On Soft Pre-α-Open Sets and Soft Pre-α-continuous Functions in Soft Topological Spaces

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Abstract:

In this paper, we introduce a new class of generalized soft open sets in soft topological spaces, namely, soft pre-α-open sets and we show that the family of all soft pre-α-open subsets of a soft topological space $(X,\tilde{\tau},E)$ form a soft topology on X which is soft finer than $\tilde{\tau}$. Soft pre-α-open sets is stronger than each of soft semi-open sets, soft α-open sets, soft pre-open sets, soft b-open sets, soft β-open sets, soft generalized semi open sets, soft generalized α-open sets and soft α-generalized open sets and weaker than soft open sets. Moreover, we use these soft sets to define and study new classes of soft functions, namely, soft pre-α-continuous functions and soft pre-α-irresolute functions in soft topological spaces and we discuss the relation between these soft functions and each of soft continuous functions and other weaker forms of soft continuous functions.

Keywords: soft pre-α-open sets, soft pre-α-closed sets, soft pre-α-clopen sets, soft pre-α-continuous functions, soft pre-α-irresolute functions.

حول المجموعات المفتوحة الميسرة من النمط-Pre−α والدوال المستمرة الميسرة من النمط-Pre−α في الفضاءات التبولوجية الميسرة

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الخلاصة:

قدمنا في هذا البحث صنفا جديد معمما من المجموعات المفتوحة الميسرة في الفضاءات التبولوجية الميسرة أسميناها بالمجموعات المفتوحة الميسرة من النمط-Pre−α ومن ثم اثبتنا ان عائلة كل المجموعات الجزئية المفتوحة الميسرة من النمط -Pre−α من الفضاء التبولوجي الميسر $(X,\tilde{\tau},E)$ تشكل تبولوجيا ميسرة على X الذي هو انعم ميسرمن $\tilde{\tau}$. المجموعات المفتوحة

الميسرة من النمط–Pre–α هي اقوى من كل من المجموعات شبة المفتوحة الميسرة ، المجموعات المفتوحة الميسرة من النمط–α، المجموعات المفتوحة الميسرة من النمط–Pre ، المجموعات المفتوحة الميسرة من النمط–b، المجموعات المفتوحة الميسرة من النمط–β ، المجموعات شبة المفتوحة المعمة الميسرة، المجموعات المفتوحة من النمط–α المعممة الميسرة والمجموعات المفتوحة المعممة من النمط–α الميسرة واضعف من المجموعات المفتوحة الميسرة. فضلا عن ذلك استخدمنا هذه المجموعات الميسرة في تعريف ودراسة اصناف جديد من الدوال الميسرة أسميناها بالدوال المستمرة الميسرة من النمط –Pre–α والدوال المحيرة الميسرة من النمط–Pre–α في الفضاءات التبولوجية الميسرة وقد ناقشنا العلاقة بين هذة الدوال الميسرة وكل من الدوال المستمرة الميسرة والصيغ الاضعف الاخرى من الدوال المستمرة الميسرة.

Introduction

Molodtsov [14] initiated a novel concept of soft set theory, which is a completely new approach for modeling vagueness and uncertainty. He successfully applied the soft set theory into several directions such as game theory, theory of measurement, Riemann Integration, smoothness of functions, and so on. In recent years the soft set theory and its applications have shown great development, because of the general nature of parameterization expressed by a soft set. Shabir and Naz [16] introduced the soft topological spaces which are defined over an initial universe with a fixed set of parameters. Chen [8], Akdag and Ozkan [3,2], Arockiarani and Arokia Lancy [6] introduced and investigated soft semi-open sets, soft α-open sets, soft b-open sets, soft pre-open sets and soft β-open sets respectively. Also, Kannan [11], Al-Salem [5], Arockiarani and Arokia Lancy [6] and Kannan and Rajalakshmi [10] introduced and studied soft generalized open sets, soft generalized α-open sets, soft α-generalized open sets, soft generalized semi-open sets and soft s*g-open sets respectively. In the present paper, we introduce a new class of generalized soft open sets in soft topological spaces, namely, soft pre-α-open sets and we show that the family of all soft pre-α-open subsets of a soft topological space $(X,\tilde{\tau},E)$ form a soft topology on X which is soft finer than $\tilde{\tau}$. This class of soft open sets is placed properly between the class of soft open sets and each of soft semi-open sets, soft α-open sets, soft pre-open sets, soft b-open sets, soft β-open sets, soft generalized semi open sets, soft generalized α-open sets and soft α-generalized open sets respectively. Also, we study the characterizations and basic properties of soft pre-α-open sets and soft pre-α-closed sets. Moreover, we use these soft sets to define and study new classes of soft functions, namely, soft pre-α-continuous functions and soft pre-α-irresolute functions in soft topological spaces and we discuss the relation between these soft functions and each of soft continuous functions and other weaker forms of soft continuous functions.

1.Preliminaries

First we recall the following definitions and theorems.

Definition(1.1)[14]: Let X be an initial universe and P(X) denote the power set of X. Let E be a set of parameters and A be a non-empty subset of E.A pair (F,A) is said to be a soft set over X, where F is a mapping given by $F: A \rightarrow P(X)$.

Definition(1.2)[13]: A soft set (F,A) over X is called a null soft set if for each $e \in A$, $F(e) = \phi$, and is denoted by $\tilde{\phi}$.

Definition(1.3)[13]: A soft set (F,A) over X is called an absolute soft set if for each $e \in A$, $F(e) = X$, and is denoted by $\tilde{X}$.

Definition(1.4)[13]: Let (F_1, A_1) and (F_2, A_2) be two soft sets over a common universe X. Then (F_1, A_1) is said to be a soft subset of (F_2, A_2), and is denoted by $(F_1, A_1) \tilde{\subseteq} (F_2, A_2)$ if $A_1 \subseteq A_2$ and $F_1(e) \subseteq F_2(e)$ for each $e \in A_1$.

Definition(1.5)[13]: Let (F_1, A_1) and (F_2, A_2) be two soft sets over a common universe X. The union of (F_1, A_1) and (F_2, A_2) is the soft set (F_3, A_3), where $A_3 = A_1 \cup A_2$, and $\forall e \in A_3$,

$$F_3(e) = \begin{cases} F_1(e) & \text{if } e \in A_1 - A_2 \\ F_2(e) & \text{if } e \in A_2 - A_1 \\ F_1(e) \cup F_2(e) & \text{if } e \in A_1 \cap A_2 \end{cases}$$

And we write $(F_3, A_3) = (F_1, A_1) \tilde{\cup} (F_2, A_2)$.

Definition(1.6)[13]: Let (F_1, A_1) and (F_2, A_2) be two soft sets over a common universe X. The intersection of (F_1, A_1) and (F_2, A_2) is the soft set (F_3, A_3), where $A_3 = A_1 \cap A_2$, and $\forall e \in A_3$, $F_3(e) = F_1(e) \cap F_2(e)$. And we write $(F_3, A_3) = (F_1, A_1) \tilde{\cap} (F_2, A_2)$.

Definition(1.7)[4]: Let (F,A) be a soft set over X. The relative complement of (F,A) is denoted by $(F, A)^c$ and is defined by $(F, A)^c = (F^c, A)$, where $F^c : A \rightarrow p(X)$ is a mapping given by $F^c(e) = X - F(e)$ for each $e \in A$.

Definition(1.8)[9]: Let (F,A) a soft set over X. Then $\tilde{x} = (e,\{x\})$ is said to be a non-empty soft element of (F,A) if $e \in A$ and $x \in F(e)$. The pair (e,ϕ) is said to be an empty soft element of (F,A). Nonempty soft elements of (F,A) and empty soft elements of (F,A) are said to be the soft elements of (F,A). If $\tilde{x}$ is a soft element of (F,A) will be denoted by $\tilde{x} \tilde{\in} (F,A)$.

Definition(1.9)[16]:. Let $\tilde{\tau}$ be the collection of soft sets over X. Then $\tilde{\tau}$ is said to be a soft topology on X if $\tilde{\tau}$ satisfies the following axioms:

i) $\tilde{\phi}$, $\tilde{X}$ belong to $\tilde{\tau}$.

ii) The union of any number of soft sets in $\tilde{\tau}$ belongs to $\tilde{\tau}$.

iii) The intersection of any two soft sets in $\tilde{\tau}$ belongs to $\tilde{\tau}$.

The triplet $(X,\tilde{\tau},E)$ is said to be a soft topological space over X. The members of $\tilde{\tau}$ are said to be soft open sets in X.

Definition(1.10)[7]: Let (A,E) be a soft subset of a soft topological space $(X,\tilde{\tau},E)$. Then:

i) The soft closure of (A,E), denoted by cl(A,E) is the intersection of all soft closed sets in $(X,\tilde{\tau},E)$ which contains (A,E).

ii) The soft interior of (A,E), denoted by int(A,E) is the union of all soft open sets in $(X,\tilde{\tau},E)$ which are contained in (A,E).

Definition(1.11): A soft subset (A,E) of a soft topological space $(X,\tilde{\tau},E)$ is said to be:

i) A soft semi-open (briefly soft s-open) set [8] if $(A,E) \tilde{\subseteq} cl(int(A,E))$.

ii) A soft α-open set [3] if $(A,E) \tilde{\subseteq} int(cl(int(A,E)))$.

iii) A soft pre-open set [6] if $(A,E) \tilde{\subseteq} int(cl(A,E))$.

iv) A soft b-open set [2] if $A \tilde{\subseteq} int(cl(A,E)) \tilde{\cup} cl(int(A,E))$.

v) A soft β-open set [6] if $(A,E) \tilde{\subseteq} cl(int(cl(A,E)))$.

Definition(1.12): Let (A,E) be a soft subset of a soft topological space $(X,\tilde{\tau},E)$. Then:

i) The soft semi-closure [12] (resp. soft α-closure [3], soft pre-closure [1]) of (A,E) is the intersection of all soft semi-closed (resp. soft α-closed, soft pre-closed) sets in $(X,\tilde{\tau},E)$ which contains (A,E) and is denoted by $scl(A,E)$ (resp. $\alpha cl(A,E)$, $pcl(A,E)$). Clearly $scl(A,E) \tilde{\subseteq} \alpha cl(A,E) \tilde{\subseteq} cl(A,E)$.

ii) The soft pre-interior of (A,E) [1], denoted by pint(A,E) is the union of all soft pre-open sets in $(X,\tilde{\tau},E)$ which are contained in (A,E).

Theorem(1.13)[1]: Let (A,E) and (B,E) be soft subsets of a soft topological space $(X,\tilde{\tau},E)$. Then:

i) $(A,E)\tilde{\subseteq}pcl(A,E)\tilde{\subseteq}cl(A,E)$.

ii) $int(A,E)\tilde{\subseteq}p\,int(A,E)\tilde{\subseteq}(A,E)$.

iii) If $(A,E)\tilde{\subseteq}(B,E)$, then $pcl(A,E)\tilde{\subseteq}pcl(B,E)$.

iv) (A,E) is soft pre-closed iff $pcl(A,E)=(A,E)$.

v) $pcl(pcl(A,E))=pcl(A,E)$.

vi) $(p\,int(A,E))^c=pcl((A,E)^c)$.

vii) $\tilde{x}\tilde{\in}pcl(A,E)$ iff for every soft pre-open set (U,E) containing $\tilde{x}$, $(U,E)\tilde{\cap}(A,E)\neq\tilde{\phi}$.

viii) $\bigcup_{\alpha\in\Lambda}pcl(A_\alpha,E)\tilde{\subseteq}pcl(\bigcup_{\alpha\in\Lambda}(A_\alpha,E))$.

Definition(1.14): A soft subset (A,E) of a soft topological space $(X,\tilde{\tau},E)$ is said to be:

i) A soft generalized closed (briefly soft g-closed) set [11] if $cl(A,E)\tilde{\subseteq}(U,E)$ whenever $(A,E)\tilde{\subseteq}(U,E)$ and (U,E) is soft open in $(X,\tilde{\tau},E)$.

ii) A soft generalized semi-closed (briefly soft gs-closed) set [6] if $scl(A,E)\tilde{\subseteq}(U,E)$ whenever $(A,E)\tilde{\subseteq}(U,E)$ and (U,E) is soft open in $(X,\tilde{\tau},E)$.

iii) A soft generalized α-closed (briefly soft gα-closed) set [5] if $\alpha cl(A,E)\tilde{\subseteq}(U,E)$ whenever $(A,E)\tilde{\subseteq}(U,E)$ and (U,E) is soft α-open in $(X,\tilde{\tau},E)$.

iv) A soft α-generalized closed (briefly soft αg-closed) set [6] if $\alpha cl(A,E)\tilde{\subseteq}(U,E)$ whenever $(A,E)\tilde{\subseteq}(U,E)$ and (U,E) is soft open in $(X,\tilde{\tau},E)$.

v) A soft s*g-closed set [10] if $cl(A,E)\tilde{\subseteq}(U,E)$ whenever $(A,E)\tilde{\subseteq}(U,E)$ and (U,E) is soft s-open in $(X,\tilde{\tau},E)$.

The complement of a soft g-closed (resp. soft gs-closed, soft gα-closed, soft αg-closed, soft s*g-closed) set is called a soft g-open (resp. soft gs-open, soft gα-open, soft αg-open, soft s*g-open) set.

Definition(1.15): A soft function $f:(X,\tilde{\tau},E_1)\to(Y,\tilde{\sigma},E_2)$ is called:

i) soft continuous [15] if $f^{-1}((V,E_2))$ is soft open in $(X,\tilde{\tau},E_1)$ for every soft open set (V,E_2) in $(Y,\tilde{\sigma},E_2)$

ii) soft semi-continuous (briefly soft s-continuous) [12] if $f^{-1}((V,E_2))$ is soft s-open in $(X,\tilde{\tau},E_1)$ for every soft open set (V,E_2) in $(Y,\tilde{\sigma},E_2)$.

iii) soft α-continuous [17] if $f^{-1}((V,E_2))$ is soft α-open in $(X,\tilde{\tau},E_1)$ for every soft open set (V,E_2) in $(Y,\tilde{\sigma},E_2)$.

iv) soft pre-continuous [17] if $f^{-1}((V,E_2))$ is soft pre-open in $(X,\tilde{\tau},E_1)$ for every soft open set (V,E_2) in $(Y,\tilde{\sigma},E_2)$.

v) soft b-continuous [2] if $f^{-1}((V,E_2))$ is soft b-open in $(X,\tilde{\tau},E_1)$ for every soft open set (V,E_2) in $(Y,\tilde{\sigma},E_2)$.

vi) soft β-continuous [17] if $f^{-1}((V,E_2))$ is soft β-open in $(X,\tilde{\tau},E_1)$ for every soft open set (V,E_2) in $(Y,\tilde{\sigma},E_2)$.

vii) soft generalized continuous (briefly soft g-continuous) if $f^{-1}((V,E_2))$ is soft g-open in $(X,\tilde{\tau},E_1)$ for every soft open set (V,E_2) in $(Y,\tilde{\sigma},E_2)$.

viii) soft generalized semi continuous (briefly soft gs-continuous) if $f^{-1}((V,E_2))$ is soft gs-open in $(X,\tilde{\tau},E_1)$ for every soft open set (V,E_2) in $(Y,\tilde{\sigma},E_2)$.

ix) soft generalized α-continuous(briefly soft gα-continuous) if $f^{-1}((V,E_2))$ is soft gα-open in $(X,\tilde{\tau},E_1)$ for every soft open set (V,E_2) in $(Y,\tilde{\sigma},E_2)$.

x) soft α-generalized continuous (briefly soft αg-continuous) if $f^{-1}((V,E_2))$ is soft αg-open in $(X,\tilde{\tau},E_1)$ for every soft open set (V,E_2) in $(Y,\tilde{\sigma},E_2)$.

xi) soft s*g-continuous if $f^{-1}((V,E_2))$ is soft s*g-open in $(X,\tilde{\tau},E_1)$ for every soft open set (V,E_2) in $(Y,\tilde{\sigma},E_2)$.

Proposition(1.16): Let $(X,\tilde{\tau},E)$ be a soft topological space. If (U,E) is a soft open set in $(X,\tilde{\tau},E)$ and (A,E) is a soft pre-open set in $(X,\tilde{\tau},E)$, then $(U,E)\,\tilde{\cap}\,(A,E)$ is a soft pre-open set in $(X,\tilde{\tau},E)$.

Proof: Since (A,E) is a soft pre-open set in $(X,\tilde{\tau},E)$, then $(A,E)\,\tilde{\subseteq}\,\text{int}(\text{cl}(A,E))$. Thus $(U,E)\,\tilde{\cap}\,(A,E)\,\tilde{\subseteq}\,(U,E)\,\tilde{\cap}\,\text{int}(\text{cl}(A,E)) = \text{int}((U,E)\,\tilde{\cap}\,\text{cl}(A,E))$ $\tilde{\subseteq}\,\text{int}(\text{cl}((U,E)\,\tilde{\cap}\,(A,E)))$. Hence $(U,E)\,\tilde{\cap}\,(A,E)$ is a soft pre-open set in $(X,\tilde{\tau},E)$.

2. Properties of Soft Pre-α-Open Sets

In this section we introduce a new class of soft open sets, namely, soft pre-α-open sets and we show that the family of all soft pre-α-open subsets of a soft topological space $(X,\tilde{\tau},E)$ form a soft topology on X which is soft finer than $\tilde{\tau}$. Also, we discuss the relation between these soft open sets and each of soft open sets and other weaker forms of soft open sets.

Definition(2.1): A soft subset (A,E) of a soft topological space $(X,\tilde{\tau},E)$ is called a soft pre-α-open set if $(A,E)\,\tilde{\subseteq}\,\text{int}(\text{pcl}(\text{int}(A,E)))$. The complement of a soft pre-α-open set is defined to be soft pre-α-closed. The family of all soft pre-α-open subsets of $(X,\tilde{\tau},E)$ is denoted by $\tilde{\tau}^{pre-\alpha}$.

Clearly, every soft open set is soft pre-α-open, but the converse is not true. Consider the following example.

Example(2.2): Let $X=\{x_1,x_2,x_3\}$, $E=\{e_1,e_2\}$ and $\tilde{\tau}=\{\tilde{X},\tilde{\phi},(F,E)\}$ be a soft topology over X, where $(F,E)=\{(e_1,F(e_1)),(e_2,F(e_2))\}=\{(e_1,\{x_1\}),(e_2,\{x_1,x_2,x_3\})\}$. Then $(A,E)=\{(e_1,A(e_1)),(e_2,A(e_2))\}=\{(e_1,\{x_1,x_2\}),(e_2,\{x_1,x_2,x_3\})\}$ is a soft pre-α-open set in $(X,\tilde{\tau},E)$, since $(A,E)\tilde{\subseteq}$ $int(pcl(int(A,E)))=int(pcl(F,E))=int(\tilde{X})=\tilde{X}$. But (A,E) is not soft open in $(X,\tilde{\tau},E)$.

Remark(2.3): soft s*g-open sets and soft pre-α-open sets are in general independent. Consider the following examples:

Example(2.4): Let $X=\{x_1,x_2,x_3\}$, $E=\{e_1,e_2,e_3\}$ and $\tilde{\tau}=\{\tilde{X},\tilde{\phi}\}$ be a soft topology over X. Then $(B,E)=\{(e_1,\{x_1,x_3\}),(e_2,\{\phi\}),(e_3,\{\phi\})\}$ is a soft s*g-open set in $(X,\tilde{\tau},E)$, but is not a soft pre-α-open set, since $(B,E)\tilde{\not\subset}$ $int(pcl(int(B,E)))=int(pcl(\tilde{\phi}))=int(\tilde{\phi})=\tilde{\phi}$. Also, in example (2.2) (A,E) is a soft pre-α-open set in $(X,\tilde{\tau},E)$, but is not soft s*g-open, since $(A,E)^c=$ $\{(e_1,\{x_3\}),(e_2,\{\phi\})\}$ is not a soft s*g-closed set in $(X,\tilde{\tau},E)$, since $(U,E)=$ $\{(e_1,\{x_1,x_3\}),(e_2,\{x_1,x_2,x_3\})\}$ is a soft semi-open set in $(X,\tilde{\tau},E)$ and $(A,E)^c\tilde{\subseteq}(U,E)$, but $cl((A,E)^c)=\{(e_1,\{x_2,x_3\}),(e_2,\{\phi\})\}\tilde{\not\subset}(U,E)$.

Theorem(2.5): Every soft pre-α-open set is soft α-open (resp. soft αg-open, soft gα-open, soft pre-open, soft b-open, soft β-open) set.

Proof: Let (A,E) be any soft pre-α-open set in $(X,\tilde{\tau},E)$, then $(A,E)\tilde{\subseteq}$ $int(pcl(int(A,E)))$. Since $int(pcl(int(A,E)))\tilde{\subseteq}int(cl(int(A,E)))$, thus (A,E) $\tilde{\subseteq}int(cl(int(A,E)))$. Therefore (A,E) is a soft α-open set in $(X,\tilde{\tau},E)$. Since every soft α-open set is soft αg-open (resp. soft gα-open, soft pre-open, soft b-open, soft β-open) set. Thus every soft pre-α-open set is soft α-open (resp. soft αg-open, soft gα-open, soft pre-open, soft b-open and soft β-open) set.

Remark(2.6): The converse of theorem (2.5) may not be true in general. In example (2.4) $(B,E)=\{(e_1,\{x_1,x_3\}),(e_2,\{\phi\}),(e_3,\{\phi\})\}$ is soft pre-open (resp. soft αg-open, soft gα-open, soft b-open, soft β-open) in $(X,\tilde{\tau},E)$, but is not soft pre-α-open set in $(X,\tilde{\tau},E)$.

Theorem(2.7): Every soft pre-α-open set is soft semi-open and soft gs-open set.

Proof: Let (A,E) be any soft pre-α-open set in $(X,\tilde{\tau},E)$, then $(A,E)\,\tilde{\subseteq}$ $\text{int}(\text{pcl}(\text{int}(A,E)))$. Since $\text{int}(\text{pcl}(\text{int}(A,E)))\,\tilde{\subseteq}\,\text{pcl}(\text{int}(A,E))\,\tilde{\subseteq}\,\text{cl}(\text{int}(A,E))$, thus $(A,E)\,\tilde{\subseteq}\,\text{cl}(\text{int}(A,E))$. Therefore (A,E) is a soft semi-open set in $(X,\tilde{\tau},E)$. Since every soft semi-open set is soft gs-open set. Thus every soft pre-α-open set is a soft semi-open and a soft gs-open set.

Remark(2.8): The converse of theorem (2.7) may not be true in general as shown in the following example.

Example(2.9): Let $X=\{x_1,x_2,x_3\}$, $E=\{e_1,e_2\}$ and $\tilde{\tau}=\{\tilde{X},\tilde{\phi},(F_1,E),(F_2,E),(F_3,E)\}$ be a soft topology over X, where $(F_1,E)=\{(e_1,\{x_1\}),(e_2,\{x_1\})\}$, $(F_2,E)=\{(e_1,\{x_2\}),(e_2,\{x_2\})\}$ and $(F_3,E)=\{(e_1,\{x_1,x_2\}),(e_2,\{x_1,x_2\})\}$. Then the soft set $(A,E)=\{(e_1,\{x_1,x_3\}),(e_2,\{x_1,x_3\})\}$ is soft semi-open and soft gs-open in $(X,\tilde{\tau},E)$, but is not soft pre-α-open, since $(A,E)\,\tilde{\not\subset}$ $\text{int}(\text{pcl}(\text{int}(A,E)))=(F_1,E)$.

Remark(2.10): soft pre-open sets and soft αg-open sets are in general independent. Consider the following examples:

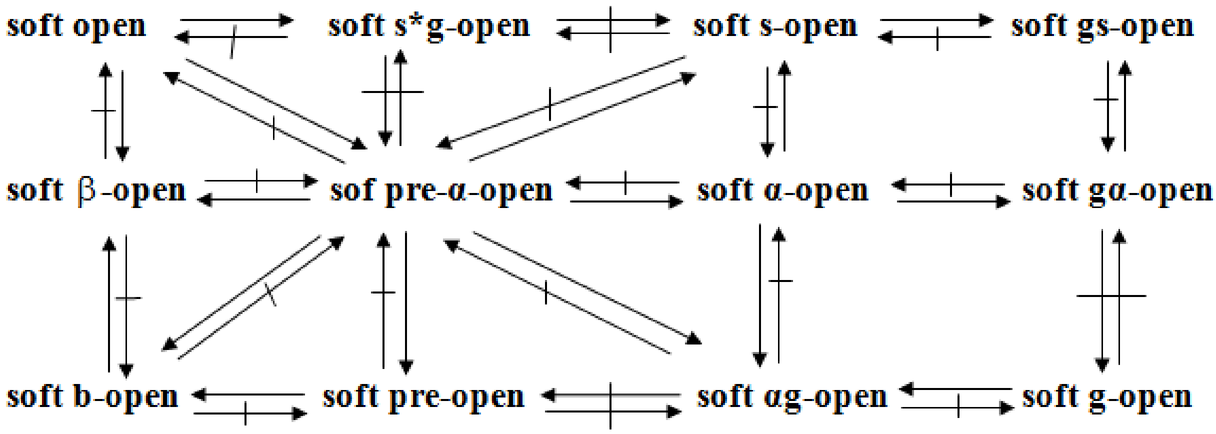
Example(2.11): Let $(\Re,\tilde{\mu},\Re)$ be the soft usual topological space. Then the soft set of all rational numbers $\tilde{Q}$ is a soft pre-open set, but is not a soft αg-open set. Also, in example (2.2) $(B,E)=\{(e_1,\{x_2\}),(e_2,\{\phi\})\}$ is a soft αg-open set in $(X,\tilde{\tau},E)$, since $(B,E)^c=\{(e_1,\{x_1,x_3\}),(e_2,\{x_1,x_2,x_3\})\}$ is a soft αg-closed set, but is not a soft pre-open set, since $(B,E)\,\tilde{\not\subset}\,\text{int}(\text{cl}(B,E))$ $=\text{int}(\{(e_1,\{x_2,x_3\}),(e_2,\{\phi\})\})=\tilde{\phi}$

Remark(2.12): soft g-open sets and soft gα-open sets are in general independent. Consider the following examples:

Example(2.13): Let $X=\{x_1,x_2,x_3\}$, $E=\{e_1,e_2\}$ and $\tilde{\tau}=\{\tilde{X},\tilde{\phi},(F_1,E),(F_2,E)\}$ be a soft topology over X, where $(F_1,E)=\{(e_1,\{x_1\}),(e_2,\{x_1,x_2,x_3\})\}$ and $(F_2,E)=\{(e_1,\{x_1,x_3\}),(e_2,\{x_1,x_2,x_3\})\}$. Then the soft set $(A,E)=$

$\{(e_1,\{x_1,x_2\}),(e_2,\{x_1,x_2,x_3\})\}$ is a soft gα-open set in $(X,\tilde{\tau},E)$, since $(A,E)^c=\{(e_1,\{x_3\}),(e_2,\{\phi\})\}$ is soft gα-closed, but is not a soft g-open set in $(X,\tilde{\tau},E)$, since $(A,E)^c$ is not soft g-closed. Also, in example (2.2) (B,E) $=\{(e_1,\{x_3\}),(e_2,\{\phi\})\}$ is a soft g-open set in $(X,\tilde{\tau},E)$, since $(B,E)^c=$ $\{(e_1,\{x_1,x_2\},(e_2,\{x_1,x_2,x_3\})\}$ is soft g-closed, but is not a soft gα-open set in $(X,\tilde{\tau},E)$, since $(B,E)^c$ is not soft gα-closed.

The following diagram shows the relationships between soft pre-α-open sets and some other soft open sets:



Proposition(2.14): A soft subset (A,E) of a soft topological space $(X,\tilde{\tau},E)$ is soft pre-α-open if and only if there exists a soft open set (U,E) in $(X,\tilde{\tau},E)$ such that $(U,E)\tilde{\subseteq}(A,E)\tilde{\subseteq}\text{int}(\text{pcl}(U,E))$.

Proof: $\Rightarrow$ Suppose that (A,E) is a soft pre-α-open set in $(X,\tilde{\tau},E)$, then $(A,E)\tilde{\subseteq}\text{int}(\text{pcl}(\text{int}(A,E)))$. Since $\text{int}(A,E)\tilde{\subseteq}(A,E)$, thus $\text{int}(A,E)\tilde{\subseteq}$ $(A,E)\tilde{\subseteq}\text{int}(\text{pcl}(\text{int}(A,E)))$. Put $(U,E)=\text{int}(A,E)$, hence there exists a soft open set (U,E) in $(X,\tilde{\tau},E)$ such that $(U,E)\tilde{\subseteq}(A,E)\tilde{\subseteq}\text{int}(\text{pcl}(U,E))$.
Conversely, suppose that there exists a soft open set (U,E) in $(X,\tilde{\tau},E)$ such that $(U,E)\tilde{\subseteq}(A,E)\tilde{\subseteq}\text{int}(\text{pcl}(U,E))$. Since $(U,E)\tilde{\subseteq}(A,E)$ $\Rightarrow$ $(U,E)\tilde{\subseteq}$ $\text{int}(A,E)\Rightarrow\text{pcl}(U,E)\tilde{\subseteq}\text{pcl}(\text{int}(A,E))\Rightarrow\text{int}(\text{pcl}(U,E))\tilde{\subseteq}\text{int}(\text{pcl}(\text{int}(A,E))$ $)$. Since $(A,E)\subseteq\text{int}(\text{pcl}(U,E))$, then $(A,E)\tilde{\subseteq}\text{int}(\text{pcl}(\text{int}(A,E)))$. Thus (A,E) is a soft pre-α-open set in $(X,\tilde{\tau},E)$.

Lemma(2.15): Let $(X,\tilde{\tau},E)$ be a soft topological space. If (U,E) is a soft open set in $(X,\tilde{\tau},E)$, then $(U,E)\tilde{\cap}\,\text{pcl}(A,E)\tilde{\subseteq}\text{pcl}((U,E)\tilde{\cap}(A,E))$ for any soft subset (A ,E) of $(X,\tilde{\tau},E)$.

Proof: Let $\tilde{x} \tilde{\in} (U,E) \tilde{\cap}$ pcl(A,E) and (V,E) be any soft pre-open set in $(X,\tilde{\tau},E)$ s.t $\tilde{x} \tilde{\in} (V,E)$. Since $\tilde{x} \tilde{\in}$ pcl(A,E), then by theorem ((1.13),vii), $(V,E) \tilde{\cap} (A,E) \neq \tilde{\phi}$. By proposition (1.16) $(U,E) \tilde{\cap} (V,E)$ is a soft pre-open set in $(X,\tilde{\tau},E)$ and $\tilde{x} \tilde{\in} (U,E) \tilde{\cap} (V,E)$, then $((V,E) \tilde{\cap} (U,E)) \tilde{\cap} (A,E) =$ $(V,E) \tilde{\cap} ((U,E) \tilde{\cap} (A,E)) \neq \tilde{\phi}$. Therefore $\tilde{x} \tilde{\in} \text{pcl}((U,E) \tilde{\cap} (A,E))$. Thus $(U,E) \tilde{\cap} \text{pcl}(A,E) \tilde{\subseteq} \text{pcl}((U,E) \tilde{\cap} (A,E))$ for any soft subset (A,E) of $(X,\tilde{\tau},E)$.

Theorem(2.16): Let $(X,\tilde{\tau},E)$ be a soft topological space. Then the family of all soft pre-α-open subsets of a soft topological space $(X,\tilde{\tau},E)$ form a soft topology on X.

Proof:(i). Since $\tilde{\phi} \subseteq \text{int}(\text{pcl}(\text{int}(\tilde{\phi})))$ and $\tilde{X} \subseteq \text{int}(\text{pcl}(\text{int}(\tilde{X})))$, then $\tilde{\phi}, \tilde{X} \tilde{\in} \tilde{\tau}^{\text{pre}-\alpha}$.

(ii). Let $(A,E),(B,E) \tilde{\in} \tilde{\tau}^{\text{pre}-\alpha}$. To prove that $(A,E) \tilde{\cap} (B,E) \tilde{\in} \tilde{\tau}^{\text{pre}-\alpha}$. By proposition (2.14), there exists $(U,E),(V,E) \tilde{\in} \tilde{\tau}$ such that $(U,E) \tilde{\subseteq} (A,E)$ $\tilde{\subseteq} \text{int}(\text{pcl}(U,E))$ and $(V,E) \tilde{\subseteq} (B,E) \tilde{\subseteq} \text{int}(\text{pcl}(V,E))$. Notice that $(U,E) \tilde{\cap} (V,E) \tilde{\in} \tilde{\tau}$ and $(U,E) \tilde{\cap} (V,E) \tilde{\subseteq} (A,E) \tilde{\cap} (B,E)$. Now,

$$\begin{aligned}(A,E) \tilde{\cap} (B,E) &\tilde{\subseteq} \text{int}(\text{pcl}(U,E)) \tilde{\cap} \text{int}(\text{pcl}(V,E)) \\ &= \text{int}(\text{int}(\text{pcl}(U,E)) \tilde{\cap} \text{pcl}(V,E)) \\ &\tilde{\subseteq} \text{int}(\text{pcl}(\text{int}(\text{pcl}(U,E)) \tilde{\cap} (V,E))) \text{ (by lemma (2.15))}. \\ &\tilde{\subseteq} \text{int}(\text{pcl}(\text{pcl}(U,E) \tilde{\cap} (V,E))) \\ &\tilde{\subseteq} \text{int}(\text{pcl}(\text{pcl}((U,E) \tilde{\cap} (V,E)))) \text{ (by lemma (2.15))}. \\ &= \text{int}(\text{pcl}((U,E) \tilde{\cap} (V,E))) \text{ (by theorem (1.13),v)}.\end{aligned}$$

Thus $(U,E) \tilde{\cap} (V,E) \tilde{\subseteq} (A,E) \tilde{\cap} (B,E) \tilde{\subseteq} \text{int}(\text{pcl}((U,E) \tilde{\cap} (V,E)))$.
Therefore by proposition (2.14), $(A,E) \tilde{\cap} (B,E) \tilde{\in} \tilde{\tau}^{\text{pre}-\alpha}$.

(iii). Let $\{(U_\alpha,E) : \alpha \in \Lambda\}$ be any family of soft pre-α-open sets in (X,τ,E), then $(U_\alpha,E) \tilde{\subseteq} \text{int}(\text{pcl}(\text{int}(U_\alpha,E)))$ for each $\alpha \in \Lambda$. Therefore by theorem ((1.13) viii), we get:

$$\begin{aligned}\bigcup_{\alpha\in\Lambda}(U_\alpha,E) &\tilde{\subseteq} \bigcup_{\alpha\in\Lambda}\text{int}(\text{pcl}(\text{int}(U_\alpha,E))) \tilde{\subseteq} \text{int}(\bigcup_{\alpha\in\Lambda}\text{pcl}(\text{int}(U_\alpha,E))) \\ &\tilde{\subseteq} \text{int}(\text{pcl}(\bigcup_{\alpha\in\Lambda}\text{int}(U_\alpha,E))) \tilde{\subseteq} \text{int}(\text{pcl}(\text{int}(\bigcup_{\alpha\in\Lambda}(U_\alpha,E)))).\end{aligned}$$

Hence $\bigcup_{\alpha\in\Lambda}(U_\alpha,E) \tilde{\in} \tilde{\tau}^{\text{pre}-\alpha}$. Thus $\tilde{\tau}^{\text{pre}-\alpha}$ is a soft topology on X.

Propositions(2.17): Let (B,E) be a soft subset of a soft topological space $(X,\tilde{\tau},E)$. Then the following statements are equivalent:

i) (B,E) is soft pre-α-closed.

ii) $cl(pint(cl(B,E))) \tilde{\subseteq} (B,E)$.

iii) There exists a soft closed subset (F,E) of $(X,\tilde{\tau},E)$ such that $cl(pint(F,E)) \tilde{\subseteq} (B,E) \tilde{\subseteq} (F,E)$.

Proof: $(i) \Rightarrow (ii)$. Since (B,E) is a soft pre-α-closed set in $(X,\tilde{\tau},E) \Rightarrow$ $(B,E)^c$ is soft pre-α-open $\Rightarrow (B,E)^c \tilde{\subseteq} int(pcl(int(B,E)^c)) \Rightarrow (B,E)^c \tilde{\subseteq}$ $int(pcl((cl(B,E))^c))$. By theorem ((1.13), vi), we get $(pint(cl(B,E)))^c =$ $pcl((cl(B,E))^c)$. Hence $(B,E)^c \tilde{\subseteq} int((pint(cl(B,E)))^c) \Rightarrow (B,E)^c \tilde{\subseteq}$ $(cl(pint(cl(B,E))))^c \Rightarrow cl(pint(cl(B,E))) \tilde{\subseteq} (B,E)$.

$(ii) \Rightarrow (iii)$.

Since $cl(pint(cl(B,E))) \tilde{\subseteq} (B,E)$ and $(B,E) \tilde{\subseteq} cl(B,E)$, then $cl(pint(cl(B,E))) \tilde{\subseteq} (B,E) \tilde{\subseteq} cl(B,E)$.Put $(F,E) = cl(B,E)$, thus there exists a soft closed subset (F,E) of $(X,\tilde{\tau},E)$ such that $cl(pint(F,E)) \tilde{\subseteq} (B,E) \tilde{\subseteq} (F,E)$.

$(iii) \Rightarrow (i)$.

Suppose that there exists a soft closed subset (F,E) of $(X,\tilde{\tau},E)$ such that $cl(pint(F,E)) \tilde{\subseteq} (B,E) \tilde{\subseteq} (F,E)$. Hence $(F,E)^c \tilde{\subseteq} (B,E)^c \tilde{\subseteq} (cl(pint(F,E)))^c =$ $int((pint(F,E))^c)$. Since $(pint(F,E))^c = pcl((F,E)^c)$, then $(F,E)^c \tilde{\subseteq} (B,E)^c \tilde{\subseteq}$ $int(pcl((F,E)^c))$. Hence $(B,E)^c$ is a soft pre-α-open set in $(X,\tilde{\tau},E)$. Thus (B,E) is a soft pre-α-closed set in $(X,\tilde{\tau},E)$.

Definition(2.18): A soft subset (A,E) of a soft topological space $(X,\tilde{\tau},E)$ is called a soft pre-α-neighborhood of a soft element $\tilde{x}$ in $\tilde{X}$ if there exists a soft pre-α-open set (U,E) in $(X,\tilde{\tau},E)$ such that $\tilde{x} \tilde{\in} (U,E) \tilde{\subseteq} (A,E)$.

Remark(2.19): Since every soft open set is a soft pre-α-open set, then every soft neighborhood of $\tilde{x}$ is a soft pre-α-neighborhood of $\tilde{x}$, but the converse is not true in general. In example (2.2) $(A,E) = \{(e_1,\{x_1,x_2\}),(e_2,$ $\{x_1,x_2,x_3\})\}$ is a soft pre-α-neighborhood of a soft element $\tilde{x} = (e_1,x_2)$, since $\tilde{x} \tilde{\in} (A,E) \tilde{\subseteq} (A,E)$. But (A,E) is not a soft neighborhood of $\tilde{x}$.

Propositions(2.20): A soft subset (A,E) of a soft topological space $(X,\tilde{\tau},E)$ is soft pre-α-open if and only if it is a soft pre-α-neighborhood of each of its soft elements.

Proof: $\Rightarrow$ If (A,E) is a soft pre-α-open set in $(X,\tilde{\tau},E)$, then $\tilde{x} \tilde{\in} (A,E) \tilde{\subseteq}$

(A,E) for each $\tilde{x}\tilde{\in}(A,E)$. Thus (A,E) is a soft pre-α-neighborhood of each of its soft elements. **Conversely,** suppose that (A,E) is a soft pre-α-neighborhood of each of its soft elements. Then for each $\tilde{x}\tilde{\in}(A,E)$, there exists a soft pre-α-open set $(U,E)_{\tilde{x}}$ in $(X,\tilde{\tau},E)$ such that $\tilde{x}\tilde{\in}(U,E)_{\tilde{x}}\tilde{\subseteq}$ (A,E). Hence $(A,E)\tilde{\subseteq}\bigcup_{\tilde{x}\tilde{\in}(A,E)}(U,E)_{\tilde{x}}\tilde{\subseteq}(A,E)$. Therefore $(A,E)=\bigcup_{\tilde{x}\tilde{\in}(A,E)}(U,E)_{\tilde{x}}$. Thus (A,E) is a soft pre-α-open set in $(X,\tilde{\tau},E)$, since it is a union of soft pre-α-open sets.

Proposition(2.21): If (A,E) is a soft pre-α-open set in a soft topological space $(X,\tilde{\tau},E)$ and $(A,E)\tilde{\subseteq}(B,E)\tilde{\subseteq}\operatorname{int}(\operatorname{pcl}(A,E))$, then (B,E) is a soft pre-α-open set in $(X,\tilde{\tau},E)$.

Proof: Since (A,E) is a soft pre-α-open set in $(X,\tilde{\tau},E)$, then by proposition (2.14), there exists a soft open set (U,E) in $(X,\tilde{\tau},E)$ such that $(U,E)\tilde{\subseteq}$ $(A,E)\tilde{\subseteq}\operatorname{int}(\operatorname{pcl}(U,E))$. Since $(A,E)\tilde{\subseteq}(B,E) \Rightarrow (U,E)\tilde{\subseteq}(B,E)$. But $\operatorname{int}(\operatorname{pcl}((A,E))\tilde{\subseteq}\operatorname{int}(\operatorname{pcl}(U,E)) \Rightarrow (U,E)\tilde{\subseteq}(B,E)\tilde{\subseteq}\operatorname{int}(\operatorname{pcl}(U,E))$. Thus (B,E) is a soft pre-α-open set in $(X,\tilde{\tau},E)$.

Corollary(2.22): If (A,E) is a soft pre-α-closed set in a soft topological space $(X,\tilde{\tau},E)$ and $\operatorname{cl}(\operatorname{pint}(A,E))\tilde{\subseteq}(B,E)\tilde{\subseteq}(A,E)$, then (B,E) is a soft pre-α-closed set in $(X,\tilde{\tau},E)$.

Proof: Since $(A,E)^c\tilde{\subseteq}(B,E)^c\tilde{\subseteq}(\operatorname{cl}(\operatorname{pint}(A,E)))^c=\operatorname{int}((\operatorname{pint}(A,E))^c)=$ $\operatorname{int}(\operatorname{pcl}((A,E)^c))$, then by proposition (2.21), $(B,E)^c$ is a soft pre-α-open set in $(X,\tilde{\tau},E)$. Thus (B,E) is a soft pre-α-closed set in $(X,\tilde{\tau},E)$.

Theorem(2.23): A soft subset (A,E) of a soft topological space $(X,\tilde{\tau},E)$ is soft clopen (soft open and soft closed) if and only if (A,E) is soft pre-α-clopen (soft pre-α-open and soft pre-α-closed).

Proof: $(\Rightarrow)$. It is a obvious.

$(\Leftarrow)$. Suppose that (A,E) is a soft pre-α-clopen set in $(X,\tilde{\tau},E)$, then (A,E) is soft pre-α-open and soft pre-α-closed in $(X,\tilde{\tau},E)$. Hence $(A,E)\tilde{\subseteq}\operatorname{int}(\operatorname{pcl}(\operatorname{int}(A,E)))$ and $\operatorname{cl}(\operatorname{pint}(\operatorname{cl}(A,E)))\tilde{\subseteq}(A,E)$. But by theorem ((1.13),i,ii) we get,
$\operatorname{pcl}(A,E)\tilde{\subseteq}\operatorname{cl}(A,E)$ and $\operatorname{int}(A,E)\tilde{\subseteq}\operatorname{pint}(A,E)$, thus:
$(A,E)\tilde{\subseteq}\operatorname{int}(\operatorname{cl}(\operatorname{int}(A,E)))$ and $\operatorname{cl}(\operatorname{int}(\operatorname{cl}(A,E)))\tilde{\subseteq}(A,E)$.
Since $\operatorname{int}(A,E)\tilde{\subseteq}(A,E) \Rightarrow \operatorname{cl}(\operatorname{int}(A,E))\tilde{\subseteq}\operatorname{cl}(A,E)$ --------(1)

Since $int(cl(int(A,E)))\tilde{\subseteq} cl(int(A,E))$, thus

$(A,E)\tilde{\subseteq} int(cl(int(A,E)))\tilde{\subseteq} cl(int(A,E)) \Rightarrow cl(A,E)\tilde{\subseteq} cl(int(A,E))$ ---------(2)

Therefore from (1) and (2), we get $cl(int(A,E)) = cl(A,E)$ ---------(3)

Similarly, since $(A,E)\tilde{\subseteq} cl(A,E) \Rightarrow int(A,E)\tilde{\subseteq} int(cl(A,E))$ --------- (4)

Now, $int(cl(A,E))\tilde{\subseteq} cl(int(cl(A,E)))\tilde{\subseteq}(A,E)$, thus

$int(cl(A,E))\tilde{\subseteq} int(A,E)$ ---------(5)

Therefore from (4) and (5), we get $int(cl(A,E)) = int(A,E)$ -------- (6)

Since $int(cl(A,E)) = int(A,E) \Rightarrow cl(int(cl(A,E))) = cl(int(A,E)) = cl(A,E)$ (by (3)). Since $cl(int(cl(A,E)))\tilde{\subseteq}(A,E)$, then $cl(A,E)\tilde{\subseteq}(A,E)$, but $(A,E)\tilde{\subseteq}$ $cl(A,E)$, therefore $(A,E) = cl(A,E)$, hence (A,E) is a soft closed set in $(X,\tilde{\tau},E)$. Similarly, since $cl(int(A,E)) = cl(A,E) \Rightarrow int(cl(int(A,E))) = int(cl(A,E)) =$ $int(A,E)$ (by (6)). Since $(A,E)\tilde{\subseteq} int(cl(int(A,E)))$, then $(A,E)\tilde{\subseteq} int(A,E)$, but $int(A,E)\tilde{\subseteq}(A,E)$, therefore $(A,E) = int(A,E)$, hence (A,E) is a soft open set in $(X,\tilde{\tau},E)$. Thus (A,E) is a soft clopen set in $(X,\tilde{\tau},E)$.

Definition(2.24): Let (A,E) be a soft subset of a soft topological space $(X,\tilde{\tau},E)$. Then:

i) The soft pre-α-closure of (A,E), denoted by p-α-cl(A,E) is the intersection of all soft pre-α-closed sets in $(X,\tilde{\tau},E)$ which contains (A,E).

ii) The soft pre-α-interior of (A, E), denoted by p-α-int(A,E) is the union of all soft pre-α-open sets in $(X,\tilde{\tau},E)$ which are contained in (A,E).

Theorem(2.25): Let (A,E) and (B,E) be soft subsets of a soft topological space $(X,\tilde{\tau},E)$.Then:

i) int(A,E)$\tilde{\subseteq}$p-α-int(A,E) $\tilde{\subseteq}$(A,E) and (A,E)$\tilde{\subseteq}$p-α-cl(A,E) $\tilde{\subseteq}$cl(A,E).

ii) p-α-int(A,E) is a soft pre-α-open set and p-α-cl(A,E) is a soft pre-α-closed set.

iii) If (A,E)$\tilde{\subseteq}$(B,E), then p-α-int(A,E)$\tilde{\subseteq}$ p-α-int(B,E) and p-α-cl(A,E)$\tilde{\subseteq}$ p-α-cl(B,E).

iv) (A,E) is soft pre-α-open iff p-α-int(A,E) = (A,E) and (A,E) is soft pre-α-closed iff p-α-cl(A,E) = (A,E).

v) p-α-int((A,E)$\tilde{\cap}$ (B,E))= p-α-int(A,E)$\tilde{\cap}$ p-α-int(B,E) and p-α-cl((A,E)$\tilde{\cup}$(B,E)) = p-α-cl(A,E) $\tilde{\cup}$p-α-cl(B,E)

vi) p-α-int(p-α-int(A,E)) = p-α-int(A,E) and p-α-cl(p-α-cl(A,E)) = p-α-cl(A,E).

vii) $\tilde{x}\,\tilde{\in}$p-α-int(A,E) iff there is a soft pre-α-open set (U,E) in $(X,\tilde{\tau},E)$ s.t $\tilde{x}\,\tilde{\in}(U,E\tilde{\subseteq}(A,E)$.

viii) $\tilde{x}\,\tilde{\in}$p-α-cl(A,E) iff for every soft pre-α-open set (U,E) containing $\tilde{x}$, $(U,E)\tilde{\cap}(A,E)\neq\tilde{\phi}$.

Proof: It is obvious.

3. Soft Pre-α-Continuous Functions and Soft Pre-α-Irresolute Functions

In this section, we introduce a new class of soft functions, namely, soft pre-α-continuous functions and soft pre-α-irresolute functions in soft topological spaces and we discuss the relation between these soft functions and each of soft continuous functions and other weaker forms of soft continuous functions.

Definition(3.1): A soft function $f:(X,\tilde{\tau},E_1)\rightarrow(Y,\tilde{\sigma},E_2)$ is called soft pre-α-continuous if $f^{-1}((V,E_2))$ is soft pre-α-open in $(X,\tilde{\tau},E_1)$ for every soft open set (V,E_2) in $(Y,\tilde{\sigma},E_2)$.

Proposition(3.2): A soft function $f:(X,\tilde{\tau},E_1)\rightarrow(Y,\tilde{\sigma},E_2)$ is soft pre-α-continuous iff $f^{-1}((V,E_2))$ is soft pre-α-closed in $(X,\tilde{\tau},E_1)$ for every soft closed set (V,E_2) in $(Y,\tilde{\sigma},E_2)$.

Proof: It is Obvious.

Proposition(3.3): Every soft continuous function is soft pre-α-continuous.

Proof: Follows from the definition (3.1) and the fact that every soft open set is soft pre-α-open.

Remark(3.4): The converse of proposition (3.3) may not be true in general as shown in the following example:

Example(3.5): Let $X=Y=\{x_1,x_2,x_3\}$ and $E=\{e_1,e_2\}$. Then $\tilde{\tau}=\{\tilde{X},\tilde{\phi},(F,E)\}$ is a soft topology over X and $\tilde{\sigma}=\{\tilde{Y},\tilde{\phi},(F,E),(F_1,E)\}$ is a soft topology overY, where $(F,E)=\{(e_1,\{x_1\}),(e_2,\{x_1,x_2,x_3\})\}$ and $(F_1,E)=\{(e_1,\{x_1,x_3\}),(e_2,\{x_1,x_2,x_3\})\}$. Define $f:(X,\tilde{\tau},E)\rightarrow(Y,\tilde{\sigma},E)$ by : $f((e_1,x_1))=(e_1,x_1)$, $f((e_1,x_2))=(e_1,x_2)$, $f((e_1,x_3))=(e_1,x_3)$, $f((e_2,x_1))=(e_2,x_1)$, $f((e_2,x_2))=(e_2,x_2)$ and $f((e_2,x_3))=(e_2,x_3)$ $\Rightarrow$ f is not soft continuous, but f is soft pre-α-continuous, since $f^{-1}(\tilde{Y})=\tilde{X}$, $f^{-1}(\tilde{\phi})=\tilde{\phi}$, $f^{-1}((F,E))=(F,E)$ and $f^{-1}((F_1,E))=(F_1,E)$ are soft pre-α-open sets in $(X,\tilde{\tau},E)$.

Remark(3.6): soft s*g-continuous functions and soft pre-α-continuous functions are in general independent. Consider the following examples:

Example(3.7): Let $X = Y = \{x_1, x_2, x_3\}$ and $E = \{e_1, e_2\}$. Then $\tilde{\tau} = \{\tilde{X}, \tilde{\phi}\}$ is a soft topology over X and $\tilde{\sigma} = \{\tilde{Y}, \tilde{\phi}, (F, E)\}$ is a soft topology over Y, where $(F, E) = \{(e_1, \{x_1\}), (e_2, \{x_1, x_2, x_3\})\}$. Define $f : (X, \tilde{\tau}, E) \to (Y, \tilde{\sigma}, E)$ by: $f((e_1, x_1)) = (e_1, x_1)$, $f((e_1, x_2)) = (e_1, x_2)$, $f((e_1, x_3)) = (e_1, x_3)$, $f((e_2, x_1)) = (e_2, x_1)$, $f((e_2, x_2)) = (e_2, x_2)$ and $f((e_2, x_3)) = (e_2, x_3) \Rightarrow$ f is soft s*g-continuous, but f is not soft pre-α-continuous, since (F, E) is soft open in $(Y, \tilde{\sigma}, E)$, but $f^{-1}((F, E)) = (F, E)$ is not soft pre-α-open in $(X, \tilde{\tau}, E)$. Also, in example (3.5) f is soft pre-α-continuous, but f is not soft s*g-continuous, since (F_1, E) is soft open set in $(Y, \tilde{\sigma}, E)$, but $f^{-1}((F_1, E)) = (F_1, E)$ is not s*g-open in $(X, \tilde{\tau}, E)$.

Theorem(3.8): Every soft pre-α-continuous function is soft α-continuous (resp. soft αg-continuous, soft gα-continuous, soft pre-continuous, soft b-continuous, soft β-continuous) function.

Proof: Follows from the theorem (2.5).

Remark(3.9): The converse of theorem (3.8) may not be true in general. Observe that in example (3.7) f is soft pre-continuous (resp. soft b-continuous, soft β-continuous, soft gα-continuous, soft αg-continuous) function, but f is not soft pre-α-continuous.

Theorem(3.10): Every soft pre-α-continuous function is soft semi-continuous function and soft gs-continuous function.

Proof: Follows from the theorem (2.7).

Remark(3.11): The converse of theorem (3.10) may not be true in general as shown in the following example:

Example(3.12): Let $X = Y = \{x_1, x_2, x_3\}$ and $E = \{e_1, e_2\}$. Then $\tilde{\tau} = \{\tilde{X}, \tilde{\phi}, (F, E), (F_1, E), (F_2, E)\}$ is a soft topology over X and $\tilde{\sigma} = \{\tilde{Y}, \tilde{\phi}, (F, E), (F_3, E)\}$ is a soft topology over Y, where $(F, E) = \{(e_1, \{x_1\}), (e_2, \{x_1\})\}$, $(F_1, E) = \{(e_1, \{x_2\}), (e_2, \{x_2\})\}$, $(F_2, E) = \{(e_1, \{x_1, x_2\}), (e_2, \{x_1, x_2\})\}$ and $(F_3, E) = \{(e_1, \{x_1, x_3\}), (e_2, \{x_1, x_3\})\}$. Define $f : (X, \tilde{\tau}, E) \to (Y, \tilde{\sigma}, E)$ by: $f((e_1, x_1)) = (e_1, x_1)$, $f((e_1, x_2)) = (e_1, x_2)$, $f((e_1, x_3)) = (e_1, x_3)$, $f((e_2, x_1)) = (e_2, x_1)$, $f((e_2, x_2)) = (e_2, x_2)$ and $f((e_2, x_3)) = (e_2, x_3) \Rightarrow$ f is soft semi-continuous and soft gs-continuous, but f is not soft pre-α-continuous, since (F_3, E) is soft open in $(Y, \tilde{\sigma}, E)$, but $f^{-1}((F_3, E)) = (F_3, E)$ is not soft pre-α-open in $(X, \tilde{\tau}, E)$, since $(F_3, E) \tilde{\not\subset} \text{int}(\text{pcl}(\text{int}(F_3, E)) = (F, E)$.

Remark(3.13): soft pre-continuous functions and soft αg-continuous functions are in general independent. Consider the following examples:

Example(3.14): Let $X = Y = \{x_1, x_2, x_3\}$ and $E = \{e_1, e_2\}$. Then $\tilde{\tau} = \{\tilde{X}, \tilde{\phi}, (F, E), (F_1, E)\}$ is a soft topology over X and $\tilde{\sigma} = \{\tilde{Y}, \tilde{\phi}, (F, E), (F_2, E), (F_3, E)\}$ is a soft topology over Y, where $(F, E) = \{(e_1, \{x_1\}), (e_2, \{x_1\})\}$, $(F_1, E) = \{(e_1, \{x_1, x_3\}), (e_2, \{x_1, x_3\})\}$, $(F_2, E) = \{(e_1, \{x_2\}), (e_2, \{x_2\})\}$ and $(F_3, E) = \{(e_1, \{x_1, x_2\}), (e_2, \{x_1, x_2\})\}$. Define $f : (X, \tilde{\tau}, E) \to (Y, \tilde{\sigma}, E)$ by: $f((e_1, x_1)) = (e_1, x_1)$, $f((e_1, x_2)) = (e_1, x_3)$, $f((e_1, x_3)) = (e_1, x_2)$, $f((e_2, x_1)) = (e_2, x_1)$, $f((e_2, x_2)) = (e_2, x_3)$ and $f((e_2, x_3)) = (e_2, x_2) \Rightarrow$ f is soft αg-continuous, since $f^{-1}(\tilde{Y}) = \tilde{X}$, $f^{-1}(\tilde{\phi}) = \tilde{\phi}$, $f^{-1}((F, E)) = (F, E)$, $f^{-1}((F_2, E)) = \{(e_1, \{x_3\}), (e_2, \{x_3\})$ and $f^{-1}((F_3, E)) = (F_1, E)$ are soft αg-open sets in $(X, \tilde{\tau}, E)$, but f is not soft pre-continuous, since $f^{-1}((F_2, E)) = \{(e_1, \{x_3\}), (e_2, \{x_3\})\}$ is not soft pre-open set in $(X, \tilde{\tau}, E)$.

Example(3.15): Let $X = Y = \mathfrak{R}$ and $E = \mathfrak{R}$. Then $(\mathfrak{R}, \tilde{\mu}, \mathfrak{R})$ is the soft usual topology over $\mathfrak{R}$ and $\tilde{\sigma} = \{\tilde{\mathfrak{R}}, \tilde{\phi}, \tilde{Q}\}$ is the soft topology over $\mathfrak{R}$. Define $f : (\mathfrak{R}, \tilde{\mu}, E) \to (\mathfrak{R}, \tilde{\sigma}, E)$ by: $f(\tilde{x}) = \tilde{x}$ for each $\tilde{x} \tilde{\in} \tilde{\mathfrak{R}}$ $\Rightarrow$ f is not soft αg-continuous, since $\tilde{Q}$ is soft open in $(\mathfrak{R}, \tilde{\sigma}, \mathfrak{R})$, but $f^{-1}(\tilde{Q}) = \tilde{Q}$ is not soft αg-open set in $(\mathfrak{R}, \tilde{\mu}, \mathfrak{R})$. But f is soft pre-continuous.

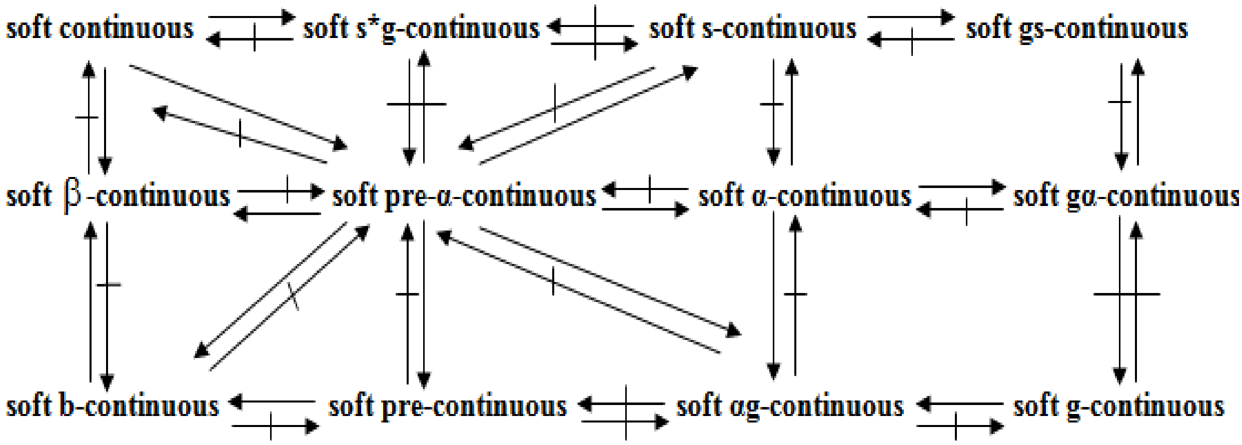
Remark(3.16): soft g-continuous functions and soft gα-continuous functions are in general independent. Consider the following examples:

Example(3.17): Let $X = Y = \{x_1, x_2, x_3\}$ and $E = \{e_1, e_2\}$. Then $\tilde{\tau} = \{\tilde{X}, \tilde{\phi}, (F, E)\}$ is a soft topology over X and $\tilde{\sigma} = \{\tilde{Y}, \tilde{\phi}, (G, E)\}$ is a soft topology over Y, where $(F, E) = \{(e_1, \{x_1\}), (e_2, \{x_1, x_2, x_3\})\}$ and $(G, E) = \{(e_1, \{x_2\}), (e_2, \{x_1, x_2, x_3\})\}$. Define $f : (X, \tilde{\tau}, E) \to (Y, \tilde{\sigma}, E)$ by: $f((e_1, x_1)) = (e_1, x_1)$, $f((e_1, x_2)) = (e_1, x_3)$, $f((e_1, x_3)) = (e_1, x_2)$, $f((e_2, x_1)) = (e_2, x_1)$, $f((e_2, x_2)) = (e_2, x_3)$ and $f((e_2, x_3)) = (e_2, x_2) \Rightarrow$ f is soft g-continuous, but f is not soft gα-continuous, since (G, E) is soft open set in $(Y, \tilde{\sigma}, E)$, but $f^{-1}((G, E)) = \{(e_1, \{x_3\}), (e_2, \{x_1, x_2, x_3\})\}$ is not soft gα-open set in $(X, \tilde{\tau}, E)$.

Example(3.18): Let $X = Y = \{x_1, x_2, x_3\}$ and $E = \{e_1, e_2\}$. Then $\tilde{\tau} = \{\tilde{X}, \tilde{\phi}, (F_1, E), (F_2, E)\}$ is a soft topology over X and $\tilde{\sigma} = \{\tilde{Y}, \tilde{\phi}, (F_3, E)\}$ is a soft topology over Y, where $(F_1, E) = \{(e_1, \{x_1\}), (e_2, \{x_1, x_2, x_3\})\}$, $(F_2, E) = \{(e_1, \{x_1, x_3\}), (e_2, \{x_1, x_2, x_3\})\}$ and $(F_3, E) = \{(e_1, \{x_2\}), (e_2, \{x_1, x_2, x_3\})\}$.

Define $f:(X,\tilde{\tau},E)\to(Y,\tilde{\sigma},E)$ by: $f((e_1,x_1))=(e_1,x_2)$, $f((e_1,x_2))=(e_1,x_2)$, $f((e_1,x_3))=(e_1,x_1)$, $f((e_2,x_1))=(e_2,x_2)$, $f((e_2,x_2))=(e_2,x_2)$ and $f((e_2,x_3))=(e_2,x_1)$ $\Rightarrow$ f is soft gα-continuous, but f is not soft g-continuous, since (F_3,E) is soft open set in $(Y,\tilde{\sigma},E)$, but $f^{-1}((F_3,E))=\{(e_1,\{x_1,x_2\}),(e_2,\{x_1,x_2,x_3\})\}$ is not soft g-open set in $(X,\tilde{\tau},E)$.

The following diagram shows the relationships between soft pre-α-continuous functions and some other soft continuous functions:



Proposition(3.19): A soft function $f:(X,\tilde{\tau},E_1)\to(Y,\tilde{\sigma},E_2)$ is soft pre-α-continuous iff $f^{-1}(\text{int}(B,E_2))\,\tilde{\subseteq}\,$p-α-int$(f^{-1}((B,E_2)))$ for every soft subset (B,E_2) of $(Y,\tilde{\sigma},E_2)$.

Proof: $\Rightarrow$ Since $\text{int}(B,E_2)\,\tilde{\subseteq}\,(B,E_2)\Rightarrow f^{-1}(\text{int}(B,E_2))\,\tilde{\subseteq}\,f^{-1}((B,E_2))$. Since $\text{int}(B,E_2)$ is a soft open set in $(Y,\tilde{\sigma},E_2)$ and f is soft pre-α-continuous, then by (3.1) $f^{-1}(\text{int}(B,E_2))$ is a soft pre-α-open set in $(X,\tilde{\tau},E_1)$ such that $f^{-1}(\text{int}(B,E_2))\,\tilde{\subseteq}\,f^{-1}((B,E_2))$. Therefore by theorem ((2.25),iv) $f^{-1}(\text{int}(B,E_2))\,\tilde{\subseteq}\,$p-α-int$(f^{-1}((B,E_2)))$ for every soft subset (B,E_2) of $(Y,\tilde{\sigma},E_2)$.

Conversely, suppose that $f^{-1}(\text{int}(B,E_2))\,\tilde{\subseteq}\,$p-α-int$(f^{-1}((B,E_2)))$ for every soft subset (B,E_2) of $(Y,\tilde{\sigma},E_2)$. To prove that f is soft pre-α-continuous. Let (U,E_2) be any soft open set in $(Y,\tilde{\sigma},E_2)$. By hypothesis $f^{-1}(\text{int}(U,E_2))=f^{-1}((U,E_2))\,\tilde{\subseteq}\,$p-α-int$(f^{-1}((U,E_2)))$. Since p-α-int$(f^{-1}((U,E_2)))\,\tilde{\subseteq}\,f^{-1}((U,E_2))$. Hence p-α-int$(f^{-1}((U,E_2)))$ $=f^{-1}((U,E_2))$. Thus f is a soft pre-α-continuous function.

Theorem(3.20): Let $f:(X,\tilde{\tau},E_1)\to(Y,\tilde{\sigma},E_2)$ be a soft function. Then the following statements are equivalent:

i) f is soft pre-α-continuous.

ii) For each $\tilde{x} \tilde{\in} \tilde{X}$ and each soft open set (V,E_2) in $(Y,\tilde{\sigma},E_2)$ with $f(\tilde{x}) \tilde{\in} (V,E_2)$, there is a soft pre-α-open set (U,E_1) in $(X,\tilde{\tau},E_1)$ such that $\tilde{x} \tilde{\in} (U,E_1)$ and $f(U,E_1) \tilde{\subseteq} (V,E_2)$.

iii) $f(\text{p-}\alpha\text{-cl}(A,E_1)) \tilde{\subseteq} cl(f(A,E_1))$ for each soft subset (A,E_1) of $(X,\tilde{\tau},E_1)$.

iv) $\text{p-}\alpha\text{-cl}(f^{-1}((B,E_2))) \tilde{\subseteq} f^{-1}(cl(B,E_2))$ for each soft subset (B,E_2) of $(Y,\tilde{\sigma},E_2)$.

Proof: $(i) \Rightarrow (ii)$. Let $f:(X,\tilde{\tau},E_1) \to (Y,\tilde{\sigma},E_2)$ be a soft pre-α-continuous function and (V,E_2) be a soft open set in $(Y,\tilde{\sigma},E_2)$ s.t $f(\tilde{x}) \tilde{\in} (V,E_2)$. To prove that, there is a soft pre-α-open set (U,E_1) in $(X,\tilde{\tau},E_1)$ s.t $\tilde{x} \tilde{\in} (U,E_1)$ and $f(U,E_1) \tilde{\subseteq} (V,E_2)$. Since f is soft pre-α-continuous, then $f^{-1}((V,E_2))$ is a soft pre-α-open set in $(X,\tilde{\tau},E_1)$ s.t $\tilde{x} \tilde{\in} f^{-1}((V,E_2))$. Let $(U,E_1) = f^{-1}((V,E_2)) \Rightarrow f(U,E_1) = f(f^{-1}((V,E_2))) \tilde{\subseteq} (V,E_2) \Rightarrow f(U,E_1) \tilde{\subseteq} (V,E_2)$.

$(ii) \Rightarrow (i)$. To prove that $f:(X,\tilde{\tau},E_1) \to (Y,\tilde{\sigma},E_2)$ is soft pre-α-continuous. Let (V,E_2) be any soft open set in $(Y,\tilde{\sigma},E_2)$. To prove that $f^{-1}((V,E_2))$ is a soft pre-α-open set in $(X,\tilde{\tau},E_1)$. Let $\tilde{x} \tilde{\in} f^{-1}((V,E_2)) \Rightarrow f(\tilde{x}) \tilde{\in} (V,E_2)$. By hypothesis there is a soft pre-α-open set (U,E_1) in $(X,\tilde{\tau},E_1)$ s.t $\tilde{x} \tilde{\in} (U,E_1)$ and $f(U,E_1) \tilde{\subseteq} (V,E_2) \Rightarrow \tilde{x} \tilde{\in} (U,E_1) \tilde{\subseteq} f^{-1}((V,E_2))$. Thus by theorem ((2.25),vii) $f^{-1}((V,E_2))$ is a soft pre-α-open set in $(X,\tilde{\tau},E_1)$. Hence f is a soft pre-α-continuous function.

$(ii) \Rightarrow (iii)$. Suppose that (ii) holds and let $\tilde{y} \tilde{\in} f(\text{p-}\alpha\text{-cl}(A,E_1))$ and let (V,E_2) be any soft open neighborhood of $\tilde{y}$ in $\tilde{Y}$. Since $\tilde{y} \tilde{\in} f(\text{p-}\alpha\text{-cl}(A,E_1)) \Rightarrow \exists\, \tilde{x} \tilde{\in} \text{p-}\alpha\text{-cl}(A,E_1)$ s.t $f(\tilde{x}) = \tilde{y}$. Since $\tilde{f}(x) \tilde{\in} (V,E_2)$, then by (ii) $\exists$ a soft pre-α-open set (U,E_1) in $(X,\tilde{\tau},E_1)$ s.t $\tilde{x} \tilde{\in} (U,E_1)$ and $f(U,E_1) \tilde{\subseteq} (V,E_2)$. Since $\tilde{x} \tilde{\in} \text{p-}\alpha\text{-cl}(A,E_1)$, then by theorem ((2.25),viii) $(U,E_1) \tilde{\cap} (A,E_1) \neq \tilde{\phi}$ and hence $f((A,E_1)) \tilde{\cap} (V,E_2) \neq \tilde{\phi}$. Therefore we have $\tilde{y} \tilde{\in} cl(f(A,E_1))$. Hence $f(\text{p-}\alpha\text{-cl}(A,E_1)) \tilde{\subseteq} cl(f(A,E_1))$.

$(iii) \Rightarrow (ii)$. Let $\tilde{x} \in \tilde{X}$ and (V,E_2) be any soft open set in $(Y,\tilde{\sigma},E_2)$ containing $f(\tilde{x})$. Let $(A,E_1) = f^{-1}((V,E_2)^c) \Rightarrow \tilde{x} \tilde{\notin} (A,E_1)$. Since $f(\text{p-}\alpha\text{-cl}(A,E_1)) \tilde{\subseteq} cl(f(A,E_1)) \tilde{\subseteq} (V,E_2)^c \Rightarrow \text{p-}\alpha\text{-cl}(A,E_1) \tilde{\subseteq} f^{-1}((V,E_2)^c) = (A,E_1)$. Since $\tilde{x} \tilde{\notin} (A,E_1) \Rightarrow \tilde{x} \tilde{\notin} \text{p-}\alpha\text{-cl}(A,E_1)$ and by theorem ((2.25), viii) there exists a soft pre-α-open set (U,E_1) containing $\tilde{x}$ such that $(U,E_1) \tilde{\cap} (A,E_1) = \tilde{\phi}$ and hence $f(U,E_1) \tilde{\subseteq} f((A,E_1)^c) \tilde{\subseteq} (V,E_2)$.

$(iii) \Rightarrow (iv)$. Suppose that (iii) holds and let (B, E_2) be any soft subset of $(Y, \tilde{\sigma}, E_2)$. Replacing (A, E_1) by $f^{-1}((B, E_2))$ we get from (iii) $f(p\text{-}\alpha\text{-}cl(f^{-1}((B, E_2)))) \tilde{\subseteq} cl(f(f^{-1}((B, E_2)))) \tilde{\subseteq} cl(B, E_2)$. Hence $p\text{-}\alpha\text{-}cl(f^{-1}((B, E_2))) \tilde{\subseteq} f^{-1}(cl(B, E_2))$.

$(iv) \Rightarrow (iii)$. Suppose that (iv) holds and let $(B, E_2) = f(A, E_1)$ where (A, E_1) is a soft subset of $(X, \tilde{\tau}, E)$. Then we get from (iv) $p\text{-}\alpha\text{-}cl(A, E_1) \tilde{\subseteq} p\text{-}\alpha\text{-}cl(f^{-1}(f(A, E_1))) \tilde{\subseteq} f^{-1}(cl(f(A, E_1)))$. Therefore $f(p\text{-}\alpha\text{-}cl(A, E_1)) \tilde{\subseteq} cl(f(A, E_1))$.

Definition(3.21): A function $f : (X, \tilde{\tau}, E_1) \to (Y, \tilde{\sigma}, E_2)$ is called soft pre-α-irresolute if the inverse image of every soft pre-α-open set in $(Y, \tilde{\sigma}, E_2)$ is a soft pre-α-open set in $(X, \tilde{\tau}, E_1)$.

Proposition(3.22): Every soft pre-α-irresolute function is soft pre-α-continuous.

Proof: It is Obvious.

Remark(3.23): The converse of proposition (3.22) may not be true in general as shown in the following example:

Example(3.24): Let $X = Y = \{x_1, x_2, x_3\}$ and $E = \{e_1, e_2\}$. Then $\tilde{\tau} = \{\tilde{X}, \tilde{\phi}, (F, E), (G, E)\}$ is a soft topology over X and $\tilde{\sigma} = \{\tilde{Y}, \tilde{\phi}, (F, E)\}$ is a soft topology over Y, where $(F, E) = \{(e_1, \{x_1\}), (e_2, \{x_1, x_2, x_3\})\}$ and $(G, E) = \{(e_1, \{x_2, x_3\}), (e_2, \{\phi\})\}$. Define $f : (X, \tilde{\tau}, E) \to (Y, \tilde{\sigma}, E)$ by: $f((e_1, x_1)) = (e_1, x_1)$, $f((e_1, x_2)) = (e_1, x_2)$, $f((e_1, x_3)) = (e_1, x_3)$, $f((e_2, x_1)) = (e_2, x_1)$, $f((e_2, x_2)) = (e_2, x_2)$ and $f((e_2, x_3)) = (e_2, x_3)$ $\Rightarrow$ f is soft pre-α-continuous, but f is not soft pre-α-irresolute, since $(A, E) = \{(e_1, \{x_1, x_2\}, (e_2, \{x_1, x_2, x_3\})\}$ is soft pre-α-open in $(Y, \tilde{\sigma}, E_2)$, but $f^{-1}((A, E)) = (A, E)$ is not soft pre-α-open in $(X, \tilde{\tau}, E)$.

Remark(3.25): soft continuous functions and soft pre-α-irresolute functions are in general independent. Consider the following examples:

Example(3.26): Let $X = Y = \{x_1, x_2, x_3\}$ and $E = \{e_1, e_2\}$. Then $\tilde{\tau} = \{\tilde{X}, \tilde{\phi}, (F, E)\}$ is a soft topology over X and $\tilde{\sigma} = \{\tilde{Y}, \tilde{\phi}, (G, E)\}$ is a soft topology over Y, where $(F, E) = \{(e_1, \{x_1\}), (e_2, \{x_1, x_2, x_3\})\}$ and $(G, E) = \{(e_1, \{x_1, x_2\}), (e_2, \{x_1, x_2, x_3\})\}$. Define $f : (X, \tilde{\tau}, E) \to (Y, \tilde{\sigma}, E)$ by: $f((e_1, x_1)) = (e_1, x_1)$, $f((e_1, x_2)) =$

(e_1, x_2), $f((e_1, x_3)) = (e_1, x_3)$, $f((e_2, x_1)) = (e_2, x_1)$, $f((e_2, x_2)) = (e_2, x_2)$ and $f((e_2, x_3)) = (e_2, x_3) \Rightarrow$ f is soft pre-α-irresolute, but f is not soft continuous, since (G, E) is soft open in $(Y, \tilde{\sigma}, E)$, but $f^{-1}((G, E)) = (G, E)$ is not soft open in $(X, \tilde{\tau}, E)$. Also, in example (3.24), f is soft continuous, but f is not soft pre-α-irresolute.

Theorem(3.27): Let $f : (X, \tilde{\tau}, E_1) \to (Y, \tilde{\sigma}, E_2)$ be a soft function. Then the following statements are equivalent:

(i) f is soft pre-α-irresolute.

(ii) For each $\tilde{x} \tilde{\in} \tilde{X}$ and each soft pre-α-neighborhood (V, E_2) of $f(\tilde{x})$ in $\tilde{Y}$, there is a soft pre-α-neighborhood (U, E_1) of $\tilde{x}$ in $\tilde{X}$ such that $f((U, E_1)) \tilde{\subseteq} (V, E_2)$.

(iii) The inverse image of every soft pre-α-closed set in $(Y, \tilde{\sigma}, E_2)$ is a soft pre-α-closed set in $(X, \tilde{\tau}, E_1)$.

iv) f(p-α-cl(A, E_1)) $\tilde{\subseteq}$ p-α-cl(f(A, E_1)) for each soft subset (A, E_1) of $(X, \tilde{\tau}, E_1)$.

v) p-α-cl($f^{-1}((B, E_2))$) $\tilde{\subseteq}$ f^{-1}(p-α-cl(B, E_2)) for each soft subset (B, E_2) of $(Y, \tilde{\sigma}, E_2)$.

Proof: $(i) \Rightarrow (ii)$. Let $f : (X, \tilde{\tau}, E_1) \to (Y, \tilde{\sigma}, E_2)$ be a soft pre-α-irresolute function and (V, E_2) be a soft pre-α-neighborhood of $f(\tilde{x})$ in $\tilde{Y}$. To prove that, there is a soft pre-α-neighborhood (U, E_1) of $\tilde{x}$ in $\tilde{X}$ such that $f((U, E_1)) \subseteq (V, E_2)$. Since f is a soft pre-α-irresolute then, $f^{-1}((V, E_2))$ is is a soft pre-α-neighborhood of $\tilde{x}$ in $\tilde{X}$. Let $(U, E_1) = f^{-1}((V, E_2)) \Rightarrow$ $f((U, E_1)) = f(f^{-1}((V, E_2))) \tilde{\subseteq} (V, E_2) \Rightarrow f((U, E_1)) \tilde{\subseteq} (V, E_2)$.

$(ii) \Rightarrow (i)$. To prove that $f : (X, \tilde{\tau}, E_1) \to (Y, \tilde{\sigma}, E_2)$ is soft pre-α-irresolute. Let (V, E_2) be a soft pre-α-open set in $(Y, \tilde{\sigma}, E_2)$. To prove that $f^{-1}((V, E_2))$ is a soft pre-α-open set in $(X, \tilde{\tau}, E_1)$. Let $\tilde{x} \tilde{\in} f^{-1}((V, E_2)) \Rightarrow$ $f(\tilde{x}) \tilde{\in} (V, E_2) \Rightarrow (V, E_2)$ is a soft pre-α-neighborhood of $f(\tilde{x})$. By hypothesis there is a soft pre-α-neighborhood $(U, E_1)_{\tilde{x}}$ of $\tilde{x}$ such that $f((U, E_1)_{\tilde{x}}) \tilde{\subseteq} (V, E_2) \Rightarrow (U, E_1)_{\tilde{x}} \tilde{\subseteq} f^{-1}((V, E_2)), \forall \tilde{x} \tilde{\in} f^{-1}((V, E_2)) \Rightarrow \exists$ a soft pre-α-open set $(W, E_1)_{\tilde{x}}$ of $\tilde{x}$ such that $(W, E_1)_{\tilde{x}} \tilde{\subseteq} (U, E_1)_{\tilde{x}} \tilde{\subseteq}$ $f^{-1}((V, E_2)), \forall \tilde{x} \tilde{\in} f^{-1}((V, E_2)) \Rightarrow \bigcup_{\tilde{x} \in f^{-1}(V, E_2)} (W, E_1)_{\tilde{x}} \tilde{\subseteq} f^{-1}((V, E_2))$. Since

$f^{-1}((V,E_2)) \tilde{\subseteq} \bigcup_{\tilde{x}\in f^{-1}(V,E_2)} (W,E_1)_{\tilde{x}} \Rightarrow f^{-1}((V,E_2)) = \bigcup_{\tilde{x}\in f^{-1}(V,E_2)} (W,E_1)_{\tilde{x}} \Rightarrow$ $f^{-1}((V,E_2))$ is a soft pre-α-open set in $(Y,\tilde{\sigma},E_2)$, since its a union of soft pre-α-open sets. Thus $f:(X,\tilde{\tau},E_1)\to(Y,\tilde{\sigma},E_2)$ is a soft pre-α-irresolute function.

(i) $\Leftrightarrow$ (iii). It is a obvious.

(iii) $\Rightarrow$ (iv). Since $f(A,E_1) \tilde{\subseteq}$ p-α-cl$(f(A,E_1))$ $\Rightarrow$ $(A,E_1) \tilde{\subseteq} f^{-1}($p-α-cl$(f(A,E_1)))$. Since p-α-cl$(f(A,E_1))$ is a soft pre-α-closed set in $(Y,\tilde{\sigma},E_2)$, then by (iii) $f^{-1}($p-α-cl$(f(A,E_1)))$ is a soft pre-α-closed set in $(X,\tilde{\tau},E_1)$ containing (A,E_1). Hence p-α-cl$(A,E_1) \tilde{\subseteq} f^{-1}($p-α-cl$(f(A,E_1)))$. Thus f(p-α-cl$(A,E_1)) \tilde{\subseteq}$ p-α-cl$(f(A,E_1))$ for each soft subset (A,E_1) of $(X,\tilde{\tau},E_1)$.

(iv) $\Rightarrow$ (v). Suppose that (iv) holds and let (B,E_2) be any soft subset of $(Y,\tilde{\sigma},E_2)$. Replacing (A,E_1) by $f^{-1}((B,E_2))$ we get from (iv) f(p-α-cl $(f^{-1}((B,E_2))))\tilde{\subseteq}$ p-α-cl$(f(f^{-1}((B,E_2)))) \tilde{\subseteq}$ p-α-cl(B,E_2). Hence p-α-cl $(f^{-1}((B,E_2))) \tilde{\subseteq} f^{-1}($p-α-cl$(B,E_2))$ for each soft subset (B,E_2) of $(Y,\tilde{\sigma},E_2)$.

(v) $\Rightarrow$ (iii). Let (F,E_2) be any soft pre-α-closed subset of $(Y,\tilde{\sigma},E_2)$. Then by hypothesis p-α-cl$(f^{-1}((F,E_2))) \tilde{\subseteq} f^{-1}($p-α-cl$(F,E_2)) = f^{-1}((F,E_2))$. Since $f^{-1}((F,E_2)) \tilde{\subseteq}$ p-α-cl$(f^{-1}((F,E_2)))$. Therefore p-α-cl$(f^{-1}((F,E_2)))$ $= f^{-1}((F,E_2))$. Thus f is soft pre-α-irresolute.

Theorem(3.28): If $f:(X,\tilde{\tau},E_1)\to(Y,\tilde{\sigma},E_2)$ and $g:(Y,\tilde{\sigma},E_2)\to(Z,\tilde{\eta},E_3)$ are soft functions, then:

i) If f and g are both soft pre-α-irresolute functions, then so is $g\circ f$.

ii) If f is soft pre-α-irresolute and g is soft pre-α-continuous, then $g\circ f$ is soft pre-α-continuous.

iii) If f is soft pre-α-continuous and g is soft continuous, then $g\circ f$ is soft pre-α-continuous.

Proof: It is a obvious.

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