A new classes of sets in Ideal nanotopological spaces ($\mathcal{U}$, $\mathcal{N}$, I)

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Abstract
The purpose of the research is to introduce a new kinds of sets called ($\mathcal{A}$-I_n and $\mathcal{A}$-α-I_n) sets also (B-I_n and B-α-I_n) sets and another types of sets in Ideal nanotopological space and study some of these sets properties also we characterize the relations between them and the related properties.

Key words: $\mathcal{A}$-I_n set, $\mathcal{A}$-α-I_n sets , B-I_n set, B-α-I_n sets

اصناف جديدة من المجموعات في الفضاءات النانوتبولوجية المثالية (($\mathcal{U}$, $\mathcal{N}$, I)

الخلاصة
الغرض من هذا البحث هو تقديم أنواع جديدة من المجموعات تسمى مجموعات ($\mathcal{A}-I_n$ and $\mathcal{A}-\alpha-I_n$) ومجموعات ($B-I_n$ and $B-\alpha-I_n$) وأنواع أخرى من المجموعات في الفضاء النانوتبولوجية المثالية و دراسة بعض خواص هذه المجموعات وتمييز العلاقات بينها وبين الخصائص المتعلقة بها.

Introduction and Preliminaries

In topological space ($\mathcal{X}$, $\mathcal{T}$) [1] the I Ideal is set not equal empty that subsets of $\mathcal{X}$ which satisfy the terms below;

I. $\mathcal{K} \in I$ and $\mathcal{H} \subset \mathcal{K}$ implying that $\mathcal{H} \in I$ and

II. $\mathcal{K} \in I$ and $\mathcal{H} \in I$ implying that $\mathcal{K} \cup \mathcal{H} \in I$.

In a space of topology ($\mathcal{X}$, $\mathcal{T}$) with Ideal. p ($\mathcal{X}$) is the family of all subsets of $\mathcal{X}$, the set operator $(.)^{\star}$:p ($\mathcal{X}$) → p ($\mathcal{X}$), referred to as a local function of $\mathcal{K}$ in relation to $\mathcal{T}$ and I is written as ; for $\mathcal{K} \subset \mathcal{X}$, $\mathcal{K}^{*}(I, \mathcal{T})$ ={ $\varkappa \in \mathcal{X}$: $\mathcal{U} \cap \mathcal{K}$ not belong to I for all $\mathcal{U} \in \mathcal{T}(\varkappa)$ } where $\mathcal{T}(\varkappa)$ ={U∈ $\mathcal{T}$: $\varkappa \in \mathcal{U}$}[2]. The operator for closure established by $CL^{\star}(.) = \mathcal{K} \cup \mathcal{K}^{*}$, [3] is a Kuratowski the operator for closure which generates a topology $\mathcal{T}^{\star}(I, \mathcal{T})$ is written as *-topology. The Ideal on $\mathcal{X}$ with topological space is topological space with Ideal or Ideal space being signified ($\mathcal{X}$, $\mathcal{T}$, I). We're able to compose $\mathcal{K}^{*}$for $\mathcal{K}^{\star}(I, \mathcal{T})$ and $\mathcal{T}^{\star}$ for $\mathcal{T}^{\star}(I, \mathcal{T})$. [4, 5] Parimala et al. Some Ideas were added in the notions of Ideal nanotopological spaces. [4]A nanotopological space ($\mathcal{U}$, $\mathcal{N}$) with the Ideal I on $\mathcal{U}$

is the Ideal nanotopological space and is denoted by (U, $\mathcal{N}$, I). G_n ($\varkappa$) ={ G_n | $\varkappa \in G_n$, $G_n \in \mathcal{N}$ } ,denotes [4] the family of nanoopen sets containing $\varkappa$.

The purpose of the research is to introduce a new kinds of sets called ($\mathcal{A}$-I_n and $\mathcal{A}$-α-I_n) sets also (B-I_n and B-α-I_n) sets and another types of sets in Ideal nanotopological space and study some of these sets properties also we characterize the relations between them and the related properties.

Definition: 1.1. [6] The finite set U be a not-empty of objects (the universe) & the equivalence relation $\mathcal{R}$ on U is the relation of Indiscernibility. Elements belonging to the same equivalence class are said to be indiscernible from each other. Space of approximation denoted by (U, $\mathcal{R}$).

Let $\mathcal{X} \subseteq$ U.

(i) $L_{\mathcal{R}}(\mathcal{X}) = S_{x\in U}\{\mathcal{R}(\varkappa) ; \mathcal{R}(\varkappa) \subseteq \mathcal{X}\}$.

(ii) $U_{\mathcal{R}}(\mathcal{X}) = S_{x\in U}\{\mathcal{R}(\varkappa) ; \mathcal{R}(\varkappa) \cap \mathcal{X} \neq \emptyset\}$.

(iii) $B_{\mathcal{R}}(\mathcal{X}) = U_{\mathcal{R}}(\mathcal{X}) - L_{\mathcal{R}}(\mathcal{X})$.

Definition: 1.٢. [6] The universe U for $\mathcal{R}$ be a relationship of equivalence on U and $\tau\mathcal{R}(\mathcal{X})$ = {U,$\emptyset$, $L_{\mathcal{R}}(\mathcal{X})$, $U_{\mathcal{R}}(\mathcal{X})$, $B_{\mathcal{R}}(\mathcal{X})$} where $\mathcal{X} \subseteq$ U, $\tau\mathcal{R}(\mathcal{X})$ complies with The axioms below are:

(i) U and $\emptyset \in \tau\mathcal{R}(\mathcal{X})$.

(ii) The (∪) of components of any $\tau\mathcal{R}(\mathcal{X})$ sub collection is in $\tau\mathcal{R}(\mathcal{X})$.

(iii) The (∩) of components for finite $\tau\mathcal{R}(\mathcal{X})$ sub collection is in $\tau\mathcal{R}(\mathcal{X})$.

That's also , $\tau\mathcal{R}(\mathcal{X})$ creates on U a topology called as the nanotopology on U according to the $\mathcal{X}$. We say (U, $\tau\mathcal{R}(\mathcal{X})$) as the space of nanotopology. The $\tau\mathcal{R}(\mathcal{X})$ components are named out as nanoopen . A set $\mathcal{K}$ is closed in the form nano and complement is open in the form nano.

Definition: 1.٣. [6] If (U, $\tau\mathcal{R}(\mathcal{X})$) is a nanotopology space according to the $\mathcal{X}$, $\mathcal{X} \subseteq$ U and if $\mathcal{K} \subseteq$ U, The nanointerior of $\mathcal{K}$ is then known as the unions $\cup_n$of all nanoopen subsets in $\mathcal{K}$, and $n$int ($\mathcal{K}$) denotes it. The biggest open subset of $\mathcal{K}$ for nano is $n$int ($\mathcal{K}$). The nanoclosure of $\mathcal{K}$ is known as the intersections $\cap_n$ of all nano closed set containing $\mathcal{K}$. We denote a nanotopological space by (U, $\mathcal{N}$) where $\mathcal{N} = \tau\mathcal{R}(\mathcal{X})$.

Definition: 1.٤. If $\mathcal{K} \subseteq$ U , the space (U, $\mathcal{N}$) is named

1) nanoα- open [6] $\mathcal{K} \subseteq n\text{int}(n\text{CL}(n\text{int}(\mathcal{K})))$.

2) nanosemi- open [6] $\mathcal{K} \subseteq n\text{CL}(n\text{int}(\mathcal{K}))$.

3) nanopre- open [6] $\mathcal{K} \subseteq n\text{int}(n\text{CL}(\mathcal{K}))$.

4) nanoregular open [7] $= n\text{int}(n\text{CL}(\mathcal{K}))$

Their respective closed sets are considered the complements of the above sets.

Definition: 1.5. [4] Let (U, $\mathcal{N}$, I) be a space. Let $(.)^*_n$ be the operator for set from$p$ (U) to p(U) (p(U) that is the collection of all items in U). For a subset $\mathcal{K} \subseteq$ U, $\mathcal{K}^*_n$ (I, $\mathcal{N}$) ={ $\varkappa \in$ U: $G_n \cap \mathcal{K}$ not belong to I, ∀ G_n∈ G_n (ϰ)} is named out as nanoopen local function of $\mathcal{K}$ with I and $\mathcal{N}$. We are clearly going to write $\mathcal{K}_n^*$ for $\mathcal{K}_n^*$ (I, $\mathcal{N}$).

Theorem: 1.6. [4] Let (U, $\mathcal{N}$, I) be a space and $\mathcal{K}$ and $\mathcal{H}$ be subsets of U. Then
1. $\mathcal{K} \subseteq \mathcal{H} \Rightarrow \mathcal{K}_n^* \subseteq \mathcal{H}_n^*$,
2. $\mathcal{K}_n^* = n\mathrm{Cl}(\mathcal{K}_n^*) \subseteq n\mathrm{Cl}(\mathcal{K})$ ($\mathcal{K}_n^*$ is a $n$closed subset of $n\mathrm{Cl}(\mathcal{K})$),
3. $(\mathcal{K}_n^*)_n^* \subseteq \mathcal{K}_n^*$,
4. $(\mathcal{K} \cup \mathcal{H})_n^* = \mathcal{K}_n^* \cup \mathcal{H}_n^*$,
5. V∈ $\mathcal{N} \Rightarrow V \cap \mathcal{K}_n^* = V \cap (V \cap \mathcal{K})_n^* \subseteq (V \cap \mathcal{K})_n^*$,
6. J ∈ I⇒ $(\mathcal{K} \cup J)_n^* = \mathcal{K}_n^* = (\mathcal{K} - J)_n^*$.
Theorem: 1.7. [4] Let (U, $\mathcal{N}$, I) be a space with an Ideal I and $\mathcal{K} \subseteq \mathcal{K}_n^*$, then $\mathcal{K}_n^* =$ CL ($\mathcal{K}_n^*$) = nCL($\mathcal{K}$).
Definition: 1.8. [4] Let (U, $\mathcal{N}$, I) be a space. The operator $n\mathrm{CL}^*$ named a nano * -closure is characterized by $n\mathrm{CL}^*(\mathcal{K}) = \mathcal{K} \cup \mathcal{K}_n^*$ when $\mathcal{K} \subseteq$ U. It can be easily observed that $n\mathrm{CL}^*(\mathcal{K}) \subseteq n\mathrm{CL}(\mathcal{K})$.
Theorem: 1.9. [5] In a space (U, $\mathcal{N}$, I), if $\mathcal{K}$ and $\mathcal{H}$ are subsets of U, then the following results are true for the set operator $n\mathrm{CL}^*$.
1. $\mathcal{K} \subseteq n\mathrm{CL}^*(\mathcal{K})$,
2. $n\mathrm{CL}^*(\varphi) = \varphi$ and $n\mathrm{CL}^*(U) = U$,
3. If $\mathcal{K} \subset \mathcal{H}$, then $n\mathrm{CL}^*(\mathcal{K}) \subseteq n\mathrm{CL}^*(\mathcal{H})$,
4. $n\mathrm{CL}^*(\mathcal{K}) \cup n\mathrm{CL}^*(\mathcal{H}) = n\mathrm{CL}^*(\mathcal{K} \cup \mathcal{H})$,
5. $n\mathrm{CL}^*(n\mathrm{CL}^*(\mathcal{K})) = n\mathrm{CL}^*(\mathcal{K})$.
Definition: 1.10. [5] If $\mathcal{K} \subseteq$ U , the space (U, $\mathcal{N}$, I) is named to be nano I-open (briefly, I_n-open) if $\mathcal{K} \subseteq n\mathrm{int}(\mathcal{K}_n^*)$.
Definition: 1.11. [9] If $\mathcal{K} \subseteq$ U , the space (U, $\mathcal{N}$, I) is named:

1. nanoα-I -open (briefly α- I_n -open) if $\mathcal{K} \subset n\mathrm{int}(n\mathrm{CL}^\star(n\mathrm{int}(\mathcal{K})))$, α-$I_n$-closed is complement for α- I_n -open set is.

2. nanosemi- I -open (briefly semi-I_n-open) if $\mathcal{K} \subset n\mathrm{CL}^\star(n\mathrm{int}(\mathcal{K}))$, semi-$I_n$ - closed is complement for a semi-I_n -open set.

3. nanopre- I -open (briefly pre-I_n -open) if $\mathcal{K} \subset n\mathrm{int}(n\mathrm{CL}^\star(\mathcal{K}))$, pre-$I_n$ - closed is complement for pre-I_n -open set.

4. nanosemi⋆- I -open (briefly semi⋆-I_n -open) set if $\mathcal{K} \subset n\mathrm{CL}(n\mathrm{int}^\star(\mathcal{K}))$. A subset $\mathcal{K}$ is said to be a semi⋆- I_n - closed set ($n\mathrm{int}(n\mathrm{CL}^\star(\mathcal{K})) \subset \mathcal{K}$ if its complement is a semi⋆ -I_n- open set.

Definition: 1.12.[10] If $\mathcal{K} \subseteq$ U , the space (U, $\mathcal{N}$, I) is named nano$\mathcal{T}$-I set (briefly, $\mathcal{T}$-I_n set) if $n\mathrm{int}(\mathcal{K}) = n\mathrm{int}(n\mathrm{CL}^\star(\mathcal{K}))$,

Theorem: 1.13. [9]In a space (U, $\mathcal{N}$,I), for a subset $\mathcal{K}$ is n -open ⇒ $\mathcal{K}$ is α- I_n-open.

2- ($\mathcal{A}$-I_n and $\mathcal{A}$-α-I_n) sets

Definition: 2.1. If $\mathcal{K} \subseteq$ U , the Ideal nanotopological space (U, $\mathcal{N}$, I) is named:

(a) an $\mathcal{A}$-I_n set if $\mathcal{K} = \mathcal{U} \cap \mathcal{V}$, In which $\mathcal{U}$ is n -open and $n\mathrm{CL}^\star(n\mathrm{int}(\mathcal{V})) = \mathcal{V}$.

(b) an $\mathcal{A}$-α-I_n set if $\mathcal{K} = \mathcal{U} \cap \mathcal{V}$, In which $\mathcal{U}$ is α-I_n -open and $n\mathrm{CL}^\star(n\mathrm{int}(\mathcal{V})) = \mathcal{V}$.

Theorem: 2.2. Every $\mathcal{A}$-I_n set is $\mathcal{A}$-α-I_n set.

Proof: It follows directly from Theorem 1.13 (for a subset $\mathcal{K}$ is n -open ⇒ $\mathcal{K}$ is α- I_n -open).
The converse of Theorem (2.2) is not true as shown in the following Example.

Example: 2.3. Let U ={ c_1 , c_2 , c_3} with U/R={{c_1 , c_3},{c_2}} and $\mathcal{X}$ ={c_3},Then $\mathcal{N}$ ={X,Ø,{c_1},{c_1 , c_3}} and I={Ø,{c_2},{c_2 , c_3}},Then A={c_1 , c_2} is $\mathcal{A}$-α-I_n set but is it not $\mathcal{A}$-I_n set.

Lemma: 2.4. Let (U, $\mathcal{N}$, I) be an Ideal nanotopological space and $\mathcal{K} \subseteq$ U. If is n-open, then $\mathcal{U} \cap n\mathrm{CL}^\star(\mathcal{K}) \subset n\mathrm{CL}^\star(\mathcal{U} \cap \mathcal{K})$.

Proof: Let $\mathcal{K} \subseteq$ U and $\mathcal{U}$ is n -open, then $\mathcal{U} \cap n\mathrm{CL}^\star(\mathcal{K}) = \mathcal{U} \cap (\mathcal{K} \cup \mathcal{K}^*) = (\mathcal{U} \cap \mathcal{K}) \cup (\mathcal{U} \cap \mathcal{K}^*) \subset (\mathcal{U} \cap \mathcal{K}) \cup (\mathcal{U} \cap \mathcal{K})^* = n\mathrm{CL}^\star(\mathcal{U} \cap \mathcal{K})$.

Proposition: 2.5. If $\mathcal{K} \subseteq$ U , Ideal nanotopological space (U, $\mathcal{N}$, I) is α-I_n-open and $\mathcal{H} \subseteq$ U is semi-I_n-open ⇒ $\mathcal{K} \cap \mathcal{H}$ is semi-I_n -open.

Proof: Thru the presumption $\mathcal{K} \subset n\mathrm{int}(n\mathrm{CL}^\star(n\mathrm{int}(\mathcal{K})))$

and $\mathcal{H} \subset n\mathrm{CL}^\star(n\mathrm{int}(\mathcal{H}))$.

The operation of Lemma 2.4 Therefore,

$$\mathcal{K} \cap \mathcal{H} \subset n\mathrm{int}(n\mathrm{CL}^\star(n\mathrm{int}(\mathcal{K}))) \cap n\mathrm{CL}^\star(n\mathrm{int}(\mathcal{H}))$$

$$\subset n\mathrm{CL}^\star(n\mathrm{int}(n\mathrm{CL}^\star(n\mathrm{int}(\mathcal{K}))) \cap n\mathrm{int}(\mathcal{H}))$$

$$\subset n\mathrm{CL}^\star(n\mathrm{CL}^\star(n\mathrm{int}(\mathcal{K}) \cap n\mathrm{int}(\mathcal{H})))$$

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$$\subset n\mathrm{CL}^\star(n\mathrm{int}(\mathcal{K} \cap \mathcal{H})).$$

This demonstrates that $\mathcal{K} \cap \mathcal{H}$ is semi-I_n- open in (U, $\mathcal{N}$, I).

Theorem: 2.6. In Ideal nanotopological space (U, $\mathcal{N}$, I), every $\mathcal{A}$-α-$I_n$set is semi-I_n -open.
Proof: Let $\mathcal{K}$ be a $\mathcal{A}$-α-$I_n$set in (U, $\mathcal{N}$, I) by definition(2.1), $\mathcal{K} = \mathcal{U} \cap \mathcal{V}$, where $\mathcal{U}$ is α -I_n-open and $n\mathrm{CL}^\star(n\mathrm{int}(\mathcal{V})) = \mathcal{V}$. Consequently, $\mathcal{V}$ is semi-I_n-open. By proposition(2.5), $\mathcal{K} = \mathcal{U} \cap \mathcal{V}$ is semi -I_n-open.

Definition: 2.7. If $\mathcal{K} \subseteq$ U , Ideal nanotopological space (U, $\mathcal{N}$, I) is named
an $\mathcal{C}$-I_n set if $\mathcal{K} = \mathcal{U} \cap \mathcal{V}$, where $\mathcal{U}$ is n -open and $n\mathrm{int}(n\mathrm{CL}^\star(n\mathrm{int}(\mathcal{V}))) = n\mathrm{int}(\mathcal{V})$.

Theorem: 2.8. Every $\mathcal{A}$-I_n set is $\mathcal{C}$-I_n set .

As seen in the following, the converse of Theorem (2.8) is not true;

Example: 2.9. B={c_2, c_3} in example 2.3 is $\mathcal{C}$-I_n set but not $\mathcal{A}$-I_n set.

Theorem: 2.10. Let (U, $\mathcal{N}$, I) be an Ideal nanotopological space and $\mathcal{K} \subset$ U. The
following conditions are equivalent:
(a) $\mathcal{K}$ is n-open.
(b) $\mathcal{K}$ is α-I_n- open and $\mathcal{A}$-I_n set.
Proof: (a) → (b) Since every n -open set is α-I_n-open and $\mathcal{K} = \mathcal{K} \cap$ U, where $\mathcal{K}$ is n -open set and $n\mathrm{CL}^\star(n\mathrm{int}(\mathrm{U})) = \mathrm{U}$

(b) → (a) $\mathcal{K}$ is α-I_n-open and $\mathcal{A}$-I_n set. Since every $\mathcal{A}$-I_n set is $\mathcal{C}$ -I_n set. This mean $\mathcal{K}$ is $\mathcal{C}$-I_n set. Therefore, it follows that $\mathcal{K}$ is n -open.

Definition: 2.11. If $\mathcal{K} \subseteq$ U , Ideal nanotopological space (U, $\mathcal{N}$, I) is named nano$\mathcal{T}$-α-I set (briefly, $\mathcal{T}$-α-I_n set) if $n$int ($\mathcal{K}$) = $n$int (nCL*($n$int ($\mathcal{K}$))).

Example: 2.12. Let U = { e_1 , e_2 , e_3 , e_4} with U /R = {{ e_2} , { e_4}, { e_1 , e_3}} and $\mathcal{X}$ = {e_3, e_4}. Then $\mathcal{N}$ = {φ, {e_4} , {e_1, e_3} , {e_1 , e_3 , e_4}, U}. Let the Ideal be I = {φ, {e_3}}. Then the set $\mathcal{K}$ = {e_2} is $\mathcal{T}$-I_n set and $\mathcal{H}$ = {e_4} is $\mathcal{T}$-α-I_n set.

Proposition: 2.13. If $\mathcal{K}$ and $\mathcal{H}$ are $\mathcal{T}$-α-I_n sets of a space (U, $\mathcal{N}$, I), then $\mathcal{K} \cap \mathcal{H}$ is $\mathcal{T}$-α-I_n set.

Proof: Let $\mathcal{K}$ and $\mathcal{H}$ be $\mathcal{T}$-α-I_n sets. We have got then

$n$int ($\mathcal{K} \cap \mathcal{H}$) ⊂ $n$int (nCL* ($n$int ($\mathcal{K} \cap \mathcal{H}$)))

⊂ $n$int [nCL* ($n$int ($\mathcal{K}$)) ∩ nCL* ($n$int ($\mathcal{H}$))]

= $n$int (nCL* ($n$int ($\mathcal{K}$))) ∩ $n$int (nCL* (int ($\mathcal{H}$)))

= $n$int ($\mathcal{K}$) ∩ $n$int ($\mathcal{H}$) = $n$int ($\mathcal{K} \cap \mathcal{H}$).

Then $n$int ($\mathcal{K} \cap \mathcal{H}$) = $n$int (nCL*($n$int ($\mathcal{K} \cap \mathcal{H}$))) ⇒ $\mathcal{K} \cap \mathcal{H}$ is a $\mathcal{T}$-α-I_n set.

- (*B*-I_n and *B*-α-I_n) sets ٣

Definition: 3.1. If $\mathcal{K} \subseteq$ U, Ideal nanotopological space (U, $\mathcal{N}$, I) is named

1. nano *B*-*I* set (briefly, *B*-I_n set) if $\mathcal{K} = \mathcal{U} \cap \mathcal{V}$, In which $\mathcal{U}$ is n -open and $\mathcal{V}$ is $\mathcal{T}$-I_n set,

2. nano *B*-α-*I* set (briefly, *B*-α-I_n set) if $\mathcal{K} = \mathcal{U} \cap \mathcal{V}$, in which $\mathcal{U}$ is n -open and $\mathcal{V}$ is $\mathcal{T}$-α-I_n set.

Example: 3.2. Let U = {e_1 , e_2 , e_3 , e_4} with U /R = {{e_2}, {e_4}, {e_1, e_3}} and $\mathcal{X}$ = {e_3 , e_4}. Then $\mathcal{N}$ = {φ, {e_4}, {e_1 , e_3} , {e_1, e_3, e_4}, U}. Let the Ideal be I = {φ, {e_3}}.Then the set C = {e_2, e_3} is *B*-I_n set and D = {e_1, e_3} is *B*-α-I_n set

Remark: 3.3. In a space (U, $\mathcal{N}$, I)

each n -open set is *B*-I_n set and each $\mathcal{T}$-I_n set is *B*-I_n-set.

The converse of remark (3.3) is not true as shown in the following;

Example: 3.4. In example (4.2) $\mathcal{K}$ = {e_2} is *B*-I_n set but not n -open set and $\mathcal{H}$ = {e_1, e_3, e_4} is *B*-I_n set but not $\mathcal{T}$-I_n set

Theorem: 3.5. For a subset $\mathcal{K}$ of a space (U, $\mathcal{N}$, I), the set $\mathcal{K}$ is n -open if and only if $\mathcal{K}$ is pre-I_n-open and *B*-I_n set.

Proof: (1) ⇨ (2): Let $\mathcal{K}$ be n -open. Then $\mathcal{K}$ = $n$int ($\mathcal{K}$) ⊂ $n$int (nCL* ($\mathcal{K}$)) and $\mathcal{K}$ is pre-I_n-open. This mean $\mathcal{K}$ is *B*-I_n set by (3.3) .

(2) ⇨ (1): let $\mathcal{K}$ is *B*-I_n set. So $\mathcal{K} = \mathcal{U} \cap \mathcal{V}$ where $\mathcal{U}$ is n -open and $n$int ($\mathcal{V}$) = $n$int (nCL* ($\mathcal{V}$)) Then $\mathcal{K} \subset \mathcal{U}$ = $n$int ($\mathcal{U}$). Also, $\mathcal{K}$ is pre-I_n- open implies $\mathcal{K}$ ⊂ $n$int (nCL* ($\mathcal{K}$)) ⊂ $n$int (nCL*($\mathcal{V}$)) = $n$int ($\mathcal{V}$) by assumption. Thus $\mathcal{K}$ ⊂ $n$int ($\mathcal{U}$) ∩ $n$int ($\mathcal{V}$) = $n$int ($\mathcal{U} \cap \mathcal{V}$) = $n$int ($\mathcal{K}$) ⇒ $\mathcal{K}$ is n -open.

Remark: 3.6. In a space (Ʋ, $\mathcal{N}$, I)

each $\mathcal{n}$ -open set is B-α-I_n set also each $\mathcal{T}$-α-I_n set is B-α-I_n set.

The converse of remark (3.6) is not true as shown in the following;

Example: 3.7. In example (3.2) the set = {e_1} is B-α-I_n set but not $\mathcal{n}$ -open set and the set $\mathcal{H}$ = {e_1, e_3, e_4} is B-α-I_n set but not T-α-I_n set.

Theorem: 3.8. For a subset $\mathcal{K}$ of a space (Ʋ, $\mathcal{N}$, I), $\mathcal{K}$ is $\mathcal{n}$ – open iff $\mathcal{K}$ is α-I_n - open and a B-α-I_n set.
Proof: (1) ⇨ (2): Let $\mathcal{K}$ be $\mathcal{n}$ -open. $\mathcal{K}$ = $\mathcal{n}$int ($\mathcal{K}$) ⊂ $\mathcal{n}$CL⋆ ($\mathcal{n}$int ($\mathcal{K}$)) and $\mathcal{K}$ = int ($\mathcal{K}$)⊂ $\mathcal{n}$int ($\mathcal{n}$CL⋆($\mathcal{n}$int ($\mathcal{K}$))) Therefore $\mathcal{K}$ is α-I_n- open. by remark(3.6) is B-α-I_n set.

(2) ⇨ (1): Given $\mathcal{K}$ is a B-α-I_n set. So $\mathcal{K}$ = $\mathcal{U}$ ∩ $\mathcal{V}$ where $\mathcal{U}$ is $\mathcal{n}$ -open and $\mathcal{n}$int ($\mathcal{V}$) = $\mathcal{n}$int ($\mathcal{n}$CL⋆ ($\mathcal{n}$int ($\mathcal{V}$))) Then $\mathcal{K}$ ⊂ $\mathcal{U}$ = $\mathcal{n}$int ($\mathcal{U}$). Also $\mathcal{K}$ is α-I_n- open implies $\mathcal{K}$ ⊂$\mathcal{n}$int ($\mathcal{n}$CL⋆($\mathcal{n}$int ($\mathcal{K}$))) ⊂ $\mathcal{n}$int ($\mathcal{n}$CL⋆($\mathcal{n}$int ($\mathcal{V}$))) = $\mathcal{n}$int ($\mathcal{V}$) by assumption. Thus $\mathcal{K}$ ⊂ $\mathcal{n}$int ($\mathcal{U}$) ∩ $\mathcal{n}$int ($\mathcal{V}$) = $\mathcal{n}$int ($\mathcal{U}$ ∩ $\mathcal{V}$) = $\mathcal{n}$int ($\mathcal{K}$) and $\mathcal{K}$ is $\mathcal{n}$ -open.

Definition: 3.9. If $\mathcal{K}$ ⊆ Ʋ, the Ideal nanotopological space (Ʋ, $\mathcal{N}$, I) is named
semi pre⋆-I_n - closed set ($\mathcal{n}$int ($\mathcal{n}$CL⋆($\mathcal{n}$int ($\mathcal{K}$))) ⊂ $\mathcal{K}$ if its complement is a semi pre ⋆ - I_n-open set.
Every semi⋆- I_n- closed set is a semi pre⋆-I_n- closed. **Theorem: 3.10.**

Proof: It follows directly from definition(3.9 and 1.11(4))

But the following example illustrates that the opposite is not true.

Example: 3.11. Consider the Ideal nanotopological space (Ʋ,$\mathcal{N}$,I) where Ʋ = {e_1 , e_2 , e_3 , e_4 }, with Ʋ/R = {{e_2}, {e_4}, {e_1, e_3}} and X = { e_3, e_4}, Then $\mathcal{N}$ = {∅, { e_4 }, { e_1 , e_3 }, {e_1 , e_3 , e_4 }, Ʋ } and I = {∅, {e_3}, {e_4}, {e_3, e_4}}. If $\mathcal{K}$ = {e_1}, then $\mathcal{n}$int ($\mathcal{n}$CL⋆($\mathcal{n}$int ($\mathcal{K}$))) = $\mathcal{n}$int ($\mathcal{n}$CL⋆ (∅)) = ∅ ⊂ $\mathcal{K}$ and $\mathcal{K}$ is semi pre⋆-I_n-closed. Since $\mathcal{n}$int ($\mathcal{n}$CL⋆($\mathcal{K}$)) = $\mathcal{n}$int ($\mathcal{n}$CL⋆ ({e_1})) = $\mathcal{n}$int ({e_1, e_2, e_3}) = {e_1, e_3} ⊈ {e_1}, $\mathcal{K}$ is not semi⋆-I_n- closed.

Theorem: 3.12. If $\mathcal{K}$ ⊆ Ʋ, Ideal nanotopological space (Ʋ, $\mathcal{N}$, I) is semi pre⋆ -I_n- closed. If $\mathcal{K}$ is semi-I_n- open, then $\mathcal{K}$ is semi⋆-I_n- closed.

Proof: If $\mathcal{K}$ is semi- I_n - open, then $\mathcal{K}$ ⊂ $\mathcal{n}$CL⋆ ($\mathcal{n}$int ($\mathcal{K}$)) and so $\mathcal{n}$CL⋆ ($\mathcal{K}$) ⊂ $\mathcal{n}$CL⋆ ($\mathcal{n}$int ($\mathcal{K}$)). Now $\mathcal{n}$int ($\mathcal{n}$CL⋆ ($\mathcal{K}$)) ⊂ $\mathcal{n}$int ($\mathcal{n}$CL⋆($\mathcal{n}$int ($\mathcal{K}$))) ⊂ $\mathcal{K}$ and so $\mathcal{K}$ is semi⋆-I_n - closed.

Theorem: 3.13. If $\mathcal{K}$ ⊆ Ʋ , Ideal nanotopological space (Ʋ, $\mathcal{N}$, I). Then both of these are equal

(1) $\mathcal{K}$ is a $\mathcal{T}$-I_n set.

(2) $\mathcal{K}$ is a semi pre⋆- I_n -closed and B-I_n set.

Proof: (2) ⇔ (1). Suppose $\mathcal{K}$ is a semi pre⋆-I_n-closed and B-I_n set.

Then $\mathcal{K}$ = $\mathcal{U}$ ∩ $\mathcal{V}$ where $\mathcal{U}$ is $\mathcal{n}$ -open and $\mathcal{V}$ is a $\mathcal{T}$-I_n set. ⇔

⇔ Now $\mathcal{n}$int ($\mathcal{n}$CL⋆ ($\mathcal{K}$)) = $\mathcal{n}$int ($\mathcal{n}$CL⋆ ($\mathcal{U}$ ∩ $\mathcal{V}$))
⊂ $\mathcal{n}$int ($\mathcal{n}$CL⋆($\mathcal{U}$) ∩ $\mathcal{n}$CL⋆($\mathcal{V}$)) = $\mathcal{n}$int ($\mathcal{n}$CL⋆($\mathcal{U}$)) ∩ $\mathcal{n}$int (CL⋆($\mathcal{V}$)) $\mathcal{n}$int ($\mathcal{n}$CL⋆ ($\mathcal{U}$)) ∩ $\mathcal{n}$int ($\mathcal{V}$) = $\mathcal{n}$int ($\mathcal{n}$CL⋆ ($\mathcal{U}$) ∩ $\mathcal{n}$int ($\mathcal{V}$)) =
⊂ $\mathcal{n}$int ($\mathcal{n}$CL⋆ ($\mathcal{U}$) ∩ $\mathcal{n}$int ($\mathcal{V}$)) = $\mathcal{n}$int ($\mathcal{n}$CL⋆ ($\mathcal{n}$int ($\mathcal{U}$ ∩ $\mathcal{V}$)))

= $n$int (nCL* ($n$int ($\mathcal{K}$))) ⊂ $\mathcal{K}$
⇔ So $n$int (nCL* ($\mathcal{K}$))⊂ $n$int ($\mathcal{K}$).

But $n$int ($\mathcal{K}$) ⊂ $n$int (nCL*($\mathcal{K}$))

So $n$int ($\mathcal{K}$) = $n$int (nCL*($\mathcal{K}$)) ⇔

⇔which demonstrates that $\mathcal{K}$ is a $\mathcal{T}$-I_n set.

Theorem: 3.14. If $\mathcal{K}$ ⊆ U, Ideal nanotopological space (U, $\mathcal{N}$, I) is semi*-I_n-open ⇔ nCL ($\mathcal{K}$) = nCL ($n$int *($\mathcal{K}$)).

Proof: If $\mathcal{K}$ is semi*-I_n-open set, then $\mathcal{K}$ ⊆ nCL ($n$int* ($\mathcal{K}$)) and nCL ($\mathcal{K}$) ⊆ nCL ($n$int *($\mathcal{K}$)). But nCL ($n$int* ($\mathcal{K}$)) ⊆ nCL ($\mathcal{K}$). Hence nCL ($\mathcal{K}$) = nCL ($n$int*($\mathcal{K}$)). Conversely, $\mathcal{K}$ ⊆ nCL ($\mathcal{K}$) = nCL ($n$int*($\mathcal{K}$)) by assumption. Consequently, $\mathcal{K}$ is semi*- I_n -open.

Corollary: 3.15. If $\mathcal{K}$ ⊆ U , Ideal nanotopological space (U, $\mathcal{N}$, I) is semi*-I_n-closed ⇔ $\mathcal{K}$ is a $\mathcal{T}$ -I_n set.

Proof: $\mathcal{K}$ is semi*-I_n-closed in U ⇔ (U - $\mathcal{K}$) semi*- I_n - open

⇔ nCL (U − $\mathcal{K}$) = nCL ($n$int* (U − $\mathcal{K}$)) by Proposition 3.14

⇔ U – ($n$int ($\mathcal{K}$)) = U – ($n$int (nCL*($\mathcal{K}$)))

⇔ $n$int ($\mathcal{K}$) = $n$int (nCL*($\mathcal{K}$))

⇔ is a $\mathcal{T}$-I_n set.

Theorem: 3.16. If $\mathcal{K}$ ⊆ U , Ideal nanotopological space (U, $\mathcal{N}$, I). Then the set ($\mathcal{K}$) is semi*-I_n-closed ⇔ $\mathcal{K}$ is semi pre*-I_n- closed and B-I_n set.

Proof: $\mathcal{K}$ is semi*- I_n -closed in U, this mean $\mathcal{K}$ is a $\mathcal{T}$-I_n set (by remark 3.6) and $\mathcal{K}$ is semi pre*-I_n -closed and B-I_n set (by theorem 3.13)

Remark: 3.17. The union of two semi*-I_n -closed (semi pre*-I_n- closed) set is not semi*-I_n-closed (semi pre*-I_n- closed) set

Example: 3.18. Desire of Ideal nanotopological space (U, $\mathcal{N}$, I) of example(3.11) If $\mathcal{K} = \{e_1, e_3\}$ and $\mathcal{H} = \{e_4\}$, $n$int (nCL*($\mathcal{K}$)) = $n$int (nCL* ({e_1, e_3})) = $n$int ({e_1, e_2, e_3}) = {e_1, e_3} = $\mathcal{K}$ and so $\mathcal{K}$ is semi*-I_n-closed and hence semi pre*-I_n-closed.

Also, $n$int (nCL* ($\mathcal{H}$)) = $n$int (nCL* ({e_4})) = $n$int ({e_4}) = {e_4} = $\mathcal{H}$. Consequently, $\mathcal{H}$ is semi*-I_n-closed and so semi pre*-I_n-closed. But $n$int (nCL*($n$int ($\mathcal{K}$ ∪ $\mathcal{H}$))) = $n$int (nCL* ($n$int ({e_1, e_3, e_4}))) = $n$int (CL*({e_1, e_3, e_4})) = $n$int (U) = U ⊈ $\mathcal{K}$ ∪ $\mathcal{H}$ and so $\mathcal{K}$ ∪ $\mathcal{H}$ is not semi pre*-I_n-closed and hence $\mathcal{K}$ ∪ $\mathcal{H}$ is not semi*-I_n-closed.

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