Salageon differential operator and Ruscheweyh differential operator in NewSubclass of Univalent Functions

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Abstract:
In this paper, we introduced some properties of the class $M_{\mathbb{A}}(k,n,ג,\alpha,\beta,z)$ by using the derivative operators which are Sălăgeon differential operator and Ruscheweyh differential operator as well as, investigated generalization of subordination theorem and weight mean on the same class . Finally this study given some Basic theory and get important results of these class by effect these operators.
Keywords.integral operator,Sălăgeon differential operator , Ruscheweyh differential operator , subordination theorem, weighted mean.

المؤثر التفاضلي سيلكون والمؤثر التفاضلي رشوية في الصف الجزئي للدوال احادية التكافوه

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الخلاصة :
في هذه البحث سوف ندرس بعض الخواص على الصف الجزئي بأستخدام المؤثر التفاضلي سيلكون والمؤثر التفاضلي رشوية للتحقيق التعميم في نظرية التعبة والوسيط الوزني في نفس الصف ودراسة بعض الخواص الهندسية للدالة بأستخدام المؤثرين التفاضلين المذكورة أعلاة.
الكلمات الدالة : تكاملات المؤثر ، المؤثر التفاضلي سيلكون، المؤثر التفاضلي رشوية ،نظرية التعميم ،الوسيط الوزني

Introduction:

In 2010 Najafzadeh [6],was introduced anew subclass of holomorphic univalent functions onSălăgeon and Ruscheweyh differential operators. In 2015 Al-Khafaji [1],was studied anew subclass which is WA(I,, $\alpha, m, n, y_1, y_2,ג$) of univalent functions defined by multiplier transformation, finally in 2018 Shabaab[8], introduced anther class is said to be p(α, B) and discussion some properties of these class in this work, we introduced and study new subclass is called $M_{\mathbb{A}}(k,n,ג,\alpha,\beta,z)$ with some properties of this class.

Let Mbethe class of the functions of the form:

$$f(z) = z + \sum_{k=t+1}^{\infty} a_k z^k, \qquad (1)$$

which are holomorphic in the open unit disk U = { z $\in \mathbb{C}: |z| < 1$}.

Here we generalized definition appear in[6] on the$M_{\mathbb{A}}(k, n, ℷ, \alpha, \beta, z)$ $class$.

Also, let $M_{\mathbb{A}}$ the subclass of M consisting of functions of the form:

$$f(z) = z - \sum_{k=t+1}^{\infty} a_k z^k \; a_k \geq 0 \qquad (2)$$

which are holomorphic (univalent) in U ,[3].

Let $n \in N \cup \{0\}$ and $ℷ \geq 0$.

By $\Omega_{ℷ}^{n} f$, we denote the operator $\Omega_{ℷ}^{n}: M_{\mathbb{A}} \rightarrow M_{\mathbb{A}}$ defined by

$$\Omega_{ℷ}^{n} f(z) = (1 - ℷ) S^n f(z) + ℷ R^n f(z), \qquad z \in U \qquad (3)$$

Where $S^n f$ is the Salageon differential operator appear in [9] and$R^n f$ is the Ruscheweyh differential operator appear in [7].

For $f(z) \in M_{\mathbb{A}}$ given by (2) we have respectively.

$$S^n f(z) = z - \sum_{k=t+1}^{\infty} K^n a_k z^k \qquad (4)$$

and

$$R^n f(z) = z - \sum_{k=t+1}^{\infty} B_K(n) a_k z^k \qquad (5)$$

Where

$$B_K(n) = \binom{k+n+1}{n} = \frac{(n+1)(n+2) \ldots \ldots (n+k-1)}{(k-1)!} \qquad (6)$$

Further by replacing (4) and (5) in (3) , we obtain

$$\Omega_{ℷ}^{n} f(z) = z - \sum_{k=t+1}^{\infty} [K^n (1 - ℷ) + ℷ B_K(n)] a_k z^k \qquad (7)$$

The following definition is generalize to definition show in [8] onthe$M_{\mathbb{A}}(k, n, ℷ, \alpha, \beta, z)$ $class$.

Definition(1) [8]: Let $f \in M_{\mathbb{A}}$ be given by (2) then the class $M_{\mathbb{A}}(k, n, ℷ, \alpha, \beta, z)$is defined by

$$\left|\frac{\frac{z\left(\Omega_{\lambda}^{n}f(z)\right)''}{\left(\Omega_{\lambda}^{n}f(z)\right)'}}{\frac{z\left(\Omega_{\lambda}^{n}f(z)\right)''}{\left(\Omega_{\lambda}^{n}f(z)\right)'}+2\alpha}\right|<\beta, \qquad (8)$$

for$z\in U, 0<\beta\leq 1\ ,\ \lambda\geq 0\ and\ 0<\alpha\leq 1.$

Now, we obtain the necessary and sufficient condition for a function f to be in the class $M_{\mathbb{A}}(k,n,\lambda,\alpha,\beta,z)$.

Several authors studied geometric properties of this function subclass for other classes, such as,Al-Khafaji[1],Atshan [2] and Najafzadeh[6].

In [1] Al- Khafaji use multiplier transformation operator on WA(□, $\alpha,m,\eta,y_1,y_2,\lambda$)classof the following theory.

Now ,the following we will use differential operators Sălăgeon and Ruscheweyhoperators on$M_{\mathbb{A}}(k,n,\lambda,\alpha,\beta,z)$ class.

Theorem (1): Let $f\in M_{\mathbb{A}}$. Then $f\in M_{\mathbb{A}}(k,n,\lambda,\alpha,\beta,z)$.if and only if

$$\sum_{k=t+1}^{\infty}[K^{n}(1-\lambda)+\lambda B_{K}(n)]k[k-1-\beta+\beta k+2\alpha\beta]a_{k}\leq 2\alpha\beta. \quad (9)$$

Where ,$0<\beta\leq 1\ ,\ \lambda\geq 0\ \ and\ \ 0<\alpha\leq 1.$

This result is strong for the following function

$$f_{k}(z)=z-\frac{2\alpha\beta}{[K^{n}(1-\lambda)+\lambda B_{K}(n)]k[k-1-\beta+\beta k+2\alpha\beta]a_{k}}z^{k}, k\in N$$

Proof: Suppose that the inequality (9) holds true and $|z|=1$.

Then from (8), we have

$$\left|z\left(\Omega_{\lambda}^{n}f(z)\right)''\right|<\beta\left|z\left(\Omega_{\lambda}^{n}f(z)\right)''+2\alpha\left(\Omega_{\lambda}^{n}f(z)\right)'\right|$$

$$\left|z\left(\Omega_{\lambda}^{n}f(z)\right)''\right|-\beta\left|z\left(\Omega_{\lambda}^{n}f(z)\right)''+2\alpha\left(\Omega_{\lambda}^{n}f(z)\right)'\right|\leq 0$$

By using (7), we have.

$$\left|-\sum_{k=t+1}^{\infty}[K^{n}(1-\lambda)+\lambda B_{K}(n)]k(k-1)a_{k}z^{k-1}\right|-$$

$$\beta\left|\sum_{k=t+1}^{\infty}[K^n(1-\lambda)+\lambda B_K(n)]k(-k+1)a_k z^{k-1}+2\alpha-2\alpha\sum_{k=t+1}^{\infty}[K^n(1-\lambda)+\lambda B_K(n)]k\,a_k z^{k-1}\right|\leq 0$$

$$\leq\sum_{k=t+1}^{\infty}[K^n(1-\lambda)+\lambda B_K(n)]k(k-1)a_k-\beta\sum_{k=t+1}^{\infty}[K^n(1-\lambda)+\lambda B_K(n)]k(-k+1)a_k-2\alpha\beta$$

$$+2\alpha\beta\sum_{k=t+1}^{\infty}[K^n(1-\lambda)+\lambda B_K(n)]k\,a_k$$

$$\leq\sum_{k=t+1}^{\infty}[K^n(1-\lambda)+\lambda B_K(n)]k[k-1-\beta+\beta k+2\alpha\beta]a_k-2\beta\alpha$$

$$=\sum_{k=t+1}^{\infty}[K^n(1-\lambda)+\lambda B_K(n)]k[k-1-\beta+\beta k+2\alpha\beta]a_k-2\beta\alpha\leq 0$$

by hypothesis.

Hence, by maximum modulus principles, $f\in M_{\mathbb{A}}(k,n,\lambda,\alpha,\beta,z)$.

Conversely, assume that $f\in M_{\mathbb{A}}(k,n,\lambda,\alpha,\beta,z)$. Then from (8) and (7), we have

$$\left|\frac{\dfrac{z\left(\Omega_\lambda^n f(z)\right)''}{\left(\Omega_\lambda^n f(z)\right)'}}{\dfrac{z\left(\Omega_\lambda^n f(z)\right)''}{\left(\Omega_\lambda^n f(z)\right)'}+2\alpha}\right|<\beta$$

$$=\left|\frac{-\sum_{k=t+1}^{\infty}[K^n(1-\lambda)+\lambda B_K(n)]k(k-1)a_k z^{k-1}}{-\sum_{k=t+1}^{\infty}[K^n(1-\lambda)+\lambda B_K(n)]k(k-1)2\alpha\,a_k z^{k-1}+2\alpha-2\alpha\sum_{k=t+1}^{\infty}[K^n(1-\lambda)+\lambda B_K(n)]k\,a_k z^{k-1}}\right|<\beta.$$

Clearly that Re (z) ≤ $|z|$ for all z (z ∈ U), then we get

$$\operatorname{Re}\left\{\frac{\sum_{k=t+1}^{\infty}[K^n(1-\lambda)+\lambda B_K(n)]k(k-1)a_k z^{k-1}}{-\sum_{k=t+1}^{\infty}[K^n(1-\lambda)+\lambda B_K(n)]k(k-1)\,a_k z^{k-1}+2\alpha-2\alpha\sum_{k=t+1}^{\infty}[K^n(1-\lambda)+\lambda B_K(n)]k\,a_k z^{k-1}}\right\}<\beta. \qquad (10)$$

We can choose the values of z on the real axis, so that $\frac{z\left(\Omega_\lambda^n f(z)\right)''}{\left(\Omega_\lambda^n f(z)\right)'}$ is real.

$$\sum_{k=t+1}^{\infty}[K^n(1-\lambda)+\lambda B_K(n)]k(k-1)a_k z^{k-1}$$

$$\leq-\beta\sum_{k=t+1}^{\infty}[K^n(1-\lambda)+\lambda B_K(n)]k(k-1)\,a_k z^{k-1}+2\alpha\beta-2\alpha\beta\sum_{k=t+1}^{\infty}[K^n(1-\lambda)+\lambda B_K(n)]k\,a_k z^{k-1}$$

Let Re z $\to 1^-$

$$\sum_{k=t+1}^{\infty}[K^n(1-λ)+λB_K(n)]k(k-1)a_k$$

$$\leq -\beta\sum_{k=t+1}^{\infty}[K^n(1-λ)+λB_K(n)]k(k-1)\,a_k+2\alpha\beta-2\alpha\beta\sum_{k=t+1}^{\infty}[K^n(1-λ)+λB_K(n)]k\,a_k$$

$$\sum_{k=t+1}^{\infty}[K^n(1-λ)+λB_K(n)]k(k-1)a_k+\beta\sum_{k=t+1}^{\infty}[K^n(1-λ)+λB_K(n)]k(k-1)\,a_k$$

$$+2\alpha\beta\sum_{k=t+1}^{\infty}[K^n(1-λ)+λB_K(n)]k\,a_k\leq 2\alpha\beta$$

we can write (10) as

$$\sum_{k=t+1}^{\infty}[K^n(1-λ)+λB_K(n)]k[k-1-\beta+\beta k+2\alpha\beta]a_k\leq 2\alpha\beta.$$

Finally, sharpness follows if we take

$$f_k(z)=z-\frac{2\alpha\beta}{[K^n(1-λ)+λB_K(n)]k[k-1-\beta+\beta k+2\alpha\beta]a_k}z^k, k=t+1,t+2, \quad (11)$$

The proof is complete.

Corollary (1): Let $f\in M_{\mathbb{A}}(k,n,λ,\alpha,\beta,z)$. Then

$$a_k\leq\frac{2\alpha\beta}{[K^n(1-λ)+λB_K(n)]k[k-1-\beta+\beta k+2\alpha\beta]a_k},\quad w\square ere\ k=t+1\ .... \qquad (12)$$

In 1925, Littlewood [4] proved the following subordination theorem.

(see also Duren [3]).

Theorem (2) [4]: if f and g are analytic in U with $f\prec g$

then for $\alpha>0$ and z = r $e^{i\vartheta}$ and (0< $r<1$)

$$\int_0^{2\pi}|f(z)|^\alpha\,d\vartheta\leq\int_0^{2\pi}|g(z)|^\alpha\,d\vartheta \qquad (13)$$

We will make use the above theorem to prove.

Theorem (3): Let $f\in M_{\mathbb{A}}(k,n,λ,\alpha,\beta,z)$and suppose that $f$is defined by

$$f(z)=z-\frac{2\alpha\beta}{[K^n(1-λ)+λB_K(n)]k[k-1-\beta+\beta k+2\alpha\beta]}z^k, \qquad (14)$$

If there exists an analytic function w given by

$$[w(z)]^k = \frac{[K^n(1-\lambda) + \lambda B_K(n)]k[k-1-\beta+\beta k+2\alpha\beta]}{2\alpha\beta} \sum_{k=t+1}^{\infty} a_k\, z^{k-1}, \qquad (15)$$

then for z = r $e^{i\vartheta}$ and (0< $r < 1$)

$$\int_0^{2\pi} \left|f\left(\mathrm{r}\, e^{i\vartheta}\right)\right|^\alpha d\vartheta \le \int_0^{2\pi} \left|f\left(\mathrm{r}\, e^{i\vartheta}\right)\right|^\alpha d\vartheta, \qquad (\alpha > 0).$$

Proof: Let $f(z)$ of the form (2) and $f_k(z)$ defined by (14), then we must show that

$$\int_0^{2\pi} \left|1 - \sum_{k=t+1}^{\infty} a_k\, z^{k-1}\right|^\alpha d\vartheta \le \int_0^{2\pi} \left|1 - \frac{2\alpha\beta}{[K^n(1-\lambda) + \lambda B_K(n)]k[k-1-\beta+\beta k+2\alpha\beta]} z^{k-1}\right|^\alpha d\vartheta.$$

By applying Littlewood's subordination theorem, it would suffice to show that

$$1 - \sum_{k=t+1}^{\infty} a_k\, z^{k-1} \prec 1 - \frac{2\alpha\beta}{[K^n(1-\lambda) + \lambda B_K(n)]k[k-1-\beta+\beta k+2\alpha\beta]} z^{k-1}.$$

By setting

$$1 - \sum_{k=t+1}^{\infty} a_k\, z^{k-1} = 1 - \frac{2\alpha\beta}{[K^n(1-\lambda) + \lambda B_K(n)]k[k-1-\beta+\beta k+2\alpha\beta]} [w(z)]^k.$$

We find that

$$[w(z)]^k = \frac{[K^n(1-\lambda) + \lambda B_K(n)]k[k-1-\beta+\beta k+2\alpha\beta]}{2\alpha\beta} \sum_{k=t+1}^{\infty} a_k\, z^{k-1},$$

Which readily yields $w(0) = 0$.

Furthermore, by using (9), we obtain

$$|[w(z)]|^k = \left|\sum_{k=t+1}^{\infty} \frac{[K^n(1-\lambda) + \lambda B_K(n)]k[k-1-\beta+\beta k+2\alpha\beta]}{2\alpha\beta} a_k z^k\right|$$

$$\le \sum_{k=t+1}^{\infty} \frac{[K^n(1-\lambda) + \lambda B_K(n)]k[k-1-\beta+\beta k+2\alpha\beta]}{2\alpha\beta} a_k |z|^k$$

$$\le |z|^{t+1} \sum_{k=t+1}^{\infty} \frac{[K^n(1-\lambda) + \lambda B_K(n)]k[k-1-\beta+\beta k+2\alpha\beta]}{2\alpha\beta} a_k$$

$$\le |z| < 1. \blacksquare$$

In [1] Al- Khafaji use multivalent functions with negative cofficients defined by linear integral operator of the following theory.

As for us we used the univalent functions on $M_{\mathbb{A}}(k,n,ℷ,\alpha,\beta,z)$ class of the following theory .

Theorem (4): Let $\alpha > 0$. If $f \in M_{\mathbb{A}}(k,n,ℷ,\alpha,\beta,z)$ and

$$f(z) = z - \frac{2\alpha\beta}{[K^n(1-ℷ)+ℷB_K(n)]k[k-1-\beta+\beta k+2\alpha\beta]} z^k, k \geq t+1,$$

then for z = r $e^{i\vartheta}$ and ($0 < r < 1$),

$$\int_0^{2\pi} \left|f'\left(\mathrm{r}\, e^{i\vartheta}\right)\right|^{\alpha} d\vartheta \leq \int_0^{2\pi} \left|f'_k\left(\mathrm{r}\, e^{i\vartheta}\right)\right|^{\alpha} d\vartheta, \qquad (16)$$

Proof : $f'(z) = 1 - \sum_{k=t+1}^{\infty} k a_k z^{k-1}$,

$$f'(z) = 1 - \frac{2\alpha\beta k}{[K^n(1-ℷ)+ℷB_K(n)]k[k-1-\beta+\beta k+2\alpha\beta]} z^{k-1}$$

$k \geq t+1.$

It is sufficient to show that

$$z - \sum_{k=t+1}^{\infty} k a_k z^k \prec z - \frac{2\alpha\beta k}{[K^n(1-ℷ)+ℷB_K(n)]k[k-1-\beta+\beta k+2\alpha\beta]} z^k.$$

By setting

$$z - \sum_{k=t+1}^{\infty} (k) a_k z^k = z - \frac{2\alpha\beta}{[K^n(1-ℷ)+ℷB_K(n)]k[k-1-\beta+\beta k+2\alpha\beta]} (k)[w(z)]^k,$$

hence

$$[w(z)]^k = \sum_{k=t+1}^{\infty} \frac{[K^n(1-ℷ)+ℷB_K(n)]k[k-1-\beta+\beta k+2\alpha\beta]}{\mu 2\alpha\beta} a_k z^k$$

Which readily yields $w(0) = 0$.

By using Theorem (1), we obtain

$$|[w(z)]|^k = \left|\sum_{k=t+1}^{\infty} \frac{[K^n(1-ℷ)+ℷB_K(n)]k[k-1-\beta+\beta k+2\alpha\beta]}{\mu 2\alpha\beta} a_k z^k\right|$$

$$\leq |z|^t \sum_{k=1}^{\infty} \frac{[K^n(1-\gimel)+\gimel B_K(n)]k[k-1-\beta+\beta k+2\alpha\beta]}{\mu 2\alpha\beta} a_k$$

$$\leq |z| < 1. \blacksquare$$

Definition (2)[5]: Let f_1 and f_2 be in the class $M_{\mathbb{A}}(k,n,\gimel,\alpha,\beta,z)$.

Then the weighted mean w_j of f_1 and f_2 is given by:

$$w_j(z) = \frac{1}{2}[(1-j)f_1(z) + (1+j)f_2(z)], \quad 0<j<1.$$

Theorem (5): Let f_1 and f_2 be in the class $M_{\mathbb{A}}(k,n,\gimel,\alpha,\beta,z)$. Then the weighted mean w_j of f_1 and f_2 is also in the class $M_{\mathbb{A}}(k,n,\gimel,\alpha,\beta,z)$

Proof: By Definition (2), we have

$$w_j(z) = \frac{1}{2}[(1-j)f_1(z) + (1+j)f_2(z)], \qquad (17)$$

$$= \frac{1}{2}\left[(1-j)\left(z-\sum_{k=t+1}^{\infty} a_{k,1} z^k\right) + (1+j)\left(z-\sum_{k=2}^{\infty} a_{k,2} z^k\right)\right]$$

$$= z - \sum_{k=t+1}^{\infty} \frac{1}{2}\left[(1-j)a_{k,1} + (1+j)a_{k,2}\right] z^k .$$

Since f_1 and f_2 are in the class $M_{\mathbb{A}}(k,n,\gimel,\alpha,\beta,z)$ so by Theorem (1), we get

$$\sum_{k=t+1}^{\infty} [K^n(1-\gimel)+\gimel B_K(n)]k[k-1-\beta+\beta k+2\alpha\beta] \ a_{k,1} \leq \mu 2\alpha\beta,$$

and

$$\sum_{k=t+1}^{\infty} [K^n(1-\gimel)+\gimel B_K(n)]k[k-1-\beta+\beta k+2\alpha\beta] \ a_{k,2} \leq \mu 2\alpha\beta,$$

Hence

$$[K^n(1-\gimel)+\gimel B_K(n)]k[k-1-\beta+\beta k+2\alpha\beta] \ \frac{1}{2}\left[(1-j)a_{k,1} + (1+j)a_{k,2}\right]$$

$$= \frac{1}{2}\ (1-j) \sum_{k=t+1}^{\infty} [K^n(1-\gimel)+\gimel B_K(n)]k[k-1-\beta+\beta k+2\alpha\beta] \ a_{k,1}$$

$$+ \frac{1}{2}(1+j) \sum_{k=t+1}^{\infty} [K^n(1-\gimel)+\gimel B_K(n)]k[k-1-\beta+\beta k+2\alpha\beta] \ a_{k,2}$$

$$\leq \frac{1}{2}(1-j)\mu 2\alpha\beta + \frac{1}{2}(1+j)\mu 2\alpha\beta = \mu 2\alpha\beta.$$

Therefore, $w_j \in M_{\mathbb{A}}(k, n, \gimel, \alpha, \beta, z)$. ■

Theorem (6): Let $f(z) \in M_{\mathbb{A}}(k, n, \gimel, \alpha, \beta, z)$, then the integral operator

$$F_\iota(z) = (1-\iota)z + \iota(1)\int_0^z \frac{f(s)}{s}\,ds \ (\iota \geq 0, z \in U),$$

is also in $M_{\mathbb{A}}(k, n, \gimel, \alpha, \beta, z)$ if $0 \leq \iota \leq t+1+k$.

Proof: If

$$f(z) = z \quad - \sum_{k=t+1}^{\infty} a_k\, z \quad ,$$

then

$$F_\iota(z) = (1-\iota)z + \iota(1)\int_0^z \left(\frac{s - \sum_{k=t+1}^{\infty} a_k\, s^k}{s}\right) ds$$

$$= (1-\iota)z + \iota(1)\left[z - \sum_{k=t+1}^{\infty} \frac{a_k z^k}{k}\right]$$

$$= z \quad - \sum_{k=t+1}^{\infty} \frac{\iota(1)}{k} a_k\, z^k$$

$$= z \quad - \sum_{k=t+1}^{\infty} g_k\, z^k$$

Where $g_k = \frac{\iota(1)}{k} a_k$. But

$$\sum_{k=t+1}^{\infty} \frac{2\alpha\beta}{[K^n(1-\gimel) + \gimel B_K(n)]k[k-1-\beta+\beta k+2\alpha\beta]} g_k$$

$$= \sum_{k=t+1}^{\infty} \frac{2\alpha\beta}{[K^n(1-\gimel) + \gimel B_K(n)]k[k-1-\beta+\beta k+2\alpha\beta]} \frac{\iota(1)}{k} a_k$$

$$\leq \sum_{k=t+1}^{\infty} \frac{2\alpha\beta}{[K^n(1-\gimel) + \gimel B_K(n)]k[k-1-\beta+\beta k+2\alpha\beta]} \frac{\iota(1)}{t+1} a_k$$

where

$$\frac{\iota(1)}{t+1} \leq 1$$

$$\leq \sum_{k=2}^{\infty} \frac{2\alpha\beta}{[K^n(1-ג) + גB_K(n)]k[k-1-\beta+\beta k+2\alpha\beta]} a_k$$

(by (9))

$$\leq 2\alpha\beta.$$

So $F_\iota(z) \in M_{\mathbb{A}}(k, n, ג, \alpha, \beta, z)$. ■

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