (4.1)

Now, by Step (2) in algorithm (4.1) , we can be obtained fuzzy linear programming problem :

Max $z^* = (2,3,1,1)\ x_1 + (4,6,2,2)\ x_2 + (5,8,3,3)$

s.to

$3x_1+6x_2 \leq 13$

٤$x_1+6x_2 \leq 12$

$x_1, x_2 \geq 0$

By using Maleki ranking function we get

Max $z = 5x_1+10x_2 +13$

s.to

$3x_1+6x_2 \leq 13$

٤$x_1+6x_2 \leq 12$

$x_1, x_2 \geq 0$

We solve the linear programming problem by using simplex method and we get the following solution : $x_1 = 0$, $x_2 = 2$, Z = 20

By using Yager ranking function we get

Max $z = 2.4\ x_1 + 4.8\ x_2 + 6.3$

s.to

$3x_1+6x_2 \leq 13$

٤$x_1+6x_2 \leq 12$

$x_1 , x_2 \geq 0$

solving the linear programming problem by using simplex method and we get the following solution: $x_1 = 0$, $x_2 = 2$, Z = 9.7

Now, by Step (4) in algorithm (4.1) , we get

Max z = $2 x_1 + 5 x_2 + 6$

s.to

$3x_1+6x_2 \leq (9,15,2,2)$

٤$x_1+6x_2 \leq (8,16,1,1)$

$x_1 , x_2 \geq 0$

By using Maleki ranking function we get

Max z = $2 x_1 + 5 x_2 + 6$

s.to

$3x_1+6x_2 \leq 24$

٤$x_1+6x_2 \leq 24$

$x_1 , x_2 \geq 0$

The solution for linear programming problem is as follows: $x_1 = 0$, $x_2 = 4$, Z = 20

By using Yager ranking function we get

Max z = $2 x_1 + 5 x_2 + 6$

s.to

$3x_1+6x_2 \leq 11.8$

٤$x_1+6x_2 \leq 11.9$

$x_1 , x_2 \geq 0$

The solution for linear programming problem is as follows: $x_1 = 0$, $x_2 = 1.9$, Z = 9.8

Now, by Step (5) in algorithm (4.1) , we get

Max $z = (2,3,1,1)\, x_1 + (4,6,2,2)\, x_2 + (5,8,3,3)$

s.to

$3x_1+6x_2 \leq (9,15,2,2)$

$٤x_1+6x_2 \leq (8,16,1,1)$

$x_1 , x_2 \geq 0$

By using Maleki ranking function we get

Max $z = 5x_1+10x_2 +13$

s.to

$3x_1+6x_2 \leq 24$

$٤x_1+6x_2 \leq 24$

$x_1 , x_2 \geq 0$

The solution for linear programming problem is as follows: $x_1 = 0$, $x_2 = 4$, Z = 40

By using Yager ranking function we get

Max $z = 2.4\, x_1 + 4.8\, x_2 + 6.3$

s.to

$3x_1+6x_2 \leq 11.8$

$٤x_1+6x_2 \leq 11.9$

$x_1 , x_2 \geq 0$

The solution for the above linear programming problem is $x_1 = 0$, $x_2 = 1.9$, Z = 9.5

Example 2:

Max $z = \dfrac{(15,16,6,8)\, x_1 + (14,17,4,7)\, x_2}{(8,10,3,4)\, x_1 + (7,8,1,2)\, x_2}$

s.to

$2x_1+3x_2 \leq 6$

$٥x_1+4x_2 \leq 10$

$x_1 , x_2 \geq 0$

By algorithm (4.1) we can write it as:

Max $z_1 = (15,16,6,8)\, x_1 + (14,17,4,7)\, x_2$

s.to

$2x_1+3x_2 \leq 6$

٥$x_1+4x_2 \leq 10$

$x_1 , x_2 \geq 0$

And

Min $z_2 = (8,10,3,4)\, x_1 + (7,8,1,2)\, x_2$

s.to

$2x_1+3x_2 \leq 6$

٥$x_1+4x_2 \leq 10$

$x_1 , x_2 \geq 0$

Now , we construct Max z^* from step (2) in algorithm (4.1)

Now, by Step (2) in algorithm (4.1) , we can be obtained fuzzy linear programming problem :

Max $z^* = (5,8,2,5)\, x_1 + (6,10,2,6)\, x_2$

s.to

$2x_1+3x_2 \leq 6$

٥$x_1+4x_2 \leq 10$

$x_1 , x_2 \geq 0$

By using Maleki ranking function we get

Max $z = 14.5x_1+18x_2$

s.to

$2x_1+3x_2 \leq 6$

٥$x_1+4x_2 \leq 10$

$x_1 , x_2 \geq 0$

We solve the linear programming problem by using simplex method and we get the following solution: $x_1 = 0.8$, $x_2 = 1.4$, Z = 38.1

By using Yager ranking function we get

Max $z = 7.3\, x_1 + 9.2\, x_2$

s.to

$2x_1+3x_2 \le 6$

٥$x_1+4x_2 \le 10$

$x_1, x_2 \ge 0$

solving the linear programming problem by using simplex method and we get the following solution : $x_1 = 0.8$, $x_2 = 1.4$, Z =19.4

Now, by Step (4) in algorithm (4.1) , we get

Max $z = 7\, x_1 + 9\, x_2$

s.to

$2x_1+3x_2 \le (5,12,2,3)$

٥$x_1+4x_2 \le (9,17,4,5)$

$x_1, x_2 \ge 0$

By using Maleki ranking function we get

Max $z = 7\, x_1 + 9\, x_2$

s.to

$2x_1+3x_2 \le 17.5$

٥$x_1+4x_2 \le 26.5$

$x_1, x_2 \ge 0$

The solution for linear programming problem is as follows

$x_1 = 1.3$, $x_2 = 4.9$, Z = 53.8

By using Yager ranking function we get

Max $z = 7\, x_1 + 9\, x_2$

s.to

$2x_1+3x_2 \le 8.7$

٥$x_1+4x_2 \le 13.0$

$x_1, x_2 \ge 0$

The solution for linear programming problem is as follows

$x_1 = 0.6$, $x_2 = 2.4$, Z = 26.7

Now, by Step (5) in algorithm (4.1) , we get

Max z = (5,8,2,5) x_1 +(6,10,2,6) x_2

s.to

$2x_1+3x_2 \leq (5,12,2,3)$

٥$x_1+4x_2 \leq (9,17,4,5)$

$x_1 , x_2 \geq 0$

By using Maleki ranking function we get

Max z = $14.5x_1+18x_2$

s.to

$2x_1+3x_2 \leq 17.5$

٥$x_1+4x_2 \leq 26.5$

$x_1 , x_2 \geq 0$

The solution for linear programming problem is as follows

$x_1 = 1.3$, $x_2 = 4.9$, Z = 108.3

By using Yager ranking function we get

Max z = $7.3 x_1 + 9.2 x_2$

s.to

$2x_1+3x_2 \leq 8.7$

٥$x_1+4x_2 \leq 13.0$

$x_1 , x_2 \geq 0$

The solution for the above linear programming problem is

$x_1 = 0.6$, $x_2 = 2.4$, Z = 27.4

5-Conclusion

In this paper present Complementry development method to convert the fuzzy fractional programming problem to fuzzy linear programming problem and solving fuzzy fractional linear programming problems by ranking function methods in which both the right – hand side and objective function coefficients are fuzzy numbers and introduce the optimal solution for this problems and we get an algorithm to solve this problems . examples are also given to illustrate the method .

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