بعض دوال الاحتمالية المميزة لحل نظام من المتباينات الخطية العشوائية

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المستخلص :

هدف البحث هو اعطاء بعض دوال الاحتمالية المميزة (دالة الارتباط، دالة الكثافة الطيفية و دالة الارتباط المتبادل) لحل نظام من المتباينات الخطية معطى بشكله القانوني مع دوال عشوائية ثابتة في الجهة اليمنى لتلك المتباينات.

الكلمات المفتاحية:

الاحتمالية المميزة، دالة الارتباط ، دالة الكثافة الطيفية ، دالة الارتباط المتبادل.

Some of the Probability Characteristics Functions of the Solution of a System of Random Linear Inequalities

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Abstract:

The aim of this paper is to given some of the probability characteristics functions (the correlation function, the spectral density function and cross-correlation function) of the solution of the system of linear inequality given in the canonical form with stationary random functions in the right hand-side of those inequalities.

Keywords:

Probability characteristics, correlation function, spectral density function, cross-correlation function.

Introduction

Many practical problems in business, science, and engineering involve systems of linear inequalities, [2]. A system of linear inequalities consists of a set of two or more linear inequalities with the same variables, [3]. When solving a system of inequalities, you should be aware that the system might have no solution or it might be represented by an unbounded region in the plane, [7].

The mathematical theory of random function has been an area of considerable activity in recent years. Applications appear in the physical sciences such as the development of designs for airplanes and rockets, the improvement of radar and other electronic devices, and the investigation of certain production and chemical processes have resulted in considerable interest in random function analysis. This recent work should not disguise the fact that the analysis of random function is one of the oldest activities of scientific man, [5].

The concept of a random function is not included in the framework of classical probability theory, and to study random function, a new mathematical apparatus has to be created. The first attempts at a mathematical investigation of probabilistic models leading to the notion of a random function appeared very really in this century, [6].

In this paper we determine some of the probability characteristics of the solution of the system of random linear inequality given in the canonical form with stationary random functions in the right hand-side of those inequalities, in section one we present the canonical form of the system of linear inequality, in section two we give the solutions of that system and the correlation function, the spectral density functions and the cross-correlation function to those solutions will illustrated that in sections three, four and five respectively.

1- The Canonical Form of the System of Random Linear Inequality

Consider the following system of random linear inequalities which are given in the canonical form

$$\frac{dy_j(t)}{dt} + \sum_{m=1}^{n} a_{jm} y_m(t) \leq \mathrm{X}_j(t), \; j = 1, 2, \ldots, n \qquad \ldots(1)$$

where a_{jm}, j, $m = 1, \ldots, n$ are constant coefficients, $\mathrm{X}_j(t)$ are stationary random functions with spectral density functions $f_{\mathrm{X}_j(t)}(\lambda)$ for all $j = 1, \ldots, n$ and they are mutually correlated $y_j(t)$ are random functions.

2- The Solution of the Canonical Form of the System of Random Linear Inequalities

To solve the system (1), that is to determine the particular integral of the random functions $y_j(t)$, $j = 1, \ldots, n$, we replace the right-hand sides of this system except one (say inequality numbered ℓ) which replaced respectively to obtain two different systems of random inequality by the following two assumptions (cases):

Case one: by the function $\beta_{j\ell}\, e^{i\lambda t}$

Case two: by the function $\beta_{j\ell}\, t^k$, $k = 0, 1,\ldots, n$

where

$$\beta_{j1} = \begin{cases} 1 & if \quad j = 1 \\ 0 & if \quad j \neq 1 \end{cases} \quad \text{for } j, \ell = 0, 1,\ldots, n. \qquad \ldots(2)$$

that is mean, we replace by zeros all the stationary random functions $X_j(t)$, $j = 1, \ldots, \ell - 1, \ell + 1, \ldots, n$ except $X_\ell(t)$ respectively by case one and then by case two [1], that is to obtain two different solutions of the given system (1) which can be rewritten under assumptions, case one and case two respectively by the following two new systems:

$$dy_{j1}(t) + \sum_{m=1}^{n} a_{jm} y_{m1}(t) \le \beta_{j1} e^{i\lambda t}, \; j, 1 = 1, \ldots, n \qquad \ldots(3)$$

and

$$dy_{j1}(t) + \sum_{m=1}^{n} a_{jm} y_{m1}(t) \le \beta_{j1} t^{k}, \; j, 1 = 1, \ldots, n, k = 0, 1, \ldots, n \qquad \ldots(4)$$

where $\beta_{j\ell}$ defined by (2).

By denoting the function number j, $j = 1, 2, \ldots, n$ which is $y_{j\ell}(t)$ by $y_{j\ell}(\lambda, t)$ where ℓ, $\ell = 0, 1,\ldots, n$ is the number of inequalities in the systems (3) or (4) which having in their right hand side the function $\beta_{j\ell}\, e^{i\lambda t}$ or the function $\beta_{j\ell}\, t^k$, the particular integral of the inequality number ℓ with the function $y_{j\ell}(\lambda, t)$ numbered by j is

$$\int_{-\infty}^{\infty} y_{j1}(\lambda, t) f_{X_1(t)}(\lambda) d\lambda \qquad \ldots(5)$$

Hence, the general solution of the systems (3) and (4) or equivalently of the given canonical system (1) will be

$$y_j(t) \leq \sum_{\ell=1}^{n} \int_{-\infty}^{\infty} y_{j\ell}(\lambda,t) f_{X_\ell(t)}(\lambda) d\lambda, \; j = 1, \ldots, n \qquad \ldots(6)$$

where $f_{X_\ell(t)}(\lambda)$ is the spectral density function of the stationary random function $X_\ell(t)$, $\ell = 1, \ldots, n$.

To complete the solution, we start by the system (3) that is by taking the Laplace transformation of both sides of this system under the initial conditions $y_{j\ell}(0) \leq 0$ which yields to the equivalent initial condition $y_{j\ell}(\lambda,0) \leq 0$. Hence

$$\mathrm{L}\left[\frac{dy_{j\ell}(t)}{dt}\right] + \sum_{m=1}^{n} a_{jm}\, \mathrm{L}\left[y_{m\ell}(t)\right] \leq \beta_{j\ell}\, \mathrm{L}\,[\, e^{i\lambda t}\,]$$

$$\mathrm{L}\left[\frac{y_{j\ell}(\lambda,t)}{dt}\right] + \sum_{m=1}^{n} a_{jm}\, \mathrm{L}\left[y_{m\ell}(t)\right] \leq \beta_{j\ell}\, \mathrm{L}\,[\, e^{i\lambda t}\,]$$

$$s\,\mathrm{L}\left[y_{j\ell}(\lambda,t)\right] - y_{j\ell}(\lambda,0) + \sum_{m=1}^{n} a_{jm}\, \mathrm{L}\left[y_{m\ell}(t)\right] \leq \beta_{j\ell}\, \mathrm{L}\,[\, e^{i\lambda t}\,]$$

or

$$s\,\mathrm{L}\left[y_{j\ell}(\lambda,t)\right] + \sum_{m=1}^{n} a_{jm}\, \mathrm{L}\left[y_{m\ell}(t)\right] \leq \beta_{j\ell}\, \mathrm{L}\,[\, e^{i\lambda t}\,] \qquad \ldots(7)$$

But by the general theory with constant coefficients we consider that the particular integral of $y_{m\ell}(t)$ is equal to $c_{m\ell} e^{i\lambda t}$ where $c_{m\ell}$ are constant, $m, \ell = 1, \ldots, n$, [4].

So that equation (7) will be,

$$s\,\mathrm{L}\left[y_{j\ell}(\lambda,t)\right] + \sum_{m=1}^{n} a_{jm}\, \mathrm{L}\left[c_{m\ell} e^{i\lambda t}\right] \leq \beta_{j\ell}\, \mathrm{L}\,[\, e^{i\lambda t}\,]$$

$$s\,\mathrm{L}\left[y_{j\ell}(\lambda,t)\right] + \sum_{m=1}^{n} a_{jm}\, c_{m\ell}\left(\frac{1}{s - i\lambda}\right) \leq \beta_{j\ell}\, \frac{1}{s - i\lambda}$$

$$\mathrm{L}\left[y_{j\ell}(\lambda,t)\right] \leq \beta_{j\ell}\, \frac{1}{s(s - i\lambda)} - \frac{1}{s}\sum_{m=1}^{n} a_{jm} c_{m\ell}\left(\frac{1}{s - i\lambda}\right)$$

$$\mathrm{L}\left[y_{j\ell}(\lambda,t)\right] \leq \frac{1}{i\lambda}\left(\beta_{j\ell} - \sum_{m=1}^{n} a_{jm} c_{m\ell}\right)\left(\frac{1}{s - i\lambda} - \frac{1}{s}\right) \qquad \ldots(8)$$

and by the inverse Laplace of transformation,

$$y_{j1}(\lambda,t) \leq \frac{1}{i\lambda}(\beta_{j1} - \sum_{m=1}^{n} a_{jm}c_{m1})(\mathrm{L}^{-1}\left[\frac{1}{s-i\lambda}\right] - \mathrm{L}^{-1}\left[\frac{1}{s}\right])$$

$$y_{j1}(\lambda,t) \leq \frac{1}{i\lambda}(\beta_{j1} - \sum_{m=1}^{n} a_{jm}c_{m1})(e^{i\lambda t}-1),\ j,1=1,2,\ldots,n \qquad \ldots(9)$$

Therefore, the particular integral of any inequality of the system (3) numbered by $1, 1=1,2,\ldots,n$ will have the form

$$\int_{-\infty}^{\infty}\frac{1}{i\lambda}(\beta_{j1} - \sum_{m=1}^{n} a_{jm}c_{m1})(e^{i\lambda t}-1)\ f_{X_1(t)}(\lambda)d\lambda \qquad \ldots(10)$$

and the general first solution of the system (3) or (1) is the sum with respect to $1=1,2,\ldots,n$ all the expressions of the form (10), that is mean (6)

$$y_1(t) \leq \sum_{1=1}^{n}\int_{-\infty}^{\infty}\frac{1}{i\lambda}(\beta_{j1} - \sum_{m=1}^{n} a_{jm}c_{m1})(e^{i\lambda t}-1)\, f_{X_1(t)}(\lambda)d\lambda,\quad j = 1,2,\ldots,n \qquad \ldots(11)$$

According to the second solution of the system (4). By the same way of solving the system (3), we take the Laplace transformation of both sides of this system under the initial conditions $y_{j1}(0) \leq 0$, $y_{j1}(\lambda,0) \leq 0$ and under the assumption that $y_{m1}(t) \leq c_{m1}t^k$ and $y_{m1}(\lambda,t) \leq c_{m1}t^k$, k = 0, 1, 2, …, n that is to get,

$$s\,\mathrm{L}\left[y_{j1}(\lambda,t)\right] + \sum_{m=1}^{n} a_{jm}c_{m1}\ \mathrm{L}\,[t^k] \leq \beta_{j\ell}\,\mathrm{L}\,[t^k]$$

$$\mathrm{L}\left[y_{j1}(\lambda,t)\right] \leq \frac{1}{s}(\beta_{j1}\frac{k!}{s^{k+1}} - \sum_{m=1}^{n} a_{jm}c_{m1}\frac{k!}{s^{k+1}})$$

or

$$\mathrm{L}\left[y_{j1}(\lambda,t)\right] \leq \beta_{j1}\frac{k!}{s^{k+2}} - \sum_{m=1}^{n} a_{jm}c_{m1}\frac{k!}{s^{k+2}} \qquad \ldots(12)$$

and by taking the inverse Laplace transformation,

$$y_{j1}(\lambda,t) \leq \beta_{j\ell}\,\mathrm{L}^{-1}\left[\frac{k!}{s^{k+2}}\right] - \sum_{m=1}^{n} a_{jm}c_{m1}\,\mathrm{L}^{-1}\left[\frac{k!}{s^{k+2}}\right]$$

$$\leq \frac{k!}{(k+1)!}\{\beta_{j\ell}\ \mathcal{L}^{-1}\left[\frac{(k+1)!}{s^{k+1+1}}\right] - \sum_{m=1}^{n} a_{jm}c_{m\ell}\ \mathcal{L}^{-1}\left[\frac{(k+1)!}{s^{k+1+1}}\right]\}$$

or

$$y_{j\ell}(\lambda,t) \leq \frac{1}{(k+1)}(\beta_{j\ell}t^{k+1} - \sum_{m=1}^{n} a_{jm}c_{m\ell}t^{k+1}), \qquad \ldots(13)$$

$$j,\ell = 1,2,\ldots,n,\ k = 0,1,\ldots,n$$

Hence, the particular integral for each inequality of the system (4) numbered by any $\ell = 1,2,\ldots,n$ will have the form

$$\int_{-\infty}^{\infty} \frac{1}{(k+1)}(\beta_{j\ell}\, t^{k+1} - \sum_{m=1}^{n} a_{jm}c_{m\ell}t^{k+1})\ f_{X_\ell(t)}(\lambda)d\lambda \qquad \ldots(14)$$

and the general second solution of the system (4) or of the canonical system (1) is the sum of all expressions (14) regarding that $\ell = 1,2,\ldots,n$

$$y_j(t) \leq \sum_{\ell=1}^{n}\int_{-\infty}^{\infty} \frac{1}{k+1}(\beta_{j\ell}\, t^{k+1} - \sum_{m=1}^{n} a_{jm}c_{m\ell}t^{k+1})\ f_{X_\ell(t)}(\lambda)d\lambda$$

or

$$y_j(t) \leq \sum_{\ell=1}^{n}\int_{-\infty}^{\infty} \frac{1}{k+1}(\beta_{j\ell} - \sum_{m=1}^{n} a_{jm}c_{m\ell})\, t^{k+1}\ f_{X_\ell(t)}(\lambda)d\lambda, \qquad \ldots(15)$$

$$j,\ell = 1,2,\ldots,n,\ \ k = 0,1,\ldots,n$$

where this relation and relation (11) represent the spectral representation of the random functions $y_j(t)$, $j = 1, 2,\ldots, n$ and $f_{X_\ell(t)}(\lambda)$ in both these two relations is the spectral density function of the stationary random functions $X_\ell(t)$.

3- Correlation Function of the Solutions of the Canonical Form of the System of Random Linear Inequalities

In this section we illustrate the correlation function. We determined the solution of the system, so we consider the correlation function of the first solution in subsection (3.1) and the correlation function of the second solution in subsection (3.2).

3.1 Correlation Function of the First Solution of the Canonical Form of the System of Random Linear Inequalities:

$$\mathrm{B}_{y_j}(t,t+h) \le \mathrm{E}\left[\overline{y_j(t)}\, y_j(t+h)\right]$$

$$\le \mathrm{E}\left\{\left[\overline{\sum_{1=1}^{n}\int_{-\infty}^{\infty}\frac{1}{i\lambda}(\beta_{j1}-\sum_{m=1}^{n}a_{jm}c_{m1})(e^{i\lambda t}-1)f_{\mathrm{X}_1}(\lambda)d\lambda}\right]\times\right.$$

$$\left.\left[\sum_{1=1}^{n}\int_{-\infty}^{\infty}\frac{1}{i\lambda}(\beta_{j1}-\sum_{m=1}^{n}a_{jm}c_{m1})(e^{i\lambda(t+h)}-1)f_{\mathrm{X}_1}(\lambda)d\lambda\right]\right\}$$

$$\le \sum_{1=1}^{n}\int_{-\infty}^{\infty}\overline{(\beta_{j1}-\sum_{m=1}^{n}a_{jm}c_{m1})}\,(\beta_{j1}-\sum_{m=1}^{n}a_{jm}c_{m1})$$

$$\overline{(\frac{e^{i\lambda t}-1}{i\lambda})}\,(\frac{e^{i\lambda(t+h)}-1}{i\lambda})\times\mathrm{E}\left[\overline{f_{\mathrm{X}_1}(\lambda)d\lambda}\, f_{\mathrm{X}_1}(\lambda)d\lambda\right]$$

Hence,

$$\mathrm{B}_{y_j}(t,t+h) \le \sum_{1=1}^{n}\left|(\beta_{j1}-\sum_{m=1}^{n}a_{jm}c_{m1})\right|^2\int_{-\infty}^{\infty}\frac{1}{\lambda^2}\left[e^{-i\lambda(t-t-h)}-e^{i\lambda t}-e^{i\lambda(t+h)}+1\right]\times$$

$$f_{\mathrm{X}_1}(\lambda)d\lambda,\ j=1,2,\ldots,n,\ \lambda>0 \qquad \ldots(16)$$

and for a given $f_{\mathrm{X}_1}(\lambda)$, the correlation function $\mathrm{B}_{y_j}(t,t+h)$ can be found completely.

3. 2 Correlation Function of the Second Solution of the Canonical Form of the System of Random Linear Inequalities:

By the same way as previous,

$$\mathrm{B}_{y_j}(t,t+h) \le \mathrm{E}\left\{\left[\overline{\sum_{1=1}^{n}\int_{-\infty}^{\infty}\frac{1}{k+1}(\beta_{j1}-\sum_{m=1}^{n}a_{jm}c_{m1})t^{k+1}f_{\mathrm{X}_1}(\lambda)d\lambda}\right]\times\right.$$

$$\left.\left[\sum_{1=1}^{n}\int_{-\infty}^{\infty}\frac{1}{k+1}(\beta_{j1}-\sum_{m=1}^{n}a_{jm}c_{m1})(t+h)^{k+1}f_{\mathrm{X}_1}(\lambda)d\lambda\right]\right\}$$

$$\mathrm{B}_{y_j}(t,t+h)\leq\sum_{l=1}^{n}\int_{-\infty}^{\infty}(\frac{1}{k+1})^2\overline{(\beta_{j1}-\sum_{m=1}^{n}a_{jm}c_{m1})}(\beta_{j1}-\sum_{m=1}^{n}a_{jm}c_{m1})\times$$
$$(t(t+h))^{k+1}\mathrm{E}\left[\overline{f_{X_1}(\lambda)d\lambda}f_{X_1}(\lambda)d\lambda\right]$$
$$\leq\sum_{l=1}^{n}(\frac{1}{k+1})^2\left|\beta_{j1}-\sum_{m=1}^{n}a_{jm}c_{m1}\right|^2(t(t+h))^{k+1}\int_{-\infty}^{\infty}f_{X_1}(\lambda)d\lambda$$

But by the property that $\int_{-\infty}^{\infty}f_{X_1}(\lambda)d\lambda$ = 1, the correlation function $\mathrm{B}_{y_j}(t,t+h)$ will be

$$\mathrm{B}_{y_j}(t,t+h)\leq\sum_{l=1}^{n}(\frac{1}{k+1})^2\left|\beta_{j1}-\sum_{m=1}^{n}a_{jm}c_{m1}\right|^2(t(t+h))^{k+1}\int_{-\infty}^{\infty}f_{X_1}(\lambda)d\lambda,$$
$$j=1,2,\ldots,n,\ k=0,1,\ldots,n \qquad \ldots(17)$$

4- Spectral Density Function of the Solutions of the Canonical Form of the System of Random Linear Inequalities

In this section, we consider the spectral density function of first and second solutions by the formula $\mathrm{B}_j(h)\leq\int_{-\infty}^{\infty}e^{i\lambda h}f_{y_j}(\lambda)d\lambda$.

4.1 The Spectral Density Function of the First Solution of the Canonical Form of the System of Random Linear Inequalities:

The relation (16) can be written as

$$\int_{-\infty}^{\infty}e^{i\lambda h}f_{y_j}(\lambda)d\lambda\leq\sum_{l=1}^{n}\left|(\beta_{j1}-\sum_{m=1}^{n}a_{jm}c_{m1})\right|^2\int_{-\infty}^{\infty}\frac{1}{\lambda^2}\left[e^{i\lambda h}-e^{i\lambda t}-e^{i\lambda(t+h)}+1\right]\times f_{X_1}(\lambda)d\lambda$$

$$f_{y_j}(\lambda)\leq\frac{1}{e^{i\lambda h}\lambda^2}\sum_{l=1}^{n}\left|(\beta_{j1}-\sum_{m=1}^{n}a_{jm}c_{m1})\right|^2\left[e^{i\lambda h}-e^{i\lambda t}-e^{i\lambda(t+h)}+1\right]f_{X_1}(\lambda),$$
$$j=1,\ldots,n,\ h>0 \qquad \ldots(18)$$

and for a given $f_{X_1}(\lambda)$, the spectral density function of first solution $y_j(t)$ can be found.

4.2 The Spectral Density Function of the Second Solution of the Canonical Form of the System of Random Linear Inequalities:

The relation (17) can be written as

$$\int_{-\infty}^{\infty} e^{i\lambda h} f_{y_j}(\lambda)d\lambda \le \sum_{1=1}^{n}(\frac{1}{k+1})^2 \left|\beta_{j1} - \sum_{m=1}^{n} a_{jm}c_{m1}\right|^2 (t(t+h))^{k+1} \int_{-\infty}^{\infty} f_{X_j}(\lambda)d\lambda$$

$$f_{y_j}(\lambda) \le \frac{1}{e^{i\lambda h}} \sum_{1=1}^{n}(\frac{1}{k+1})^2 \left|\beta_{j1} - \sum_{m=1}^{n} a_{jm}c_{m1}\right|^2 (t(t+h))^{k+1} f_{X_j}(\lambda),$$

$$j = 1,\ldots,n,\ k = 0,\ldots,n,\ h > 0 \qquad \ldots(19)$$

and for a given $f_{X_1}(\lambda)$, the spectral density function of second solution $y_j(t)$ can be found.

5- Cross Correlation Function of the Solutions of the Canonical Form of the System of Random Linear Inequalities:

We have in [2]

$$B_{y_j y_w}(t, t+h) \le E\left\{\left[\overline{y_j(t) - E(y_j(t)}\right]\left[y_w(t+h) - E(y_w(t+h)\right]\right\}$$

$$B_{y_j y_w}(t, t+h) \le E\left[\overline{y_j(t)}y_w(t+h)\right] - E\left[\overline{y_j(t)}E(y_w(t+h))\right] -$$

$$E\left[E\overline{y_j(t)}y_w(t+h)\right] + E\left[E\overline{y_j(t)}\,E(y_w(t+h))\right]$$

$$\le E\left[\overline{y_j(t)}y_w(t+h)\right] - E\left[\overline{y_j(t)}\right]E\left[y_w(t+h)\right] -$$

$$E\left[\overline{y_j(t)}\right]E\left[y_w(t+h)\right] + E\left[\overline{y_j(t)}\right]E\left[y_w(t+h)\right]$$

Hence

$$B_{y_j y_w}(t, t+h) \le E\left[\overline{y_j(t)}y_w(t+h)\right] - E\left[\overline{y_j(t)}\right]E\left[y_w(t+h)\right]$$

$$j, w = 1, 2\ldots, n,\ h > 0 \qquad \ldots(20)$$

where $y_j(t)$, $y_w(t+h)$ defined by equation (11).

5.1 Cross Correlation Function of the First Solution of the Canonical Form of the System of Random Linear Inequalities:

By (20), we have

$$\mathrm{E}\left[\overline{y_j(t)}y_w(t+h)\right]\le \mathrm{E}\left\{\left[\overline{\sum_{l=1}^{n}\int_{-\infty}^{\infty}\frac{1}{i\lambda}(\mathrm{B}_{j1}-\sum_{m=1}^{n}a_{jm}c_{m1})(e^{i\lambda t}-1)f_{\mathrm{X}_1}(\lambda)d\lambda}\right]\times\right.$$

$$\left.\left[\sum_{l=1}^{n}\int_{-\infty}^{\infty}\frac{1}{i\lambda}(\mathrm{B}_{w1}-\sum_{m=1}^{n}a_{wm}c_{m1})(e^{i\lambda(t+h)}-1)f_{\mathrm{X}_1}(\lambda)d\lambda\right]\right\}$$

$$\le \sum_{l=1}^{n}\int_{-\infty}^{\infty}\overline{(\mathrm{B}_{j1}-\sum_{m=1}^{n}a_{jm}c_{m1})}\,(\mathrm{B}_{w1}-\sum_{m=1}^{n}a_{wm}c_{m1})\,\overline{(\frac{e^{i\lambda t}-1}{i\lambda})}\times$$

$$(\frac{e^{i\lambda(t+h)}-1}{i\lambda})\mathrm{E}\left[\overline{f_{\mathrm{X}_1}(\lambda)d\lambda}\,f_{\mathrm{X}_1}(\lambda)d\lambda\right]$$

$$\le \sum_{l=1}^{n}\overline{(\mathrm{B}_{j1}-\sum_{m=1}^{n}a_{jm}c_{m1})}(\mathrm{B}_{w1}-\sum_{m=1}^{n}a_{wm}c_{m1})\int_{-\infty}^{\infty}\frac{1}{\lambda^2}\overline{(e^{i\lambda t}-1)}\times$$

$$(e^{i\lambda(t+h)}-1)f_{\mathrm{X}_1}(\lambda)d\lambda$$

$$\mathrm{E}\left[\overline{y_j(t)}\right]\mathrm{E}\left[y_w(t+h)\right]\le \mathrm{E}\left[\overline{\sum_{l=1}^{n}\int_{-\infty}^{\infty}\frac{1}{i\lambda}(\mathrm{B}_{j1}-\sum_{m=1}^{n}a_{jm}c_{m1})(e^{i\lambda t}-1)f_{\mathrm{X}_1}(\lambda)d\lambda}\right]\times$$

$$\mathrm{E}\left[\sum_{s=1}^{n}\int_{-\infty}^{\infty}\frac{1}{i\lambda}(\mathrm{B}_{js}-\sum_{m=1}^{n}a_{wm}c_{ms})(e^{i\lambda(t+h)}-1)f_{\mathrm{X}_s}(\lambda)d\lambda\right]$$

$$\le \sum_{\substack{l=1\\ l\ne s}}^{n}\sum_{s=1}^{n}\int_{-\infty}^{\infty}\overline{(\mathrm{B}_{j1}-\sum_{m=1}^{n}a_{jm}c_{m1})}(\mathrm{B}_{js}-\sum_{m=1}^{n}a_{wm}c_{ms})\times$$

$$\overline{(\frac{e^{i\lambda t}-1}{i\lambda})}(\frac{e^{i\lambda(t+h)}-1}{i\lambda})\mathrm{E}\left[\overline{f_{\mathrm{X}_1}(\lambda)d\lambda}\,f_{\mathrm{X}_s}(\lambda)d\lambda\right]$$

$$\le \sum_{\substack{l=1\\ l\ne s}}^{n}\sum_{s=1}^{n}\overline{(\mathrm{B}_{j1}-\sum_{m=1}^{n}a_{jm}c_{m1})}(\mathrm{B}_{js}-\sum_{m=1}^{n}a_{wm}c_{ms})\times$$

$$\int_{-\infty}^{\infty}\frac{1}{\lambda^2}(e^{-i\lambda t}-1)(e^{i\lambda(t+h)}-1)f_{\mathrm{X}_1,\mathrm{X}_s}(\lambda)d\lambda$$

Hence,

$$B_{y_j y_w}(t, t+h) \le E\left[\overline{y_j(t)} y_w(t+h)\right] - E\left[\overline{y_j(t)}\right] E\left[y_w(t+h)\right]$$

$$\le \int_{-\infty}^{\infty} \frac{1}{\lambda^2} \overline{(e^{i\lambda t}-1)} (e^{i\lambda(t+h)}-1) \left\{ \sum_{1=1}^{n} \overline{(B_{j1} - \sum_{m=1}^{n} a_{jm} c_{m1})} \right.$$

$$(B_{w1} - \sum_{m=1}^{n} a_{wm} c_{m1}) f_{X_1}(\lambda) - \sum_{\substack{1=1 \\ 1 \ne s}}^{n} \sum_{s=1}^{n} \overline{(B_{j1} - \sum_{m=1}^{n} a_{jm} c_{m1})}$$

$$\left. (B_{js} - \sum_{m=1}^{n} a_{wm} c_{ms}) f_{X_1, X_s}(\lambda) \right\} d\lambda, \quad j, w = 1, 2, \ldots, n$$

5.2 Cross Correlation Function of the Second Solution of the Canonical Form of the System of Random Linear Inequalities::

By taking $y_j(t)$, $y_w(t+h)$ defined by equation (15) and by using inequality (20), which is

$$B_{y_j y_w}(t, t+h) \le E\left[\overline{y_j(t)} y_w(t+h)\right] - E\left[\overline{y_j(t)}\right] E\left[y_w(t+h)\right]$$

where

$$E\left[\overline{y_j(t)} y_w(t+h)\right] \le E\left\{ \left[\overline{\sum_{1=1}^{n} \int_{-\infty}^{\infty} \frac{1}{k+1} (B_{j1} - \sum_{m=1}^{n} a_{jm} c_{m1}) t^{k+1} f_{X_1}(\lambda) d\lambda} \right] \times \right.$$

$$\left. \left[\sum_{1=1}^{n} \int_{-\infty}^{\infty} \frac{1}{k+1} (B_{w1} - \sum_{m=1}^{n} a_{wm} c_{m1}) (t+h)^{k+1} f_{X_1}(\lambda) d\lambda \right] \right\}$$

$$E\left[\overline{y_j(t)} y_w(t+h)\right] \le \sum_{1=1}^{n} \left(\frac{1}{k+1}\right)^2 \overline{(B_{j1} - \sum_{m=1}^{n} a_{jm} c_{m1})} (B_{w1} - \sum_{m=1}^{n} a_{wm} c_{m1}) \times$$

$$(t(t+h))^{k+1} \int_{-\infty}^{\infty} f_{X_1}(\lambda) d\lambda$$

or

$$E\left[\overline{y_j(t)} y_w(t+h)\right] \le \sum_{1=1}^{n} \left(\frac{1}{k+1}\right)^2 \overline{(B_{j1} - \sum_{m=1}^{n} a_{jm} c_{m1})} (B_{w1} - \sum_{m=1}^{n} a_{wm} c_{m1}) \times (t(t+h))^{k+1}$$

$$E\left[\overline{y_j(t)}\right]E\left[y_w(t+h)\right] \le E\left[\overline{\sum_{l=1}^{n}\int_{-\infty}^{\infty}\frac{1}{k+1}(B_{jl}-\sum_{m=1}^{n}a_{jm}c_{ml})t^{k+1}f_{X_l}(\lambda)d\lambda}\right]\times$$

$$E\left[\sum_{s=1}^{n}\int_{-\infty}^{\infty}\frac{1}{k+1}(B_{js}-\sum_{m=1}^{n}a_{wm}c_{ms})(t+h)^{k+1}f_{X_s}(\lambda)d\lambda\right]$$

$$\le \sum_{\substack{l=1\\ l\neq s}}^{n}\sum_{s=1}^{n}(\frac{1}{k+1})^2\ \overline{(B_{jl}-\sum_{m=1}^{n}a_{jm}c_{ml})}(B_{js}-\sum_{m=1}^{n}a_{wm}c_{ms})\times$$

$$(t(t+h))^{k+1}\int_{-\infty}^{\infty}f_{X_l,X_s}(\lambda)d\lambda$$

but $\int_{-\infty}^{\infty}f_{X_l,X_s}(\lambda)d\lambda = 1$, hence

$$B_{y_jy_w}(t,t+h) \le E\left[\overline{y_j(t)}y_w(t+h)\right]-E\left[\overline{y_j(t)}\right]E\left[y_w(t+h)\right]$$

$$\le (\frac{1}{k+1})^2(t(t+h))^{k+1}\left\{\sum_{l=1}^{n}\overline{(B_{jl}-\sum_{m=1}^{n}a_{jm}c_{ml})}(B_{wl}-\sum_{m=1}^{n}a_{wm}c_{ml})-\right.$$

$$\left.\sum_{\substack{l=1\\ l\neq s}}^{n}\sum_{s=1}^{n}\overline{(B_{jl}-\sum_{m=1}^{n}a_{jm}c_{ml})}(B_{js}-\sum_{m=1}^{n}a_{wm}c_{ms})\right\}$$

j, *w* = 1, 2,…, *n* , *k* = 0, …, *n*.

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