

Fredkin Gate acts on the Quantum Bits for a System Includes Four Particles

¹Mezher B. Saleh
mezherbaker@yahoo.com

²Loay E. George
loayedwar57@scbaghdad.edu.iq

¹Department of Physics/College of Science/Al- Mustansiriyah University

²Department of Computer Science/College of Science/ Baghdad University

Abstract

In this research, the effect of Fredkin gate on a system consists of four particles is considered. The number of states of this system is $n=2^4=16$, these states are $(|0000\rangle, |0001\rangle, |0010\rangle, |0011\rangle, |0100\rangle, |0101\rangle, |0110\rangle, |0111\rangle, |1000\rangle, |1001\rangle, |1010\rangle, |1011\rangle, |1100\rangle, |1101\rangle, |1110\rangle, |1111\rangle)$. When Fredkin gate is applied on these states most of them remains without change except some of the states will be changed. The features of the gate process can be utilized in quantum information computations.

Keywords: Qubit, Quantum states, Fredkin Gate, Quantum circuits.

مزهر باقر صالح ١ لؤي ادور جورج ٢
١ قسم الفيزياء /كلية العلوم/ الجامعة المستنصرية
٢ قسم علوم الحاسبات/كلية العلوم/جامعة بغداد

الخلاصة

في هذا البحث، تم دراسة تأثير بوابة فريديكين Fredkin Gate على نظام يتكون من أربعة جسيمات. ان عدد الحالات في هذا النظام هو $n = 2^4 = 16$ ، وهذه الحالات هي $(|0000\rangle, |0001\rangle, |0010\rangle, |0011\rangle, |0100\rangle, |0101\rangle, |0110\rangle, |0111\rangle, |1000\rangle, |1001\rangle, |1010\rangle, |1011\rangle, |1100\rangle, |1101\rangle, |1110\rangle, |1111\rangle)$. عندما يتم تطبيق بوابة Fredkin على هذه الحالات معظمها يبقى دون تغيير باستثناء بعض الحالات سوف تتغير. ميزة عمل البوابة انه يمكن استخدامها في حسابات المعلومات الكمية.

I. Introduction

Many three qubit gates have been introduced in the literature; one of them is the Fredkin Gate. In some previous studies the Fredkin gate was used in different fields of quantum information processing [1, 2].

In 1995, Chau and Wilczek discussed as Fredkin 3-bit gate as a universal. They referred that it is impossible to do universal computation using reversible 2-bit gates only. They constructed the Fredkin gate using a combination of two one-body and seven two-body reversible (quantum) operators [3].

In 2009, Qiang, Li and Shou presented a scheme for implementing a Fredkin gate on three modes of a cavity. The scheme is based on the dispersive atom-cavity interaction. By modulating the cavity frequency and the atomic transition frequency, appropriately, an effective form of nonlinear interaction can be obtained between photons in the three-mode cavity. This availability is testified via numerical analysis. Also, they considered the situations with and without dissipation [4].

In 2010, QiuBo showed that a local unitary (LU) equivalent Fredkin gate can be obtained from the free evolution of three mixed-spin qubits by virtue of numerical simulation with only one step. The spin-1 qubit acts as the control qubit. The two spin-1/2 qubits interact with the spin-1 qubit via the first neighbor spin interaction, respectively; they play the role of target qubits. Also, they examined the imperfect Fredkin gate operation by considering the effects of non-identical coupling constants, uniform and inhomogeneous magnetic fields [5].

In 2014, Ozaydin *et al.* proposed an optical scheme to prepare large-scale entangled networks of W states. The scheme works by simultaneously fusing three polarization-encoded W states of arbitrary size via accessing only one qubit of each W state. It is composed of a Fredkin gate (controlled-swap gate), and an H-polarized ancilla photon. Starting with three n-qubit W states, the scheme prepares a new W state with $3(n-1)$ qubits after post selection if both fusion gates operate successfully (i.e., a fourfold coincidence at the detectors). The proposed scheme reduces the cost of creating arbitrarily large W states considerably when compared to previously reported schemes [6].

The aim of this research to find the effect of Fredkin gate on a system consists of four particles.

II. Theory

The Fredkin Gate is one of the important gates in quantum computer. Fredkin gate is three qubits gate with matrixes (8x8), each column in this matrix represents a state [7, 8]. Fredkin-gate is defined as:

$$\text{Fredkin Gate} = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix} \quad (1)$$

C	I ₁	I ₂	C	O ₁	O ₂
0	0	0	0	0	0
0	0	1	0	0	1
0	1	0	0	1	0
0	1	1	0	1	1
1	0	0	1	0	0
1	0	1	1	1	0
1	1	0	1	0	1
1	1	1	1	1	1

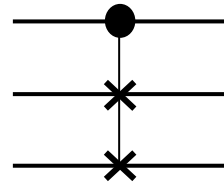


Fig. (1): The Truth table and circuit of Fredkin Gate[1]

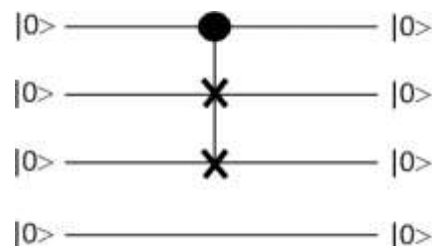
The basic Fredkin gate is a controlled swap gate that maps three inputs (C, I₁, I₂) onto three outputs (C, O₁, O₂). The C input is mapped directly to the C output. If C = 0, no swap is performed; I₁ maps to O₁, and I₂ maps to O₂. Otherwise, the two outputs are swapped so that I₁ is mapped to O₂, and I₂ maps to O₁. This work of Fredkin gate can be used for any quantum system consists of three qubits or more [9].

II. Calculations

The system consists of four particles has sixteen states (|0000>, |0001>, |0010>, |0011>, |0100>, |0101>, |0110>, |0111>, |1000>, |1001>, |1010>, |1011>, |1100>, |1101>, |1110>, |1111>). The acts of Fredkin gate on three qubits for each above states as following:

Case-1 (|0000>): figure (2) presents the act of Fredkin gate on the state |0000>.

Fig. (2): the input/output of Fredkin gate for the state |0000>



$$\begin{aligned}
 \text{Fredkin_Gate}|000\rangle|0\rangle &= \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix} |0\rangle \\
 &= \begin{bmatrix} 1 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix} |0\rangle
 \end{aligned}$$

$$\text{Fredkin_Gate}|000\rangle|0\rangle = |000\rangle|0\rangle = |0000\rangle \tag{2}$$

Case-2 (|0001>): figure (3) presents the act of Fredkin gate on the input state |0001>.

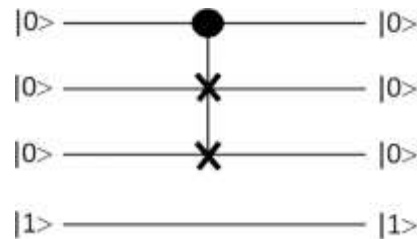


Fig. (3): the input/output of Fredkin gate for the state |0001>

Since the first three qubits in left for the state |0001> are similar to the first three qubits in state |0001> so that there are no changes in these qubits; that is:

$$\text{Fredkin_Gate}|0001\rangle = \text{Fredkin_Gate}|000\rangle|1\rangle = |0001\rangle \tag{3}$$

Case-3 (|0010>): figure (4) presents the act of Fredkin gate on the input state |0010>.

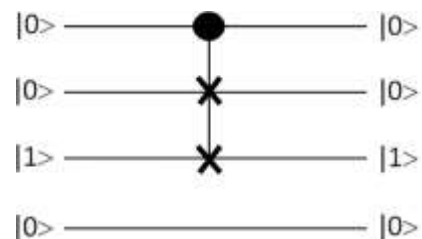


Fig. (4): the input/output of Fredkin gate for the state |0010>

$$\begin{aligned}
 \text{Fredkin_Gate}|001\rangle|0\rangle &= \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 0 \\ 1 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix} |0\rangle \\
 &= \begin{bmatrix} 0 \\ 1 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix} |0\rangle
 \end{aligned}$$

$$\text{Fredkin_Gate}|001\rangle|0\rangle = |001\rangle|0\rangle = |0010\rangle \tag{4}$$

Case-4 (|0011>): figure (5) presents the act of Fredkin gate on the input state |0011>.

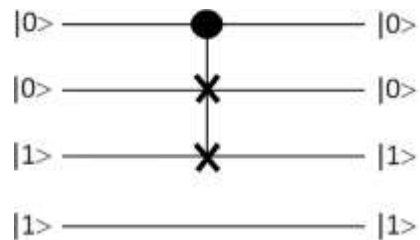


Fig. (5): the input/output of Fredkin gate for the state |0011>

Since the first three qubits in left for state |0010> are similar to the first three qubits in state |0011> so that there are no changes in these qubits; that is:

$$\text{Fredkin_Gate}|0011\rangle = \text{Fredkin_Gate}|001\rangle|1\rangle = |0011\rangle \tag{5}$$

Case-5 (|0100>): figure (6) presents the act of Fredkin gate on the input state |0100>.

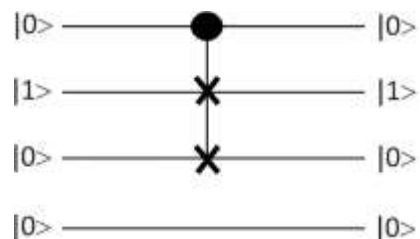


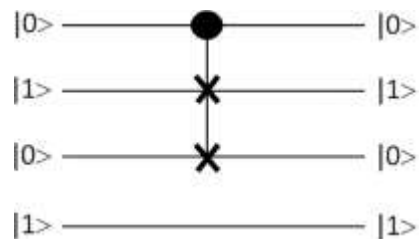
Fig. (6): the input/output of Fredkin gate for the state |0100>

$$\begin{aligned}
 \text{Fredkin_Gate}|010\rangle|0\rangle &= \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} 0 \\ 0 \\ 1 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix} |0\rangle \\
 &= \begin{bmatrix} 0 \\ 0 \\ 1 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix} |0\rangle
 \end{aligned}$$

$$\text{Fredkin_Gate}|010\rangle|0\rangle = |010\rangle|0\rangle = |0100\rangle \tag{6}$$

Case-6 (|0101>): figure (7) presents the act of Fredkin gate on the input state |0101>.

Fig. (7): the input/output of Fredkin gate for the state |0101>

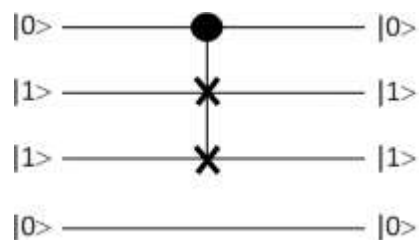


Since the first three qubits in left for state |0101> are similar to the first three qubits in state |0101> so that there are no changes in this qubits; that is:

$$\text{Fredkin_Gate}|0101\rangle = \text{Fredkin_Gate}|010\rangle|1\rangle = |0101\rangle \tag{7}$$

Case-7 (|0110>): figure (8) presents the act of Fredkin gate on the input state |0110>.

Fig. (8): the input/output of Fredkin gate for the state |0110>



$$\begin{aligned}
 \text{Fredkin_Gate}|011\rangle|0\rangle &= \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 0 \\ 0 \\ 0 \\ 1 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix} |0\rangle \\
 &= \begin{bmatrix} 0 \\ 0 \\ 0 \\ 1 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix} |0\rangle
 \end{aligned}$$

$$\text{Fredkin_Gate}|011\rangle|0\rangle = |011\rangle|0\rangle = |0110\rangle \tag{8}$$

Case-8 (|0111>): figure (9) presents the act of Fredkin gate on the input state |0111>.

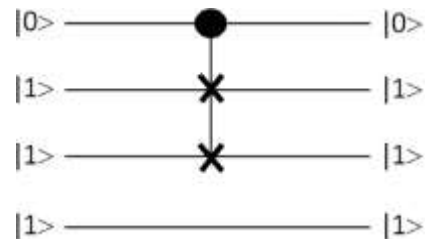


Fig. (9): the input/output of Fredkin gate for the state |0111>

Since the first three qubits in left for state |0111> are similar to the first three qubits in state |0111> so that there are no changes in these qubits.

$$\text{Fredkin_Gate}|0111\rangle = \text{Fredkin_Gate}|011\rangle|1\rangle = |0111\rangle \tag{9}$$

Case-9 (|1000>): the first qubit at left is 1 so that when applying Fredkin gate on this state and according to the work of this gate the first and third qubit for left will be swapped. Figure (10) presents the act of Fredkin gate on the input state |1000>.

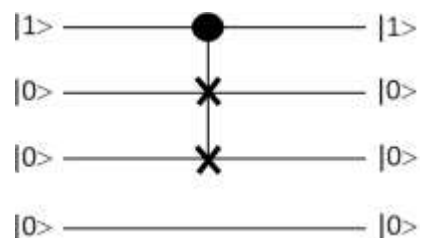


Fig. (10): the input/output of Fredkin gate for the state |1000>

$$\begin{aligned}
 \text{Fredkin_Gate}|100\rangle|0\rangle &= \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 1 \\ 0 \\ 0 \\ 0 \end{bmatrix} |0\rangle \\
 &= \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 1 \\ 0 \\ 0 \\ 0 \end{bmatrix} |0\rangle
 \end{aligned}$$

$$\text{Fredkin_Gate}|100\rangle|0\rangle = |100\rangle|0\rangle = |1000\rangle \tag{10}$$

Since the second and third qubits has the same value 0 so that there is no change in state $|1000\rangle$ as depicted in equation (10).

Case-10 ($|1001\rangle$): figure (11) presents the act of Fredkin gate on the input state $|1001\rangle$.

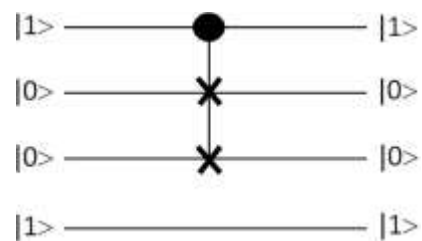


Fig. (11): the input/output of Fredkin gate for the state $|1001\rangle$

Since the first three qubits in left for state $|1001\rangle$ are similar to the first three qubits in state $|1001\rangle$ so that there are no changes in these qubits.

$$\text{Fredkin_Gate}|1001\rangle = \text{Fredkin_Gate}|100\rangle|1\rangle = |1001\rangle \tag{11}$$

Case-11 ($|1010\rangle$): the first qubit is 1, the second and third qubits from left has a different values so that when applying Fredkin gate on this state the output state becomes $|1100\rangle$. Figure (12) presents the act of Fredkin gate on the input state $|1010\rangle$.

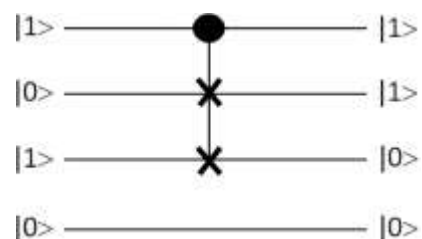


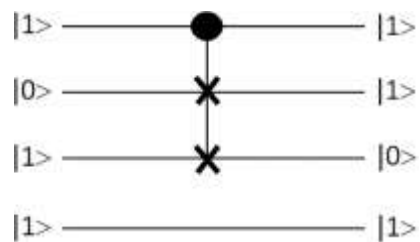
Fig. (12): the input/output of Fredkin gate for the state $|1010\rangle$

$$\begin{aligned}
 \text{Fredkin_Gate}|101\rangle|0\rangle &= \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 1 \\ 0 \\ 0 \end{bmatrix} |0\rangle \\
 &= \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 1 \\ 0 \end{bmatrix} |0\rangle
 \end{aligned}$$

$$\text{Fredkin_Gate}|101\rangle|0\rangle = |110\rangle|0\rangle = |1100\rangle \tag{12}$$

Case-12 (|1011>): when Fredkin gate acts on this state it will be changed to state |1101>. Since the first three qubits at left is similar to the first three qubits in state |1011>, so that the result for Fredkin gate on act the state |1011> is similar to that for state |1010>, as depicted in figure (13).

Fig. (13): the input/output of Fredkin gate for the state |1011>



$$\text{Fredkin_Gate}|1011\rangle = \text{Fredkin_Gate}|101\rangle|1\rangle = |1101\rangle \tag{13}$$

Case-13 (|1100>): in this state the first qubit is 1, the second and third qubits from left has different values, so that when Fredkin gate is applied on this state the output state becomes |1010>. Figure (14) presents the act of Fredkin gate on the input state |1100>.

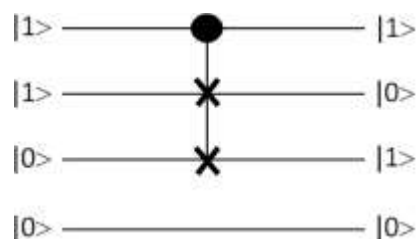


Fig. (14): the input/output of Fredkin gate for the state $|1100\rangle$

$$\begin{aligned}
 \text{Fredkin_Gate}|110\rangle|0\rangle &= \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 1 \\ 0 \end{bmatrix} |0\rangle \\
 &= \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 1 \\ 0 \\ 0 \end{bmatrix} |0\rangle
 \end{aligned}$$

$$\text{Fredkin_Gate}|110\rangle|0\rangle = |101\rangle|0\rangle = |1010\rangle \tag{14}$$

Case-14 ($|1101\rangle$): when Fredkin gate act on this state the state it will be changed to the $|1011\rangle$. Since the first three qubits at left are similar to the first three qubits in state $|1100\rangle$, so that the result for Fredkin gate act on state $|1101\rangle$ is similar to that for state $|1100\rangle$; as depicted in figure (15).

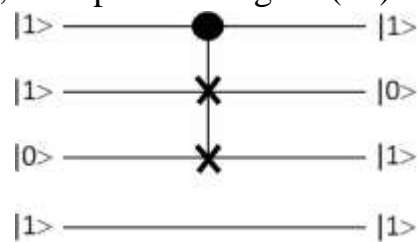


Fig. (15): the input/output of Fredkin gate for the state $|1101\rangle$

$$\text{Fredkin_Gate}|1101\rangle = \text{Fredkin_Gate}|110\rangle|1\rangle = |1011\rangle \tag{15}$$

Case-15 ($|1110\rangle$): since the first qubit is 1 and the second and third qubits from left side have same value, so the application of Fredkin gate on this state will cause no change in it; see figure (16).

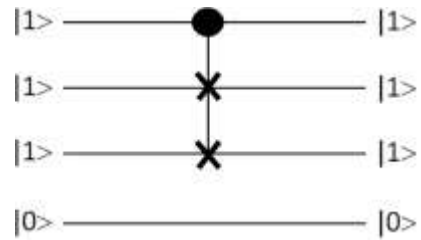


Fig. (16): the input/output of Fredkin gate for the state $|1110\rangle$

$$\begin{aligned}
 \text{Fredkin_Gate}|111\rangle|0\rangle &= \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 1 \end{bmatrix} |0\rangle \\
 &= \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 1 \end{bmatrix} |0\rangle
 \end{aligned}$$

$$\text{Fredkin_Gate}|111\rangle|0\rangle = |111\rangle|0\rangle = |1110\rangle \tag{16}$$

Case-16 ($|1111\rangle$): when Fredkin gate is applied on this state no change is occurred, because the 2nd and 3rd qubits have same values (1); see figure (17).

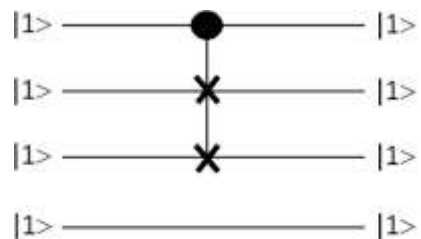


Fig. (17): the input/output of Fredkin gate for the state $|1110\rangle$

$$\text{Fredkin_Gate}|1111\rangle = \text{Fredkin_Gate}|111\rangle|1\rangle = |1111\rangle \tag{17}$$

In general the input and output states for Fredkin gate is showing in table (1), four states are change but another states will be unchange.

Table (1) The input/output states for Fredkin gate

No.	Input	Output	No.	Input	Output
0	0000>	0000>	8	1000>	1000>
1	0001>	0001>	9	1001>	1001>
2	0010>	0010>	10	1010>	1100>
3	0011>	0011>	11	1011>	1101>
4	0100>	0100>	12	1100>	1010>
5	0101>	0101>	13	1101>	1011>
6	0110>	0110>	14	1110>	1110>
7	0111>	0111>	15	1111>	1111>

When draw a graphic relationship between the output and input for act the Fredkin gate on this system, It will be the shape as shown in Figure (17).

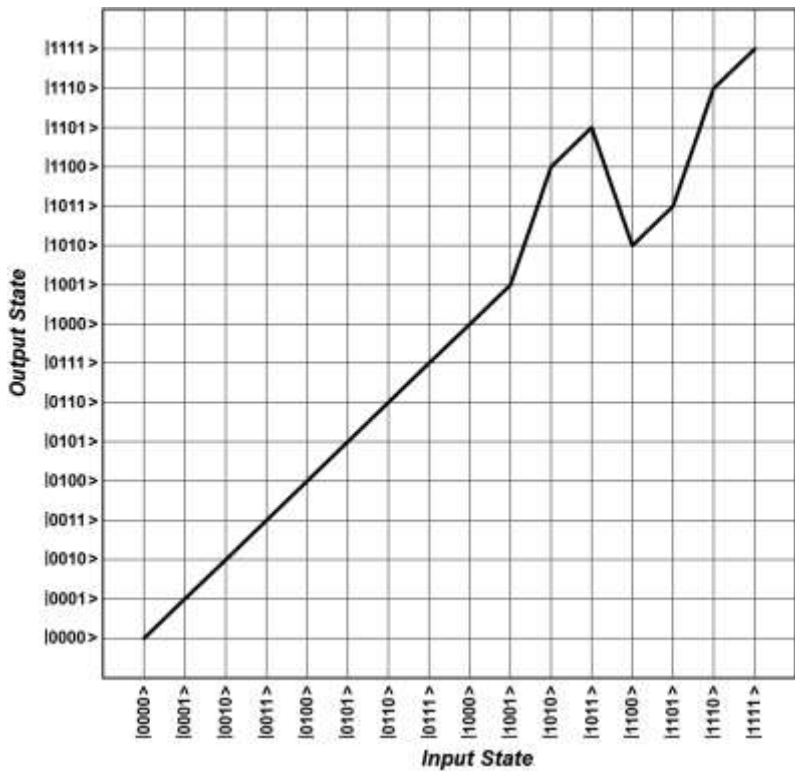


Fig. (17): The relationship between the input and output of Fredkin gate applies on a system consists of four particle

VI. Conclusion

In this research we found the state of Quantum system can be change to another state by using the Fredkin gate. The fredkin effect of a system consists of four particles with sixteen states was investigated. It was found that most of system states did not changed, except four of them, they change pairwise. There are two state raising cases ($|1010\rangle$ and $|1011\rangle$) and other pair of lowering states ($|1100\rangle$ and $|1101\rangle$) .

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