Numerical Computations of Quality Factor and Attenuation Coefficient for TE_{mnl}- Modes in Gyrotron Tube

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Abstract

In the present work, the quality factor of the Gyrotron tube as a function of cavitythe diameter and lengthwas studied. Attenuation coefficients as a function of both frequency (GHz) and cavity radius was also studied. A computer program was costructed and designed for compute the quality factor and attenuation coefficients for only TE_{mnl} modes. The same program calculates the cut off frequencies of the lowest order modes. Results shows good agreement with experimental and theoritical studies.

Keywords: *quality factor, attenuation coefficient, TE modes, Gyrotron tube.*

الحسابات العددية لعامل الجودة ومعامل التوهين للأنماط الكهربائية المستعرضة TE_{mnl} في إنبوب الجايروترون

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المستخلص

في البحث الحالي، تم دراسة عامل الجودة لإنبوبة الجايروترون كدالة لقطر وطول التجويف ودراسة معاملات التوهين كدالة لنصف قطر التجويف والتردد (بوحدات كيكا هيرتز) في إنبوبة الجايروترون. ونتيجةً لذلك، تم تصميم وإنشاء برنامج لحساب عامل الجودة ومعاملات التوهين للأنماط TE_{mnl} فقط لدليل الموجة الدائري. إضافة الى ذلك، يجيز البرنامج إمكانية حساب ترددات القطع للأنماط ذا الرتب الأدنى. لقد بيّنت النتائج توافقاً جيداً مع النتائج العملية والبيانات النظرية المنشورة.

Introduction:

We supposed a hollow metalic wave guide,in which, multimodes of electromagnetic waves are propagating it'sfrequency range is up to Giga Hertz in range. In order to be propagating take place along(z) axis of it, the field vector should exhibit a dependence of the function ($e^{\mp \gamma z}$). The parameter (γ) is called the propagation constant,and it has a complex value:

$\gamma = \alpha + \mathrm{j}\beta \cdots\cdots\cdots\cdots (1)$

Where: α is the modal attenuation constant, $j = \sqrt{-1}$, β is the phase constant, which determines the change of wave phase during the propagation in side the wave guide [1].

Furthermore, α _parameter in eq(1) represents a sumation of two components.

$\alpha = \alpha_{\mathrm{d}} + \alpha_{\mathrm{c}} \cdots\cdots\cdots\cdots (2)$

Where α_{d} and α_{c} are the(losses) attenuation constants due to dielectric and metal ,respecitivily.In metal hollow, α_{d} is negligible, such that: $\alpha \cong \alpha_c \cdots\cdots\cdots\cdots (3)$

It is useful to specify the attenuation constant for mode I term of decibel per meter ($^{dB}/_{m}$) [2]. The modal of field distribution over the wave guide cross

section is slightly disturbed. Instead, aperturbation approach can be used to estimat (α_{c}), except for modes with frequencies that propagating very close to the cut off frequency [3]. Noting that the transmitted power inside the wave guide decay exponentially, according to the equation:

$P = P_o\, e^{-2\alpha_{\mathrm{c}} z} \cdots\cdots\cdots\cdots (4)$,

where P_o is the power in the (z= 0) location. Differentation for eq(4) with respect to (z), solving for α_{c} ,and defining P_L we obtain:

$$\alpha = \frac{1}{2}\frac{P_L}{P} \cdots\cdots\cdots\cdots (5)$$

Noting that: $\frac{dp}{dz} = -P_L$

recognizing that transmitted power P is integral of the a verge pointing vector over the wave guide cross section .

During ago decades ,Yeap , et al. have discussed and developed a novel technique to compute the attenuation for propagating waves in circular wave guides, with loss and also with superconductor [3]. Our earlier two numerical studies we study the quality factor for TM_{mn0}-modes in gyrotron tube [6] and also for TM_{mnl} [7].

Numerical method:

Consider a wave propagating along the longitudinal z-axis in a cylindrical wave guide with cross section radius(r). To drive the longitudinal electric (E_Z) or magnetic (H_Z) fields one should start with Helmholtz wave equ[8]. Decomposing the last equation into two parts, radial and longitudenal,in the cylindrical coordinate system, resulting in the following equ

$$\nabla_{rad}^2 E(r,\emptyset,z) + \frac{\partial^2 E(r,\emptyset,z)}{\partial Z^2} + k^2 E(r,\emptyset,z) = 0 \cdots\cdots\cdots\cdots (6)$$

Where Laplasian operator, in this system, depends only on to(r,∅),
Such that:

$$\nabla_{rad}^2 E(r,\emptyset) = \frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial E(r,\emptyset)}{\partial r}\right) + \frac{1}{r^2}\frac{\partial^2 E(r,\emptyset)}{\partial^2 \phi} + k^2 E(r,\emptyset) = 0 \cdots\cdots\cdots (7)$$

Where:$k^2 = \frac{\omega^2}{c^2} = \varepsilon_\circ \mu_\circ \omega^2$

ω is the angular frequency and $\varepsilon_\circ \mu_\circ$ are the electric permittivity and magnetic permeability for vacum,respectivity the plane wave solution for the electric field take form:

$$\mathrm{E}(\mathrm{r},\phi,\mathrm{z}) = E(r,\phi)exp[i(\omega t - \mathrm{k}_Z)] \cdots\cdots\cdots\cdots (8)$$

Equation (8) can be split in to two equation (separation of variable methode) to be in the following form:

$$\mathrm{E}(\mathrm{r},\phi,\mathrm{z}) = U(r)\phi(\phi)Z(z) \cdots\cdots\cdots\cdots (9)$$

With neglecting the part $\phi(\phi)$,the remaining are two equations:

$$r^2\frac{\partial^2 u(r)}{\partial r^2} + r\frac{\partial u(r)}{\partial r} + [(k_c r - m^2)]v(r) = 0 \cdots\cdots\cdots\cdots (10)$$

$$\frac{\partial^2 \phi}{\partial \phi^2} + m^2\phi(\phi) = 0 \cdots\cdots\cdots\cdots (11)$$

Equation (11) represents a simple harmonic oscillator equation while Eq. (10) represents one form of the Bessel's equations, whose solutions are the Bessel's functions [10].

The following set of field equations are:

$$H_z = c_m^{\grave{}} J_m(k_c)\ \sin(m\phi) \cdots\cdots\cdots\cdots (12)$$

$$E_z = c_m J_m(k_c r)\ \cos(m\phi) \cdots\cdots\cdots\cdots (13)$$

Where c_m and $c_m^{\grave{}}$ denote the coefficients of the longitudinal fields, r is the radial distance, $J_m(k_c r)$ is called the Bessel function of the first kind and m is the order of the Bessel function.

The propagation constant γ is a complex variable which constitutes a phase constant β and an attenuation α, where: $\gamma = \beta - j\alpha$.

Results and Discussion:

A Fortran program was written and designed (as shown in appendix_1) to evaluate the attenuation coefficient of TE waves which based on equation [10].

$$\propto_{TE_{mn}} = \frac{R_S}{a\eta}\frac{1}{\sqrt{1-(f_c/f)^2}}\left[(\frac{f_c}{f})^2 + \frac{m^2}{(x_{mn}^{`2} - m^2)}\right] \cdots\cdots\cdots\cdots (14)$$

Where R_S is the surface resistance of the metal $R_S = \sqrt{\frac{\mu\pi f}{a}}$ for silver R_S=0.029Ω ,μ is the permeability of the dielectric and η is the intrinsic impedance of the dielectric (377Ω for air), $f = \omega/2\pi$ is the operating frequency in *Hz* unit.

The dominant mode in a circular wave guide is theTE_{11}-mode which has acutoff frequency given by:

$$f_{c,11} = \frac{0.293}{a\sqrt{\mu\varepsilon}}$$

Were μ *and* ε *are* the magnetic permeability and the permittivity of the dielectric respectively.

The configuration of the electric and magnetic fields of TE_{11}-mode are shown in Fig.(2).

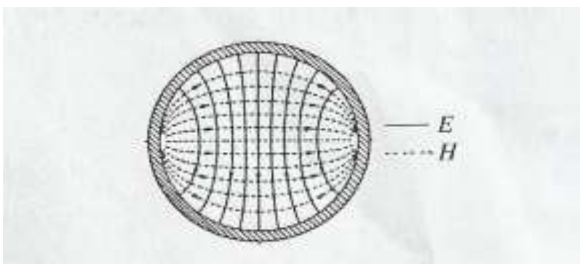

Fig.1: Field configuration for the TE_{11} (dominant) mode in a circular waveguide.

We have been used equation: [11]

$$Q\frac{\delta}{\lambda} = \frac{\left[1-(\frac{m}{X_{mn}})^2\right][X_{mn}^2 + p^2R^2]^{\frac{3}{2}}}{2\pi\left[X_{mn}{}^2 + p^2R^3 + (1-R)\left(\frac{pRm}{X_{mn}}\right)^2\right]} \cdots\cdots\cdots (14)$$

where R=D/L and p=$\pi\ell/2$

To study the quality factor (Q) and (D/L) for different TE $_{mnl}$ modes, Computer program was designed and written to this purpose. It can be seen from Fig.3 or Eq.(14) that for a given TE_{mnl}-mode, $Q(\delta/\lambda)$ is a maximum for a circular cylinder that is, D=L, because size is limited to minimize losses , for which losses are above those of large in diameter.

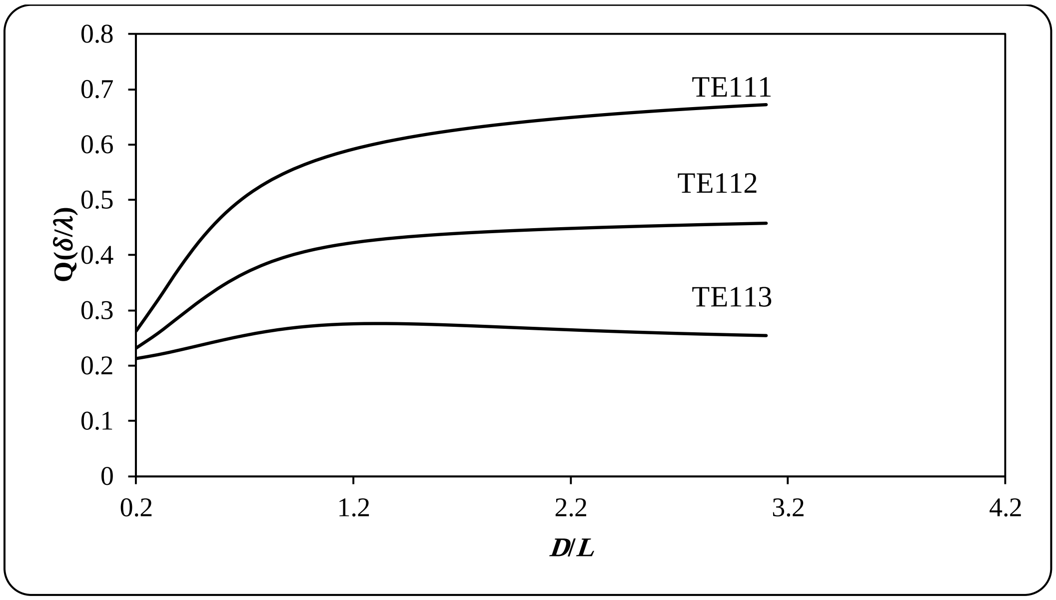

Fig.(3-a): $Q(\delta/\lambda)$ versus (D/L) for the TE_{111},TE_{112},TE_{113}.

Equation(14) is represented graphically in Figs.(3), where $Q(\delta/\lambda)$ values are plotted for several TE-modes as a function of D/L, where R=D/L . The quantity $Q(\delta/\lambda)$ is commonly tabulated instead of Q ,since this quantity is a function of only the mode and shape of the cavity. $Q(\delta/\lambda)$ is a maximum for a circular cylinder that is, D=L, because size is limited to minimize losses , for which losses are above those of large in diameter.

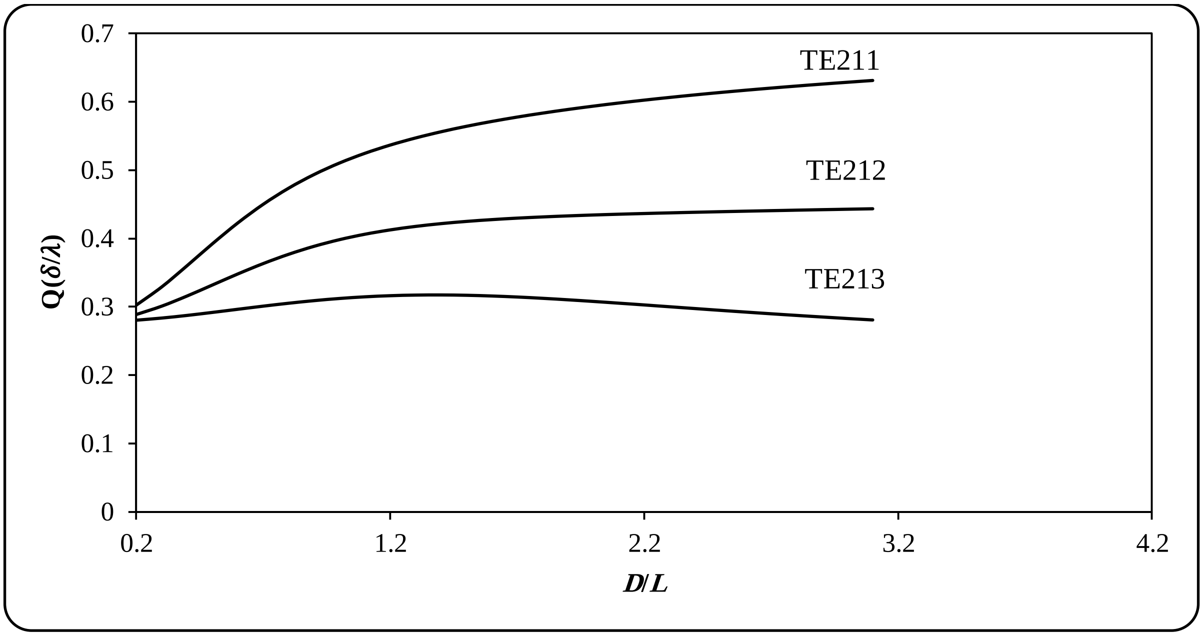

Fig.(3-b): $Q(\delta/\lambda)$ versus (D/L) for the TE_{211},TE_{212},TE_{213}.

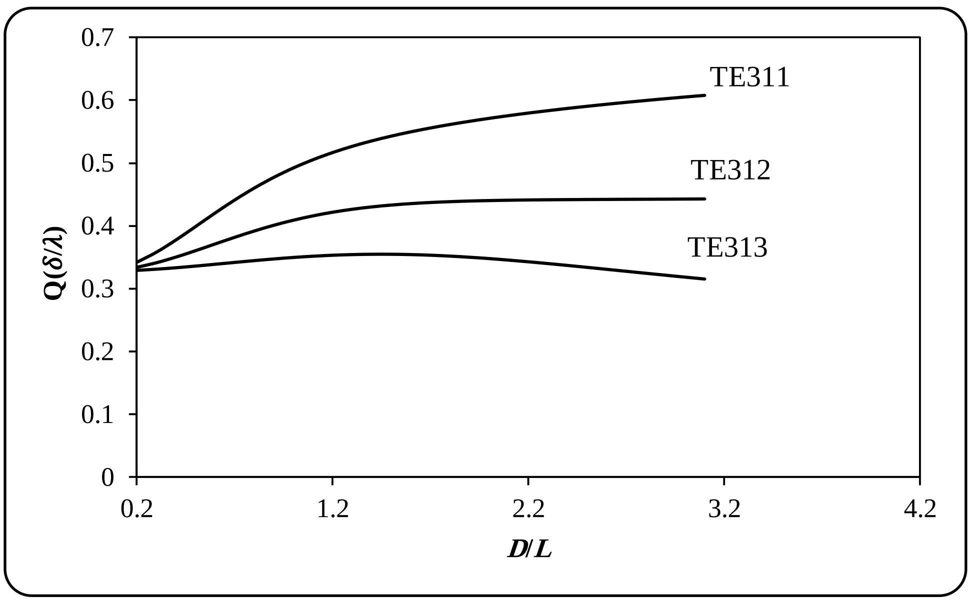

Fig.(3-c): $Q(\delta/\lambda)$ versus (D/L) for the TE_{311},TE_{312},TE_{313}.

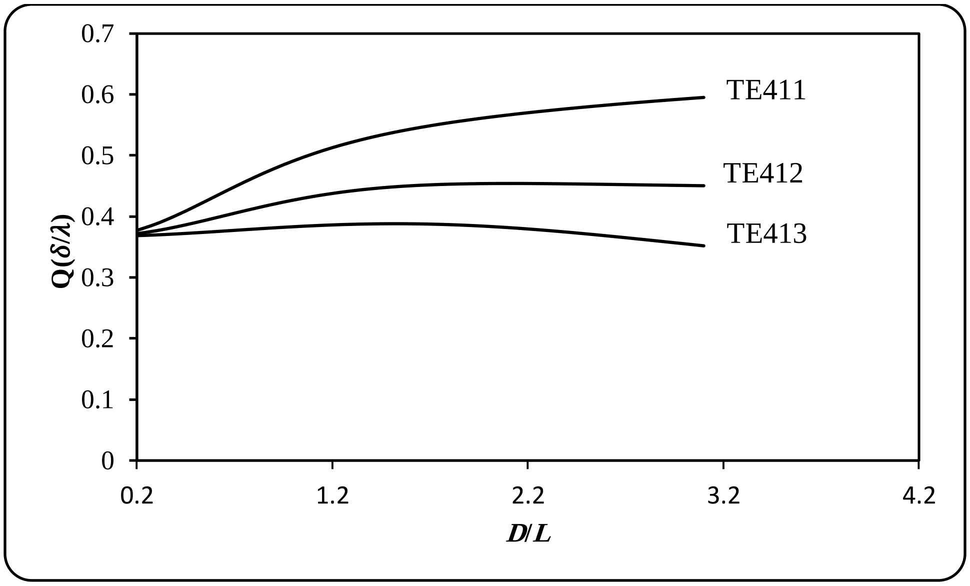

Fig.(3-d) ($Q(\delta/\lambda)$) versus (D/L) for the TE_{411},TE_{412},TE_{413}

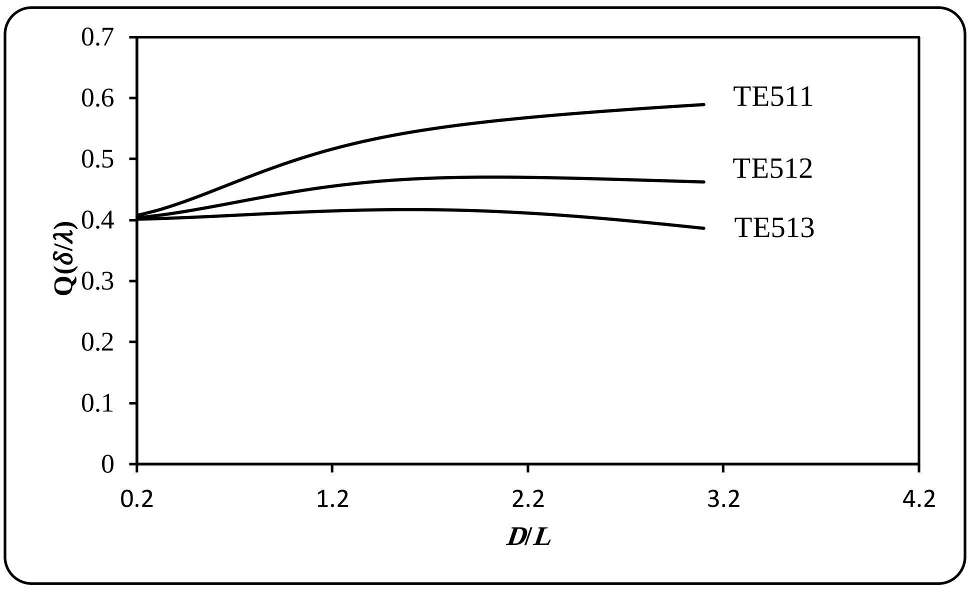

Fig.(3-e): $Q(\delta/\lambda)$ versus (D/L) for the TE_{511},TE_{512},TE_{513}.

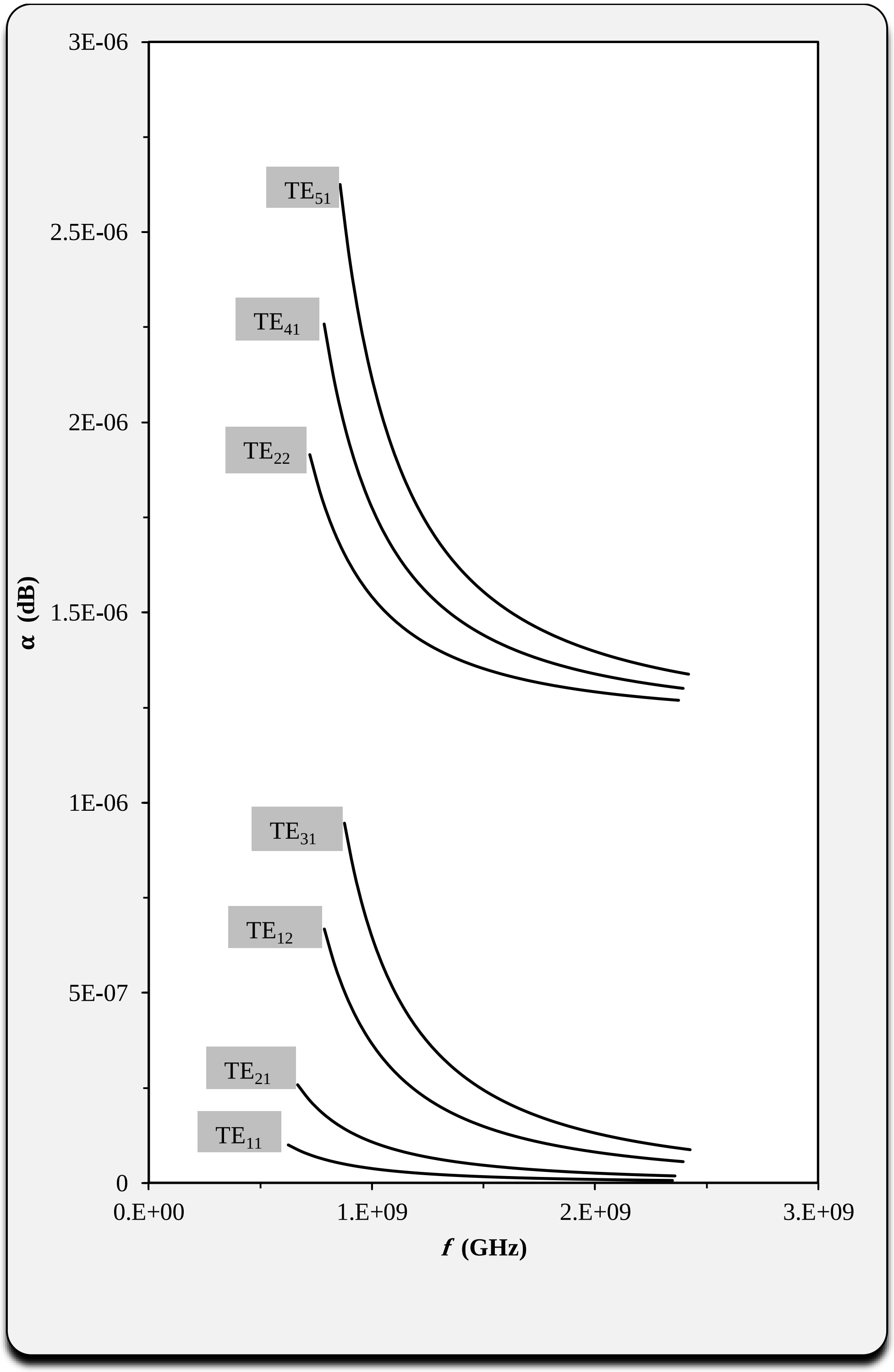

Fig.4: The attenuation constant of several higher order modes in circular waveguide.

Table.1: show the cutoff frequencies of the high order circular waveguide modes.

Modes	Mode Normalized type f_c		X_{mn}
	Our Results	Ref.2	
TE_{11}	1.0	1.0	1.841
TE_{21}	1.677	1.658	3.054
TE_{31}	2.306	2.455	4.201
TE_{41}	2.919	2.888	5.318
TE_{51}	3.486	------	6.415
TE_{12}	2.927	2.895	5.331
TE_{22}	3.682	3.641	6.706

Equation (7) is represented graphically in Fig.4 here attenuation coeffcient is plotted for several modes as a function of frequency. In these figures compare the dissipative attenuation in this TE_{11} wave (silver guide) with the reactive attenuation in the next higher order wave. For different TE_{11}, TE_{12} , TE_{21}, TE_{31}, TE_{41}, TE_{51} and TE_{22} in a guide of fixed diameter, attenuation is plotted verses frequency. Moreover, to use the potentially very low attenuation constant, the operating frequency must be well above cutoff frequency, so many more modest than above are in the propagating range. Numbers of our results cutoff frequency compared with experimental values [2] for high order circular waveguide modes were listed in table-1-.

Conclusions:

In order to have propagation, the wave frequency should be greater than the cut off frequency $f > f_{c,mn}$.

The $TE_{mn\_}$ mode yields a decreasing attenuation for a pipe of fixed size as frequency is increased.

The modeTE_{11}that able to propagate at the lowest frequency at energy propagates as wave is function of waveguide size. The quality factor has a maximum value a circular cylinder when D=L, because of size that limited in order to minimize losses.

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Appendix1:

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PROGRAM MODE_
PARAMETER (IMAX=15, JMAX=15)
COMMON/MES/X(0:IMAX+1,0:JAMX+1)
OPEN (UNIT=2,FILE='XM.DAT')
OPEN (UNIT=8,FILE ='OUTAM.OUT')
OPEN (UNIT=1,FILE='DATA1.OUT')
OPEN (UNIT=3, FILE='DATA2.OUT')
OPEN (UNIT=5, FILE='DIL.OUT')
OPEN (UNIT=6, FILE='DATA3.OUT')
X(1,1)=3.832
X(2,1)=5.136
X(3,1)=4.201
X(4,1)=5.318
X(5,1)=6.415
X(2,2)=8.417
X(1,2)=5.338
WRITE (6,*)  X(1,1), X(2,1), X(3,1), X(4,1), X(5,1),(2,2),(1,2)
DO 1 I=1, 13
READ (2,*) M, N
XMN1=X (M,N)
!.................................................................................................................
 CALL Q_Factor (M1, N1, XMN1,  DOL1, QDOL1, α, F)
!.................................................................................................................
END DO
!
END PROGRAM
```