New oscillation results of Fourth order Differential Equations

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Abstract : The oscillation criteria was tested for all four order nonlinear neutral differential equation solutions in this paper . Examples are given to explain our key findings.

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Introduction :

In this article, we are considering fourth order neutral differential equations of the form:

$$\left[\mathcal{P}(\sigma)+\mathcal{S}(\sigma)\mathcal{P}\big(u(\sigma)\big)\right]^{(4)}+\mathcal{Q}(\sigma)\mathcal{T}\Big(\mathcal{P}\big(v(\sigma)\big)\Big)=0,\sigma\geq\sigma_0 \qquad (1)$$

Where $\mathcal{S}(\sigma),\mathcal{Q}(\sigma)\in C([\sigma_0,\infty);R)$,$u(\sigma),v(\sigma)\in C[\sigma_0,\infty);R),\lim_{\sigma\to\infty}u(\sigma)=\infty,\lim_{\sigma\to\infty}v(\sigma)=\infty, 0<\mathcal{S}(\sigma)<\mathcal{S},\mathcal{T}(a)\in C(R;R)$, For $\mathcal{T}(ab)\geq\mathcal{T}(a)\mathcal{T}(b)$, and $|\mathcal{T}(\sigma)|\geq m|\sigma|, m>0$.

By solving (1), we represent a function $\mathcal{P}(\sigma)$, where $\mathcal{P}(\sigma)+\mathcal{S}(\sigma)\mathcal{P}\big(u(\sigma)\big)$ is three times continuously differentiable and $\mathcal{P}(\sigma)$ satisfies (1) and $\mathcal{P}(\sigma)$ satisfies (1) on $[\sigma_0,\infty]$.

A nontrivial solution $\mathcal{P}(\sigma)$ is said to be oscillatory if it changes sign on $(\boldsymbol{\gamma},\infty)$**.** where $\boldsymbol{\gamma}$ is any number, otherwise it is said to be nonoscillatory, that. Equation (1) is said to be oscillatory if every solution of (1) is oscillatory.

Several research papers on the oscillatory behavior of neutral delay differential with fourth and n order equation solutions have appeared in the last several years For more information, one can refer to [1-5,7-8].

In this article, our aim is to supply new conditions for ensuring that every solution of(1)is an oscillating.

Mains Outcomes :

In that part, we establish same new conditions for every solution of (1) is an oscillating Define for

$$\mathfrak{D}(\sigma)=\mathcal{P}(\sigma)+\mathcal{S}(\sigma)\mathcal{P}\big(u(\sigma)\big) \qquad (2)$$

Then applying (2) in (1) we obtain

$$\mathfrak{D}^{(4)}(\sigma)=-\mathcal{Q}(\sigma)\mathcal{T}\Big(\mathcal{P}\big(v(\sigma)\big)\Big) \qquad (3)$$

Theorem1 : Assume that,$v(\sigma)\geq\sigma$,$u(\sigma)<\sigma,\mathcal{Q}(\sigma)\geq 0$ and

$$\liminf_{\sigma\to\infty}\int_\sigma^{v(\sigma)}\int_\sigma^{\varrho} m(\omega-\sigma)^2\mathcal{Q}(\omega)\mathcal{T}\big((1-\mathcal{S})\big)\,d\omega>\frac{2}{e} \qquad (4)$$

$$\limsup_{\sigma\to\infty}\sigma^3\int_\sigma^\infty m|\mathcal{Q}(\varrho)|\mathcal{T}(1-\mathcal{S})d\varrho>3 \qquad (5)$$

Then every solution of equation (1) oscillates.

Proof : Assume for the sake of contradiction that equation (1) has a nonoscillatory solution, let $\mathcal{P}(\sigma)$ be eventually positive(the case when $\mathcal{P}(\sigma)$ be eventually negative is similar and we omitted) solution is of (1) and $\mathcal{P}(u(\sigma)), \mathcal{P}(v(\sigma)) > 0$ for $\sigma \geq \sigma_1 \geq \sigma_0$.

From (3) it follows that

$\mathfrak{D}^{(4)}(\sigma) \leq 0,\ \sigma \geq \sigma_1$

Since $\mathfrak{D}(\sigma) \geq \mathcal{P}(\sigma) > 0$ for $\sigma_1 \geq \sigma_0$

Then $\mathfrak{D}^{(3)}(\sigma)$ is monotone , and eventually of ones sign,we calm that $\mathfrak{D}^{(3)}(\sigma) > 0$

Otherwise $\mathfrak{D}^{(3)}(\sigma) < 0$,leas to$\mathfrak{D}''(\sigma) < 0, \mathfrak{D}'(\sigma) < 0, \mathfrak{D}(\sigma) < 0$ for $\sigma \geq \sigma_2 \geq \sigma_1$

Which a contradiction then $\mathfrak{D}^{(3)}(\sigma) > 0$ leads to $\mathfrak{D}''(\sigma) < 0$ or $\mathfrak{D}''(\sigma) > 0$

First suppose that $\mathfrak{D}''(\sigma) < 0$ implies that $\mathfrak{D}'(\sigma) < 0$ or $\mathfrak{D}'(\sigma) > 0$

If $\mathfrak{D}'(\sigma) < 0$ we must that $\mathfrak{D}(\sigma) < 0$ which a contradiction .then $\mathfrak{D}'(\sigma) > 0$

By (2) we get

$\mathcal{P}(\sigma) = \mathfrak{D}(\sigma) - \mathcal{S}(\sigma)\mathcal{P}(u(\sigma)) > (1 - \mathcal{S})\mathfrak{D}(\sigma)$

By integral inequality

$$\mathfrak{D}^{(\varsigma)}(\sigma) = \sum_{i=\varsigma}^{\tau-1} (-1)^{i-\varsigma} \frac{(\varrho - \sigma)^{i-\varsigma}}{(i-\varsigma)!} \mathfrak{D}^{(i)}(\varrho) + \frac{(-1)^{i-\varsigma}}{(\tau - 1 - \varsigma)!} \int_{\sigma}^{\varrho} (\omega - \sigma)^{(\tau-1-\varsigma)} \mathfrak{D}^{(\tau)}(\omega) d\omega$$

Take ς =1, τ =4 we get

$$\mathfrak{D}'(\sigma) = \mathfrak{D}'(\varrho) - (\varrho - \sigma)\mathfrak{D}''(\varrho) + \frac{(\varrho - \sigma)^2}{2}\mathfrak{D}^{(3)}(\varrho) + \frac{(-1)^3}{2}\int_{\sigma}^{\varrho} (\omega - \sigma)^2 \mathfrak{D}^{(4)}(\omega)\, d\omega$$

So

$$\boldsymbol{\mathfrak{D}'(\sigma) \geq \frac{-1}{2}\int_{\sigma}^{\varrho} (\omega - \sigma)^2 \mathfrak{D}^{(4)}(\omega)\, d\omega} \qquad \textbf{(6)}$$

By (3) we have

$$-\mathfrak{D}^{(4)}(\sigma) = \mathcal{Q}(\sigma)\mathcal{T}\left(\mathcal{P}(v(\sigma))\right)$$

Substation in (6) we get

$$\mathfrak{D}'(\sigma) \geq \frac{1}{2}\int_{\sigma}^{\varrho} (\omega - \sigma)^2 \mathcal{Q}(\omega)\mathcal{T}\left(\mathcal{P}(v(\omega))\right) d\omega$$

$$\mathfrak{D}'(\sigma) \geq \frac{1}{2}\int_{\sigma}^{\varrho} m(\omega - \sigma)^2 \mathcal{Q}(\omega)\mathcal{T}\big((1 - \mathcal{S})\big)\,\mathfrak{D}(v(\omega)) d\omega$$

Since $\mathfrak{D}(v(\sigma))$ increasing then

$$\mathfrak{D}'(\sigma) - \frac{1}{2}\mathfrak{D}(v(\sigma))\int_{\sigma}^{\varrho} m(\omega - \sigma)^2 \mathcal{Q}(\omega)\mathcal{T}\big((1 - \mathcal{S})\big)\, d\omega \geq 0$$

By condition (4) and by a well-known result in [[6], Theorem 2.3.4.] which is contradiction .

Now suppose that $\mathfrak{D}''(\sigma) > 0$ implies that $\mathfrak{D}'(\sigma) > 0$

And by (2) $\mathcal{P}(\sigma) > (1 - \mathcal{S})\mathfrak{D}(\sigma)$

By (3) we get $\mathfrak{D}^{(4)}(\sigma) \leq -mQ(\sigma)\mathcal{T}(1-\mathcal{S})\mathfrak{D}(\upsilon(\sigma))$

Integrating from σ to ∞ we get

$$-\mathfrak{D}^{(3)}(\sigma) \leq -\int_{\sigma}^{\infty} mQ(\varrho)\mathcal{T}(1-\mathcal{S})\mathfrak{D}(\upsilon(\varrho))d\varrho$$

$$\mathfrak{D}^{(3)}(\sigma) \geq \int_{\sigma}^{\infty} mQ(\varrho)\mathcal{T}(1-\mathcal{S})\mathfrak{D}(\upsilon(\varrho))d\varrho$$

$$\mathfrak{D}^{(3)}(\sigma) \geq \mathfrak{D}(\upsilon(\sigma))\int_{\sigma}^{\infty} mQ(\varrho)\mathcal{T}(1-\mathcal{S})d\varrho$$

Since $\mathfrak{D}(\upsilon(\sigma)) \geq \mathfrak{D}(\sigma)$ then

$$\boldsymbol{\mathfrak{D}^{(3)}(\sigma) \geq \mathfrak{D}(\sigma)\int_{\sigma}^{\infty} mQ(\varrho)\mathcal{T}(1-\mathcal{S})d\varrho} \qquad \textbf{(7)}$$

Let

$$\int_{\sigma_2}^{\sigma} \varrho\mathfrak{D}^{(4)}(\varrho)\,d\varrho = \sigma\mathfrak{D}^{(3)}(\sigma) - \sigma_2\mathfrak{D}^{(3)}(\sigma_2) - \mathfrak{D}^{(2)}(\sigma) + \mathfrak{D}^{(2)}(\sigma_2)$$

as $\sigma \to \infty$ the last inequality yields to

$$\lim_{\sigma\to\infty}[\sigma\mathfrak{D}^{(3)}(\sigma) - \sigma_2\mathfrak{D}^{(3)}(\sigma_2) - \mathfrak{D}^{(2)}(\sigma) + \mathfrak{D}^{(2)}(\sigma_2)] = \infty$$

Then

$$\lim_{\sigma\to\infty}[\sigma\mathfrak{D}^{(3)}(t) - \mathfrak{D}^{(2)}(\sigma)] = \infty$$

Hence

$$\mathfrak{D}^{(2)}(\sigma) \geq \sigma\mathfrak{D}^{(3)}(\sigma)$$

similary we can find $\mathfrak{D}(\sigma) \geq \frac{\sigma^3}{3}\mathfrak{D}^{(3)}(\sigma)$

then equation (7) become $\mathfrak{D}(\sigma) \geq \frac{\sigma^3}{3}\mathfrak{D}^{(3)}(\sigma) \geq \frac{\sigma^3}{3}\mathfrak{D}(\sigma)\int_{\sigma}^{\infty} mQ(\varrho)\mathcal{T}(1-\mathcal{S})d\varrho$

then $3 \geq \sigma^3\int_{\sigma}^{\infty} mQ(\varrho)\mathcal{T}(1-\mathcal{S})d\varrho$

which is acontraduction of (5).

Theorem 2: Assume that,$\upsilon(\sigma) \geq \sigma, Q(\sigma) \leq 0$ $\upsilon(\omega(\sigma)) > \sigma$, $\upsilon(u^{-1}(\sigma)) > \sigma$ and

$$\boldsymbol{\limsup_{\sigma\to\infty} \sigma^3\int_{\sigma}^{\infty} mQ(\varrho)\mathcal{T}(1-\mathcal{S})d\varrho > 3} \qquad \textbf{(8)}$$

$$\boldsymbol{\liminf_{\sigma\to\infty} \int_{\sigma}^{\upsilon(\omega(\sigma))} m|Q(s)|\mathcal{T}\left(\frac{1}{2}\int_{\omega(s)}^{s}(\upsilon(s)-\varsigma)^2 d\varsigma\right) > \frac{1}{e}} \qquad \textbf{(9)}$$

$$\boldsymbol{\liminf_{\sigma\to\infty} \int_{\sigma}^{\upsilon(u^{-1}(\sigma))}\int_{\zeta}^{\infty}\int_{\omega}^{\infty} m|Q(\varsigma)|\mathcal{T}\left(\frac{1}{\mathcal{S}}\left(1-\frac{1}{\mathcal{S}}\right)\right)\upsilon\left(u^{-1}(\varsigma)\right)d\varsigma > \frac{1}{e}} \qquad \textbf{(10)}$$

Then every solution of equation (1) oscillates.

Proof : Assume for the sake of contradiction that equation (1) has a nonoscillatory solution, let $\mathcal{P}(\sigma)$ be eventually positive(the case when $\mathcal{P}(\sigma)$ be eventually negative is similar and we omitted) solution is of eq(1) and $\mathcal{P}(u(\sigma)), \mathcal{P}(\upsilon(\sigma)) > 0$ for $\sigma \geq \sigma_1 \geq \sigma_0$.

From (3) it follows that

$\mathfrak{D}^{(4)}(\sigma) \geq 0,\ \sigma \geq \sigma_1$

Since $\mathfrak{D}(\sigma) \geq \mathcal{P}(\sigma) > 0$ for $\sigma_1 \geq \sigma_0$

Thus for $\sigma \geq \sigma_1$ either $\mathfrak{D}^{(3)}(\sigma) > 0$ or $\mathfrak{D}^{(3)}(\sigma) < 0$ for $\sigma \geq \sigma_1 \geq \sigma_0$.

First let $\mathfrak{D}^{(3)}(\sigma) > 0$,thus implies that $\mathfrak{D}''(\sigma) > 0,\ \mathfrak{D}'(\sigma) > 0$,

By (3) we have $\mathfrak{D}^{(4)}(\sigma) = |\mathcal{Q}(\sigma)|\mathcal{T}\left(\mathcal{P}\big(\mathcal{v}(\sigma)\big)\right)$

since $\mathcal{P}(\sigma) > (1-\mathcal{S})\mathfrak{D}(\sigma))$

Then $\mathfrak{D}^{(4)}(\sigma) \geq m|\mathcal{Q}(\sigma)|\mathcal{T}(1-\mathcal{S})\mathfrak{D}\big(\mathcal{v}(\sigma)\big)$

Integrating above inequality from $\mathcal{v}^{-1}(\sigma)$ to σ we get

$$\mathfrak{D}^{(3)}(\sigma) \geq \int_{\mathcal{v}^{-1}(\sigma)}^{\sigma} m|\mathcal{Q}(\varrho)|\mathcal{T}(1-\mathcal{S})\mathfrak{D}\big(\mathcal{v}(\varrho)\big)d\varrho$$

$\mathfrak{D}^{(3)}(\sigma) \geq \mathfrak{D}\big(\mathcal{v}(\sigma)\big)\int_{\mathcal{v}^{-1}(\sigma)}^{\sigma} m|\mathcal{Q}(\varrho)|\mathcal{T}(1-\mathcal{S})d\varrho$ (11)

No by using $\mathfrak{D}(\sigma) \geq \frac{\sigma^3}{3}\mathfrak{D}^{(3)}(\sigma)$ then

$$\mathfrak{D}(\sigma) \geq \frac{\sigma^3}{3}\mathfrak{D}^{(3)}(\sigma) \geq \frac{\sigma^3}{3}\mathfrak{D}(\sigma)\int_{\mathcal{v}^{-1}(\sigma)}^{\sigma} m|\mathcal{Q}(\varrho)|\mathcal{T}(1-\mathcal{S})d\varrho$$

$$3 \geq \sigma^3\int_{\mathcal{v}^{-1}(\sigma)}^{\sigma} m|\mathcal{Q}(\varrho)|\mathcal{T}(1-\mathcal{S})d\varrho$$

Which is acontraduction (8)

Next if $\mathfrak{D}^{(3)}(\sigma) < 0$ for $\sigma \geq \sigma_2 \geq \sigma_1$,thus implies that $\mathfrak{D}''(\sigma) > 0$ or $\mathfrak{D}''(\sigma) < 0$ for all $\sigma \geq \sigma_2$,if $\mathfrak{D}''(\sigma) > 0$ then either $\mathfrak{D}'(\sigma) > 0$ or $\mathfrak{D}'(\sigma) < 0$

If $\mathfrak{D}'(\sigma) > 0$

By inequality

$$\mathfrak{D}(\sigma) = \sum_{i=0}^{\tau-1}\frac{(\xi-\sigma)^i}{i!}\mathfrak{D}^{(i)}(\xi) + \frac{1}{(\tau-1)!}\int_{\xi}^{\sigma}(\sigma-\varsigma)^{(\tau-1)}\mathfrak{D}^{(\tau)}(\varsigma)d\varsigma$$

Take τ=3 we have

$$\mathfrak{D}(\sigma) = \mathfrak{D}(\xi) + \mathfrak{D}'(\xi) + \frac{1}{2}\int_{\xi}^{\sigma}(\sigma-\varsigma)^2\mathfrak{D}^{(3)}(\varsigma)d\varsigma$$

$$\mathfrak{D}(\sigma) \geq \frac{1}{2}\int_{\xi}^{\sigma}(\sigma-\varsigma)^2\mathfrak{D}^{(3)}(\varsigma)d\varsigma$$

Let $\xi = \omega(\sigma)$ then

$$\mathfrak{D}(\sigma) \geq \frac{1}{2}\int_{\omega(\sigma)}^{\sigma}(\sigma-\varsigma)^2\mathfrak{D}^{(3)}(\varsigma)d\varsigma$$

Since $\mathfrak{D}^{(3)}(\sigma)$ increasing then

$$\mathfrak{D}(\sigma) \geq \frac{\mathfrak{D}^{(3)}(\omega(\sigma))}{2} \int_{\omega(\sigma)}^{\sigma} (\sigma - \varsigma)^2 d\varsigma$$

So

$$\mathfrak{D}\big(\boldsymbol{v}(\boldsymbol{\sigma})\big) \geq \frac{\mathfrak{D}^{(3)}(\boldsymbol{v}(\boldsymbol{\omega}(\boldsymbol{\sigma})))}{\mathbf{2}} \int_{\boldsymbol{\omega}(\boldsymbol{\sigma})}^{\boldsymbol{\sigma}} (\boldsymbol{v}(\boldsymbol{\sigma}) - \boldsymbol{\varsigma})^{\mathbf{2}} \mathbf{d}\boldsymbol{\varsigma} \qquad \mathbf{(12)}$$

eq(3) can be writing as the from $\mathfrak{D}^{(4)}(\sigma) \geq |\mathcal{Q}(\sigma)|\mathcal{T}\left(\mathfrak{D}\big(v(\sigma)\big)\right)$

suppose that $\varphi(\sigma) = \mathfrak{D}^{(3)}(\sigma)$ then $\varphi'(\sigma) \geq m|\mathcal{Q}(\sigma)|\mathcal{T}\left(\mathfrak{D}\big(v(\sigma)\big)\right)$

by using (12) we get

$$\varphi'(\sigma) \geq m|\mathcal{Q}(\sigma)|\varphi\big(v(\omega(\sigma))\big)\mathcal{T}\left(\frac{1}{2}\int_{\omega(\sigma)}^{\sigma} (v(\sigma) - \varsigma)^2 d\varsigma\right)$$

$$\varphi'(\sigma) - m|\mathcal{Q}(\sigma)|\varphi\big(v(\omega(\sigma))\big)\mathcal{T}\left(\frac{1}{2}\int_{\omega(\sigma)}^{\sigma} (v(\sigma) - \varsigma)^2 d\varsigma\right) \geq 0$$

By condition (9) and by a well-known result in ([6], Theorem 2.3.4.) which contraction .

Next if $\mathfrak{D}'(\sigma) < 0$,then equation (2) can by writen as the from

$\mathcal{S}(\sigma)\mathcal{P}\big(u(\sigma)\big) = \mathfrak{D}(\sigma) - \mathcal{P}(\sigma)$

And $\quad \mathcal{P}(\sigma) > \frac{1}{\mathcal{S}}(1 - \frac{1}{\mathcal{S}})\mathfrak{D}\big(u^{-1}(\sigma)\big)$

No from eq (3) and above inequality we have

$\mathfrak{D}^{(4)}(\sigma) \geq m|\mathcal{Q}(\sigma)|\mathcal{T}\left(\frac{1}{\mathcal{S}}(1 - \frac{1}{\mathcal{S}})\right)\mathfrak{D}\left(v\big(u^{-1}(\sigma)\big)\right)$

Since $\mathfrak{D}\left(v\big(u^{-1}(\sigma)\big)\right) \geq v\big(u^{-1}(\sigma)\big)\mathfrak{D}'\left(v\big(u^{-1}(\sigma)\big)\right)$

Then $\mathfrak{D}^{(4)}(\sigma) \geq m|\mathcal{Q}(\sigma)|\mathcal{T}\left(\frac{1}{\mathcal{S}}(1 - \frac{1}{\mathcal{S}})\right)v\big(u^{-1}(\sigma)\big)\mathfrak{D}'\left(v\big(u^{-1}(\sigma)\big)\right)$

Integrating above inequality from σ to ∞ we get

$$-\mathfrak{D}^{(3)}(\sigma) \geq \int_{\sigma}^{\infty} m|\mathcal{Q}(\varsigma)|\mathcal{T}\left(\frac{1}{\mathcal{S}}(1 - \frac{1}{\mathcal{S}})\right)v\big(u^{-1}(\varsigma)\big)\mathfrak{D}'\left(v\big(u^{-1}(\varsigma)\big)\right)d\varsigma$$

Since $\mathfrak{D}'\left(v\big(u^{-1}(\sigma)\big)\right)$ increasing then

$$\mathfrak{D}''(\sigma) \geq \mathfrak{D}'\left(v\big(u^{-1}(\sigma)\big)\right)\int_{\sigma}^{\infty} m|\mathcal{Q}(\varsigma)|\mathcal{T}\left(\frac{1}{\mathcal{S}}(1 - \frac{1}{\mathcal{S}})\right)v\big(u^{-1}(\varsigma)\big)d\varsigma$$

Integrating above inquality from σ to ∞ we get

$$\mathfrak{D}''(\sigma) \geq \mathfrak{D}'\left(v\big(u^{-1}(\sigma)\big)\right)\int_{\sigma}^{\infty}\int_{\omega}^{\infty} m|\mathcal{Q}(\varsigma)|\mathcal{T}\left(\frac{1}{\mathcal{S}}(1 - \frac{1}{\mathcal{S}})\right)v\big(u^{-1}(\varsigma)\big)d\varsigma$$

Let $\varphi(\sigma) = \mathfrak{D}'(\sigma)$ then

$$\varphi'(\sigma)-\varphi\left(\mathcal{v}\left(\mathcal{u}^{-1}(\sigma)\right)\right)\int_{\sigma}^{\infty}\int_{\omega}^{\infty} m|\mathcal{Q}(\varsigma)|\mathcal{T}\left(\frac{1}{\mathcal{S}}(1-\frac{1}{\mathcal{S}})\right)\mathcal{v}\left(\mathcal{u}^{-1}(\varsigma)\right)d\varsigma \geq 0$$

By condition (10) and by a well-known result in [[6], Theorem 2.3.4.]Which is contradiction.

Examples : In this section, We're presenting some examples that explain the essential results

Example 1. Consider the equation:

$$\left[\mathcal{P}(\sigma)+\tfrac{1}{3}\mathcal{P}(\sigma-\pi)\right]^{(4)}+\tfrac{2}{3}\mathcal{T}\left(\mathcal{P}(\sigma+3\pi)\right)=0$$

$\mathcal{S}(\sigma)=\dfrac{1}{3}$ $,\mathcal{u}(\sigma)=\sigma-\pi, \mathcal{Q}(\sigma)=\dfrac{2}{3}$ $,\mathcal{T}\left(\mathcal{P}(\sigma+3\pi)\right)=\mathcal{P}(\sigma+3\pi),$

$\mathcal{v}(\sigma)=\sigma+3\pi, m=0.5$

We will find the conditions of Th (1) are satisfies also We're even going to find a oscillation solution by $\mathcal{P}(\sigma)=\cos\sigma$ from this equation.

Example 2. consider the equation:

$$\left[\mathcal{P}(\sigma)+\tfrac{4}{5}\mathcal{P}(\sigma-2\pi)\right]^{(4)}-\tfrac{9}{5}\mathcal{T}\left(\mathcal{P}(\sigma+4\pi)\right)=0$$

$\mathcal{S}(\sigma)=\dfrac{4}{5}$ $,\mathcal{u}(\sigma)=\sigma-2\pi, \mathcal{Q}(\sigma)=-\dfrac{9}{5}$ $,\mathcal{T}\left(\mathcal{P}(\sigma+4\pi)\right)=\mathcal{P}(\sigma+4\pi),$

$\mathcal{v}(\sigma)=\sigma+4\pi, m=0.8$

We will find the conditions of Th (2) are satisfies also We're even going to find a oscillation solution by $\mathcal{P}(\sigma)=\sin\sigma$ from this equation.

Conclusion: In the present paper, A wide variety of new conditions, This is obtained to ensure the oscillation activity is for every solution of nutral differential equation for fourth order.

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