Imaging Performance of square apertures: Point spread function

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المستخلص:

يتضمن هذا البحث دراسة نظرية لمنظومة بصرية متضمنة فتحة مربعة والتي تلقي اهتماما كبيرا في كثير من المجالات التطبيقية. تم اشتقاق علاقة لدالة الانتشار النقطية (والتي تعتبر من المعايير الرئيسية لتقييم كفاءة المنظومات البصرية) لمنظومة بصرية تعاني من الحيود. تم ايضا ايجاد علاقة لدالة الانتشار النقطية لمنظومة بصرية تعاني خطأ بؤري .تم مناقشة النتائج المستحصلة في هذا البحث.

I. Introduction:

The imaging properties of optical system with square pupils have received a great deal of attention [1]. This is due to the fact that the square pupil design stands out for another optical design like circular pupil in comparison best performance and simplicity [2]. The present project deals with the theoretical study of the intensity distribution within the image. In this work, a special formula has been de derived called the point spread function (P.S.F.) by using pupil function technique. The work contained here deals with the evaluation of aberration free diffraction patterns. Finally some typical numerical results are presented and discussed.

2. Mathematical Formulation:

Consider a spherical wave which emerges from a square aperture and converges toward the focal plane. The complex amplitude at any point in the image plane [3] is given by:

$$F(u,v)=\frac{1}{A}\int_{y}\int_{x} f(x,y)e^{i2\pi(ux+vy)} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots(1)$$

where:

(u, v) = entrance pupil coordinate.
(x, y) = exit pupil coordinate.
A = Area of exit pupil.
f(x, y) is the pupil function [4], which has the form:

$$f(x,y)=\tau(x,y)e^{ikw(x,y)}$$

where $\tau(x, y)$ is the real amplitude distribution across wave front. In most cases this is uniform and is therefore put equal to unity.
k, the propagation constant, is $2\pi/\lambda$.
w (x, y) [1] is the wave aberration function

$$w_2 = w(x,y) = \sum_{n=1}^{N} w_{2n}(x^2+y^2)^n \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots(2)$$

where w_2 is the aberration function.
The spread function (distribution of illuminance in image plane due to point source) G(u,v) is given by the squared modulus of the complex amplitude [5] is:

$$G(u,v) = N\left|\int_y\int_x f(x,y)e^{i2\pi(ux+vy)}\,dx\,dy\right|^2$$

$$\text{Put } z = 2\pi u\ ,\ m = 2\pi v$$

$$\therefore G(z,m) = N\left|\int_{-1}^{1}\int_{-1}^{1} f(x,y)e^{i(zx+my)}\,dx\,dy\right|^2$$

N = normalizing constant.
When m = 0.

$$G(z) = N\left|\int_{-1}^{1}\int_{-1}^{1} f(x,y)e^{izx}\,dx\,dy\right|^2 \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots(4)$$

For a diffraction - limited system, where w(x, y)=0.

$$G(z) = N\left|\int_{-1}^{1}\int_{-1}^{1} e^{izx}\,dx\,dy\right|^2 \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots(5)$$

The normalizing constant N is such that G(0)=l

$$1 = N\left|\int_{-1}^{1}\int_{-1}^{1} dx\,dy\right|^2$$

$$N = \frac{1}{16}$$

Substituting N in eq.(5), we obtain

$$G(z) = \frac{1}{16} \left| \int_{-1}^{1}\int_{-1}^{1} e^{izx}\, dx\, dy \right|^2$$

$$G(z) = \frac{1}{16}\left[\int_{-1}^{1}\int_{-1}^{1} \cos(zx)\, dx\, dy\right]^2 \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots(6)$$

Now consider the same aberration-free system, this time having a longitudinal focal shift $w(x, y) = w_2 (x^2 + y^2)$. Following the procedures outlined for the un aberated case, the intensity is therefore given by:

$$G(z) = \frac{1}{16}\left\{\left[\int_{-1}^{1}\int_{-1}^{1} \cos(2\pi.w_2(x^2 + y^2) + zx)\, dx\, dy\right]^2 + \right.$$

$$\left.\left[\int_{-1}^{1}\int_{-1}^{1} \sin(2\pi.w_2(x^2 + y^2) + zx)\, dx\, dy\right]^2\right\} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots(7)$$

3. Numerical Results and Discussion:

24-point Gains quadrature has been used to evaluate the integral in eq. (6). The result thus obtained for the normalized intensity are given in table 1. (Fig. l) show how the integrity distribution in the diffraction pattern for square pupil varies with the distance from image center (z).

A comparison between the intensity distribution for system having circular pupil [5,6] and those obtained here from Fig. (1), we note that the radius of the intensity distribution for square pupil is smaller than for circular pupil. Thus the central peak of the intensity is sharper for the square than for the circular pupil. Therefore, we conclude that the resolution of the square pupil should be better than of the circular pupil.

The manner in which the P.S.F. of an ideal system having square pupil varies with focal shift is illustrated in figures 2 (a), (b), (c), (d). Even as small an amount of w_2 as 0.51 has an appreciable influence on the intensity distribution [7]. On the focal plane (z = 0), the second term of the integral in eq. (7) has vanish. Therefore the intensity on the focal plane is given by:

$$G(Z) = \frac{1}{16}\left\{\left[\int_{-1}^{1}\int_{-1}^{1} \cos(2\pi.w_2(x^2 + y^2))\, dx\, dy\right]^2 + \right.$$

$$\left.\left[\int_{-1}^{1}\int_{-1}^{1} \sin(2\pi.w_2(x^2 + y^2))\, dx\, dy\right]^2\right\}.$$

Figure (3) shows the variation of the central intensity with shift of focus.

Table 1: ***Intensity distribution for aberration free square aperture.***

Z	G (Z)
0	1
0.1	0.997
0.2	0.987
0.3	0.97
0.4	0.918
0.5	0.919
0.6	0.886
0.7	0.847
0.8	0.804
0.9	0.758
1	0.708
1.1	0.656
1.3	0.549
1.5	0.442
1.7	0.34
1.9	0.248
2.3	0.105
2.9	0.007
3.1	0
3.6	0.015
4.1	0.04
5.1	0.033
5.6	0.013
6.6	0.002
7.1	0.011

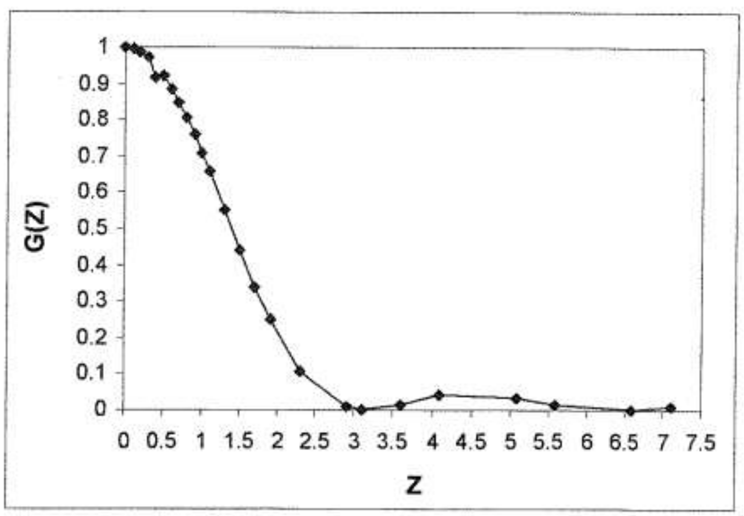

Fig. (1): Variation of the intensity distribution for system having square aperture.

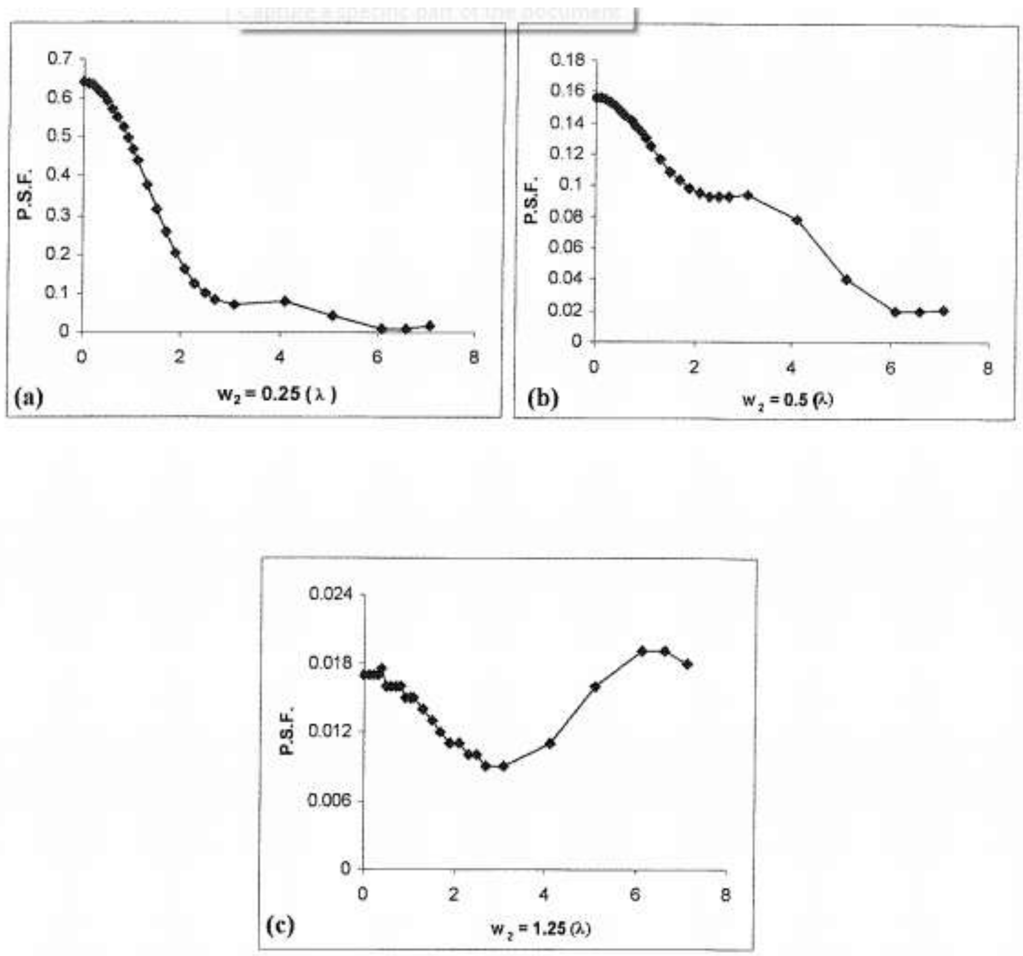

Fig. (2): (a), (b), (c) Variation of the P.S.F. for different longitudinal focal shifts.

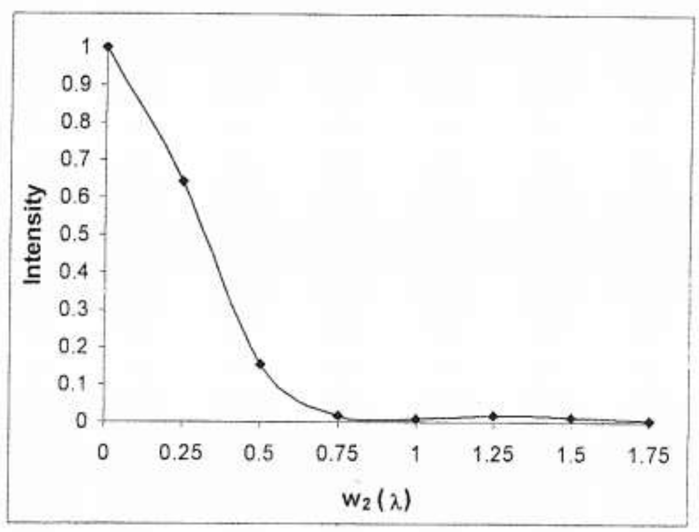

Fig. (3): The axial intensity distribution for aberration – free system.

4. Conclusion

We have derived a formalism to obtain a suitable expression for the intensity distribution in the diffraction pattern due to an optical system that uses a square pupil.

Examining the numerous graphical illustrations many in interesting conclusions have been drawn. The system having square pupil have a higher resolving power than the system contained circular pupil. It may be noted that, for higher values of 2 there is no much variation of the intensity. The intensity distribution diffraction pattern appears to be much effected by the defocus.

References:

1- Eric C. Kinter, J. Modern opt. 46: 1031-1042 (1999).

2- B. P. Mathur, N. N. Kundu, N. Gupta, IEEE Transactions on electron devices,35 (3): 294 - 297 (1988).

3- P. Horitz, Appl. Opt. 15: 167 - 171 (1976).

4- R. Bamkat, Opt. Commun. 156 (4): 235 - 239 (1998).

5- A. Refigier, RG. Mcmahon, DJ. Helfand, J. Opt. Soc. Am. 17 (7): lI85-l 191 (2000).

6- M. Born and Wolf, Principles of optics (Pergamon, London 1970).

7- De Nicolas, S. Denicolas, Pure Appl. Opt. 1(5): 1249 – 1259 (1998).