New types of Contra _ (1, 2)*_open Functions

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Abstract:

The aim in this article is to present and study several kinds of contra _(1,2)* _open functions namely [contra _(1,2)*_ sg* _open functions , contra _(1,2)* _ sg** _open functions , contra _(1, 2)*_ sg*** _open functions , contra _(1,2)* _ (sg*, g) _ open functions , contra _(1,2)*_ (g , sg*) _open functions] in bitopological spaces. Also some of their propositions are proven and we will discuss the relationship between these functions.

Keywords: (1,2)* _ sg* _open functions, contra _(1,2)* _open functions , contra _(1,2)* _ sg* _open functions, contra _(1,2)* _ sg** _open functions, contra _(1,2)* _ sg*** _open functions.

انواع جديدة من الدوال الضد _*(1,2)_ المفتوحة

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الخلاصة

الهدف من البحث هو تقديم و دراسة العديد من أنواع الدوال الضد_*(1,2)_ المفتوحة تدعى [الدوال الضد_*(1,2)_ المفتوحة من النوع–*sg ،الدوال الضد_*(1,2)_ المفتوحة من النوع– **sg ، الدوال الضد_*(1,2)_ المفتوحة من النوع – ***sg ، الدوال الضد_*(1,2)_ المفتوحة من النوع – (*g,sg) والنوع –(sg*,g)] في الفضاءات التبولوجية الثنائية . ايضا سوف نبرهن بعض من خصائصها وسوف نناقش العلاقات بين تلك الدوال .

الكلمات المفتاحية: الدالة المفتوحة*sg_*(1,2) ، الدالة ضد المفتوحة*(1,2) ، الدالة ضد المفتوحة*sg_*(1,2) ، الدالة ضد المفتوحة **sg_*(1,2)، الدالة ضد المفتوحة***sg–*(1,2).

1. Introduction:

In 1963, Kelly [1] introduced the notion of bitopological spaces .In [4] O. Ravi;etal introduced and studied in(2011)the concept of (1,2)*_g_closed maps. In (2011) ,O. Ravi ;etal [5] studied and investigated the properties of (1,2)*_sg*_homeomorphisms but he studied in (2015) , [7] (1,2)*-g#-continuous function .In (2016) Dunya and Messaa[2]studied some types of (1,2)*-M-πgb – closedmapping . In (2017)some properties of t_1t_2-δsemiopen and closedin bitopological spaces set s was introduced by M. Arunmaran;etal .While , the concepts of contra_(1,2)*_$M_{\delta\pi}$-continuous functions given and discussion by (Mohana and Arockiarani)[9] and in (2018)Mohammed ;etal. Studied a new type of contra-continuity via $\delta - \beta - open$ set s[10] .

The aim of this paper is to introduce some new types of contra_(1,2)*_open functions. Also, we given the relationships between these type of functions and study some their properties in bitopological spaces.

Throughout this paper, H ,M and N denote bitopological spaces (H ,$\mathcal{T}_1$,$\mathcal{T}_2$),(M, ρ_1 ,ρ_2) and (N, ξ_1 ,ξ_2) respectively .

2. preliminaries:

Definition (2,1) , [3]:- Let K ⊆ (X ,$\mathcal{T}_1$,$\mathcal{T}_2$), then K is $\mathcal{T}_{1,2}$_open (or (1,2)*_open), if K = E ∪ F, where E∈$\mathcal{T}_1$ and F ∈$\mathcal{T}_2$. ($\mathcal{T}_{1,2}$_open)^c is $\mathcal{T}_{1,2}$_closed (or (1,2)*_closed) .

Definition (2, 2) ,[3]**:** Let K be a subset of a bitopological space (H ,$\mathcal{T}_1$,$\mathcal{T}_2$), then

(1) ∩ {F:K ⊆ F : F is $\mathcal{T}_{1,2}$_closed } *is* $\mathcal{T}_{1,2}$_closure of K

(2) ∪ {E:E ⊆ K : E is $\mathcal{T}_{1,2}$_open } is $\mathcal{T}_{1,2}$_Interior of K .

Remark(2,3) , [4]:

$\mathcal{T}_{1,2}$_open subset s of (H ,$\mathcal{T}_1$,$\mathcal{T}_2$),it is not necessary form a topology.

Example (2,4) : Let H ={p , q , r} and let $\mathcal{T}_1$ = {H , ϕ , {p, r} } and $\mathcal{T}_2$ = {H , ϕ,{q ,r}} ,then $\mathcal{T}_{1,2}$_*open* set in(H ,$\mathcal{T}_1$,$\mathcal{T}_2$) ={ H , ϕ , {p, r} ,{q ,r}} . It clear that $\mathcal{T}_{1,2}$_open subset s of (H ,$\mathcal{T}_1$,$\mathcal{T}_2$) is not form topology.

Definition(2,5), [3]:A subset K of a bitopological space(H ,$\mathcal{T}_1$,$\mathcal{T}_2$)is(1, 2)*_semi_open if K⊂$\tau_1\tau_2$_cl($\tau_1\tau_2$_int (K)),the set (1, 2)*_semi_closedis the complement of (1, 2)* _semi_open set **.** And the intersection of all (1,2)* _semi_closed set s of H containing K is (1,2)* _semi_closure and symbolize it (1,2)* _scl(K) .

Definition(2,6) : Let (H ,$\mathcal{T}_1$,$\mathcal{T}_2$) be a bitopological space and S ⊆ H, then S is:

1. (1,2)* _Generalized closed s [6] ((1, 2)*_g_closed set) if $\mathcal{T}_{1,2}$_cl(S) ⊂ W whenever S ⊂ W and W ∈ (1, 2)*-open set in (H,$\mathcal{T}_1$,$\mathcal{T}_2$).
2. (1,2)* _semi_Generalized_star_closed set [5] ((1, 2)*_sg*_closed set) if $\mathcal{T}_{1,2}$_cl(S) ⊂ W s.t. S ⊂ W , U is (1, 2)*_ semiopen set in (H ,$\mathcal{T}_1$,$\mathcal{T}_2$) .

Remark(2,7) : In [5], [6] it is proved that in M bitopological spaces H

(i) Every $\mathcal{T}_{1,2}$_closed (resp. $\mathcal{T}_{1,2}$_open) set in H is (1, 2)* _sg*_closed (resp. (1, 2)* _sg*_open) set

(ii) Every$\mathcal{T}_{1,2}$_ closed (resp. $\mathcal{T}_{1,2}$_ open) set in H is (1, 2)* _g_closed (resp. (1, 2)* _g_open) set

(iii) Every (1,2)*_sg*_closed(resp. (1,2)*_sg*_open)set in H is (1,2)* _g_closed (resp. (1,2)*_g_open)set in H .

The family of all (1,2)* _g_closed (resp.(1,2)* _g_open) set s and (1,2)*_ sg*_closed(resp. (1,2)* _sg*_open) set s of (H ,$\mathcal{T}_1$,$\mathcal{T}_2$) will be denoted by (1,2)*_gC(H) (resp. (1,2)*_gO(H))and (1,2)* _Sg*C(H) (resp. (1,2)* _Sg*O(H)) .

Definition (2,8): A function λ: (H ,$\mathcal{T}_1$,$\mathcal{T}_2$) ⟶ (M, ρ_1 ,ρ_2) is:

- (1,2)* _open (resp. (1, 2)* _closed) function [3] if for every $\mathcal{T}_{1,2}$_open (resp. $\mathcal{T}_{1,2}$_closed) set S in H λ (S) is $\rho_1\rho_2$_open (resp. $\rho_1\rho_2$_ closed) set in M

- (1,2)* _g_open (resp. (1,2)* _g_closed)function [4] if for every $\mathcal{T}_{1,2}$_open (resp. $\mathcal{T}_{1,2}$ – closed) set S in H , λ (S) is (1,2)* _g_open (resp.(1,2)* _g_closed)set in M.
- (1,2)*_sg*_open (resp.(1,2)*_sg*_closed)function[5] if for every $\mathcal{T}_{1,2}$_open (resp. $\mathcal{T}_{1,2}$_closed) set S in H , λ (S) is (1,2)*_sg*_open (resp.(1,2)* _sg*_closed)set in M.
- pre_(1,2)*_ sg*_open (resp. pre_(1,2)*_sg*closed function[5] if for every (1,2)*_sg * _open (resp.(1,2)*_sg*_closed)set S in H , λ (S) is (1,2)*_sg*_open (resp.(1,2)*_sg*_closed) in M.
- Contra_(1, 2)*_open (resp. Contra_(1, 2)*_closed) function if for every $\mathcal{T}_{1,2}$_open (resp.$\mathcal{T}_{1,2}$_closed) set S in H , λ (S) is $\rho_1\,\rho_2$_closed (resp. $\rho_1\,\rho_2$_open) set in M.

Definition(2,9),[4],[5]: A bitopological space (H ,$\mathcal{T}_1$,$\mathcal{T}_2$) is called :

(1) (1,2)*_$\mathcal{T}_{1/2}$_space if every (1, 2)*_g_closed(resp. (1,2)*_g_open) set in H is $\mathcal{T}_{1,2}$_closed(resp. $\mathcal{T}_{1,2}$_open)

(2) RM_ space if any subs in (H ,$\mathcal{T}_1$,$\mathcal{T}_2$) is either $\mathcal{T}_{1,2}$_$open$ or $\mathcal{T}_{1,2}$_$closed$.

Theorem(2,10),[5]: In RM−space H every(1, 2)* _sg*_closed(resp. (1, 2)* _sg*_open) set in H is $\mathcal{T}_{1,2}$_closed (resp.$\mathcal{T}_{1,2}$_open) set .

3. Certain Kinds of Contra _ (1, 2)*_open nctions :

In this section , we define and study some new types of contra_(1,2)*_open functions in bitopological spaces . Now , we will introduce first type of contra_(1,2)*_open functions in the following definition:

Definition (3,1): A function λ :(H ,$\mathcal{T}_1$,$\mathcal{T}_2$) ⟶ (M, ρ_1 ,ρ_2) is said to be contra _(1,2)* _**sg***_open functionif for every $\mathcal{T}_{1,2}$_open set S in (H ,$\mathcal{T}_1$,$\mathcal{T}_2$), λ (S) is (1,2)* _sg*_closedset in (M, ρ_1 , ρ_2) .

Example (3,2): Suppose H =M={p, q , r} and $\mathcal{T}_1$ = {H , ϕ , {p}} , $\mathcal{T}_2$ = {H ,ϕ}, ρ_1 ={M , ϕ,{q} } , ρ_2 ={ M, ϕ, {p} , {p ,q} }. Then the set s in { H , ϕ , {p} } are called $\mathcal{T}_{1,2}$_open set s in H , the set s in ={ M, ϕ , {p} , {q} , {p ,q} } are called $\rho_{1,2}$ _open set s in (M, ρ_1 , ρ_2) , and Sg*C (M, ρ_1 , ρ_2)={ M, ϕ , {r} , {p , r} , {q ,r} } . Define λ : (H,$\mathcal{T}_1$,$\mathcal{T}_2$) ⟶ (M, ρ_1 , ρ_2) by λ (p)= r , λ (q)=q and λ (r)=p , clearly that λ is contra _(1,2)* _sg*_open function .

Proposition (3,3): Every contra_(1,2)*_open function is contra_(1,2)* _sg*_open .

Proof: Let λ :(H,$\mathcal{T}_1$,$\mathcal{T}_2$) ⟶ (M, ρ_1 , ρ_2) be a contra_(1,2)*_open function and let S is $\mathcal{T}_{1,2}$_open set in H , since λ is a contra_(1,2)*_open . Thus, λ (S) is $\rho_{1,2}$ – closed in M and by (2,7) step-1- , we get , λ (S) is (1,2)* _sg*_ closedset in M . Hence , λ is contra _(1,2)* _sg*_open function.

To demonstrate that the inverse of the proposition(3,3) is not always correct we have Example (3,4):

Example (3,4): Let H = M ={p , q , r} with the topologies $\mathcal{T}_1$ = {H , ϕ , {p, r}}, $\mathcal{T}_2$ = {H , ϕ}, ρ_1 ={ H , ϕ,{p}} and ρ_2 ={ M, ϕ ,{q ,r}} ,then $\mathcal{T}_{1,2}$_open set in(H ,$\mathcal{T}_1$,$\mathcal{T}_2$) ={ H , ϕ , {p, r} } , $\rho_{1,2}$ _open in (M, ρ_1 , ρ_2)={M, ϕ,{p}, {q ,r}} , $\rho_{1,2}$ _closedin (M,ρ_1 , ρ_2)={M,ϕ,{ p},{q ,r}}, and Sg*C(M, ρ_1 , ρ_2)={M,ϕ, {p},{q},{r} ,{p, q},{p, r},{q ,r} } . Define λ : (H ,$\mathcal{T}_1$,$\mathcal{T}_2$) ⟶ (M, ρ_1 , ρ_2) by λ (p)= p , λ (q)=q and λ (r)=r , clearly that λ is contra

_(1,2)* _sg*_open function , but is not contra _(1,2)*_open . Since , for $\mathcal{T}_{1,2}$_open set S={p, r} in H , λ (S)= λ ({p, r})={p ,r} is not $\rho_{1,2}$ _closed set s in M .

To make the converse true we give the following proposition :

Proposition (3,5): If λ :(H,$\mathcal{T}_1$,$\mathcal{T}_2$) ⟶ (M, ρ_1 ,ρ_2) is a contra_(1,2)* _sg*_open function and M is a RM-space , then λ is contra_(1,2)*_open .

Proof:-

Let S be a $\mathcal{T}_{1,2}$_open s in H . Since λ is contra_(1,2)* _sg*_open function .Thus , λ (S) is (1,2)* _sg*_closed set in M , According to the assumption M is RM-space . Hence , λ (S) is $\rho_{1,2}$ _closed in M Therefore , λ is contra _(1,2)*_open function .

Remark(3,6): The composition of two contra_(1,2)* _sg*_open functions doesn't have to be contra_(1,2)*_sg*_open :

Example (3,7): Let H =M=N={p,q,r} and let $\mathcal{T}_1$ = {H,ϕ,{p},{p,q}, $\mathcal{T}_2$ ={H,ϕ,{q}}, ρ_1 ={M ,ϕ,{p}} , ρ_2 ={N,ϕ,{q ,r}}, ξ_1={N,ϕ}, ξ_2={N,ϕ, {p ,r}}, then $\mathcal{T}_{1,2}$ −open in(H,$\mathcal{T}_1$,$\mathcal{T}_2$)={H,ϕ,{p},{q},{p, q}}, $\rho_{1,2}$ −open set s in (M, ρ_1 , ρ_2)={M,ϕ,{p},{q ,r}}, $\rho_{1,2}$ _closedin (M,ρ_1 , ρ_2)={M,ϕ,{p},{q,r}}and $\xi_{1,2}$−open set s in (N , ξ_1, ξ_2) ={N, ϕ ,{p ,r}} , define a function λ : (H,$\mathcal{T}_1$,$\mathcal{T}_2$) ⟶ (M, ρ_1 ,ρ_2) by λ (p)= p , λ (q)=q and λ (r)=r and γ:(M,ρ_1 ,ρ_2) ⟶(N, ξ_1, ξ_2)by γ(p)=q , γ(q)=r and γ(r)=p, It is observe that function λ and γ are contra_(1,2)*_sg*_open functions, but $\gamma \circ \lambda$:(H,$\mathcal{T}_1$,$\mathcal{T}_2$) ⟶(N, ξ_1, ξ_2) is not contra_(1,2)*_sg*_open , since for $\mathcal{T}_{1,2}$ − open set S={q} in H , $\gamma \circ \lambda$ (S)= $\gamma \circ \lambda$ ({q})= γ (λ ({q})= γ (q)=r is not (1,2)*_sg*_closedin N .

Proposition(3,8):

Let λ :(H , $\mathcal{T}_1$,$\mathcal{T}_2$) ⟶ (M,ρ_1 ,ρ_2) be contra _(1,2)*_sg*_open function and γ :(M, ρ_1 ,ρ_2) ⟶(N , ξ_1, ξ_2) be a pre__(1,2)*_sg*_closedfunction, then $\gamma \circ \lambda$:(H,$\mathcal{T}_1$,$\mathcal{T}_2$)⟶(N , ξ_1, ξ_2) is contra_(1,2)* _sg*_open function .

Proof:- Let S is $\mathcal{T}_{1,2}$_open set in H,. Thus , λ (S) is (1,2)*_sg*_closedin M. Also, since γ is a pre _(1,2)*_sg*_closed, then γ(λ (S))= $\gamma \circ \lambda$ (S) is (1,2)* _sg*_closedin N .Therefore , $\gamma \circ \lambda$:(H,$\mathcal{T}_1$,$\mathcal{T}_2$) ⟶(N, ξ_1, ξ_2) is contra_(1,2)* _sg*_ open function.

Proposition(3,9):

Let λ :(H,$\mathcal{T}_1$,$\mathcal{T}_2$)⟶(M, ρ_1 ,ρ_2) be(1,2)* _open function and γ :(M, ρ_1 ,ρ_2) ⟶(N , ξ_1, ξ_2) be a contra_(1,2)*_sg*_open function , then $\gamma \circ \lambda$:(H,$\mathcal{T}_1$,$\mathcal{T}_2$) ⟶(N , ξ_1, ξ_2) is contra_(1,2)*_sg*_open .

Proof :- Suppose S is $\mathcal{T}_{1,2}$_open in H . Thus , λ (S) is $\rho_{1,2}$ _open set in M, since γ is a contra_(1,2)_sg*_open then γ (λ (S))= $\gamma \circ \lambda$ (S) is (1,2)*_sg*_*closed* in N . Therefore , $\gamma \circ \lambda$:(H,$\mathcal{T}_1$,$\mathcal{T}_2$) ⟶(N , ξ_1, ξ_2) is contra(1,2)* _sg*_open function .

Proposition (3,10): Let λ : (H ,$\mathcal{T}_1$,$\mathcal{T}_2$) ⟶ (M, ρ_1 ,ρ_2) be any function and γ :(M, ρ_1 ,ρ_2) ⟶(N , ξ_1, ξ_2) be a contra _(1,2)* _sg*_open function , then $\gamma \circ \lambda$:(H,$\mathcal{T}_1$,$\mathcal{T}_2$) ⟶(N , ξ_1, ξ_2) is contra(1,2)* _sg*_open if M is RM-space and λ is

- **(i)** (1,2)*_sg*_open .
- **(ii)** pre _(1,2)*_sg*_open .

Proof

(i):- Let S be a $\mathcal{T}_{1,2}$_open s in H . Thus , λ (S) is a (1,2)*_sg*_open in M, by hypothesis M is RM-space. Then , λ (S) is a $\rho_{1,2}$ _open set in M , since γ is a contra _(1,2) _sg*_open , then γ (λ

(S))= $\gamma \circ \lambda$ (S) is a (1,2)* _sg*_closed in N . Therefore , $\gamma \circ \lambda$:(H,$\mathcal{T}_1$,$\mathcal{T}_2$) ⟶(N , ξ_1, ξ_2) is contra(1,2)* _sg*_open function.

The proof of step-ii- similar to step-i- .

In the following another type of contra_(1,2) _sg*_$open$:

Definition (3, 11):

A function λ :(H,$\mathcal{T}_1$,$\mathcal{T}_2$) ⟶ (M, ρ_1 ,ρ_2) is contra_(1,2)* _sg**_open function if for every (1,2)* _sg*_open set S in (H,$\mathcal{T}_1$,$\mathcal{T}_2$), λ (S) is (1,2)* _sg*_closedset in (M, ρ_1 , ρ_2) .

Example (3,12): Let H=M={p, q,r}and $\mathcal{T}_1$={H,ϕ}and $\mathcal{T}_2$ ={H,ϕ,{p}}, ρ_1 ={M, ϕ},and ρ_2 ={M,ϕ,{p, r}},then $\mathcal{T}_{1,2}$_open in(H ,$\mathcal{T}_1$,$\mathcal{T}_2$)=Sg*O(H,$\mathcal{T}_1$,$\mathcal{T}_2$)={H, ϕ ,{p}} , $\rho_{1,2}$ _$open$ set s in (M, ρ_1 , ρ_2)={M,ϕ,{p ,r}} , $\rho_{1,2}$ _closed set s in (M, ρ_1 , ρ_2)={M,ϕ,{q}}, and Sg*C(M, ρ_1 , ρ_2)={M,ϕ,{q}, {p, q}, {q ,r}}, define λ :(H,$\mathcal{T}_1$,$\mathcal{T}_2$)⟶(M,ρ_1 , ρ_2)by λ (p)= q , λ (q)=b and λ (r)= , thus λ is contra (1,2)*_sg**_open function .

Proposition (3,13): Every contra_(1,2)*_sg**_open function contra (1,2)*_sg*_open .

Proof: Let λ :(H,$\mathcal{T}_1$,$\mathcal{T}_2$) ⟶ (M, ρ_1 , ρ_2) be a contra_(1,2)*_–sg**_open function and let S be a $\mathcal{T}_{1,2}$_open set in H, by Remark(2,7) step-i we get S is (1,2)*_sg*_open in H. Also, we have λ is contra_(1,2)_sg**_open function. Thus , λ (S) is(1,2)*_sg*_closed set in M . Hence , λ is contra (1,2)* _sg**_open function .

The converse of above proposition needn't be true in general :

Example (3,14):- Suppose that H =M={p , q , r} , $\mathcal{T}_1$ = {H , ϕ} , $\mathcal{T}_2$ = {H , ϕ,{p ,r}}, ρ_1 ={M, ϕ, {q}} , and ρ_2 ={M, ϕ ,{p}, {p, q}}, then $\mathcal{T}_{1,2}$_open set s in(H ,$\mathcal{T}_1$,$\mathcal{T}_2$)={H ,ϕ,{p ,r}}, Sg*O(H ,$\mathcal{T}_1$,$\mathcal{T}_2$)={H,ϕ ,{p}, {r},{p ,r}} , $\rho_{1,2}$ _open set s in (M, ρ_1 , ρ_2) = { M ,ϕ , {p} , {q} , {p ,q} } and $\rho_{1,2}$ –closed set s in (M,ρ_1 , ρ_2)=Sg*C(M, ρ_1 , ρ_2)={M,ϕ, {r},{p ,r},{q , r}}. define λ :(H ,$\mathcal{T}_1$,$\mathcal{T}_2$)⟶(M, ρ_1 , ρ_2) by λ (p)= q , λ (q)=b and λ (r)=r. It is clear that λ is contra_(1,2)* _sg*_open , but λ is not contra_(1,2)*_sg**_open , since for (1,2)*_sg*_open set S={p} in H , f(S)=f({p})=q is not (1,2)* _sg*_closed set in M.

To make the converse true we give the following proposition :

Proposition (3,15): If λ :(H ,$\mathcal{T}_1$,$\mathcal{T}_2$) ⟶ (M, ρ_1 , ρ_2) is a contra_(1,2)* _sg*_open function and H is RM-space , then λ is contra _(1,2)* _sg**_open .

Proof :- suppose S is (1,2)* _sg*_open in H , we have H is RM-space , then by using Theorem(2,9) we get , S is $\mathcal{T}_{1,2}$_open in H . Also , since λ is a contra(1,2)* _sg*_open function . Thus , λ (S) is (1,2)*_ sg*_ closed in M . Hence λ is contra_(1,2)*_sg**_open function.

Remark(3,16): The concepts of contra_(1,2)*_open function and contra _(1,2)* _sg**_open function are independent .

Example (3,17): Let H =M={p , q , r} and let $\mathcal{T}_1$ = {H , ϕ ,{p}} and $\mathcal{T}_2$ = {H , ϕ,{p ,r}}, ρ_1 ={M , ϕ} , ρ_2 ={ M, ϕ, {p ,r}} , then $\mathcal{T}_{1,2}$_open set s in(H ,$\mathcal{T}_1$,$\mathcal{T}_2$)=Sg*O(H ,$\mathcal{T}_1$,$\mathcal{T}_2$)={H , ϕ ,{p} ,{p ,r}}, $\rho_{1,2}$ open set s in (H, ρ_1 , ρ_2)= {H,ϕ,{p ,r}} , $\rho_{1,2}$ closed set s in (M,ρ_1 , ρ_2)={M,ϕ,{q}} and Sg*C (M, ρ_1 , ρ_2) = { M,ϕ,{q} ,{p, q} , {q ,r} }, define λ :(H,$\mathcal{T}_1$,$\mathcal{T}_2$)⟶(M, ρ_1 , ρ_2)by λ (p)= q , λ (q)=b and λ (r)=r , clearly λ is contra_(1,2)*_sg*_open function , but λ is not contra_(1,2)*_open , since for $\mathcal{T}_{1,2}$ –open set S={p ,r} in H , λ (S)= λ ({p ,r})={q ,r} is not $\rho_{1,2}$ _closed set in M .

Example (3,18): Suppose H=M={p , q , r} , $\mathcal{T}_1$ = {H, ϕ} , $\mathcal{T}_2$= {H, ϕ,{p ,r}}, ρ_1 ={M , ϕ,{p},{p ,r}} ,and ρ_2 ={ M, ϕ, {q ,r}}, then $\mathcal{T}_{1,2}$_open set s in(H ,$\mathcal{T}_1$,$\mathcal{T}_2$)={ H,ϕ,{p ,r} }, $\rho_{1,2}$ _open set s in (M, ρ_1 , ρ_2)={M,ϕ, {p}, {p ,r},{q ,r} } , $\rho_{1,2}$ _closed set s in (M,ρ_1 , ρ_2)={M,ϕ,{p},{q},{q

$,r\}\}$, Sg*O(H $,\mathcal{T}_1,\mathcal{T}_2$)={H , ϕ $\{p\},\{r\},\{p,r\}\}$ and Sg*C (M, ρ_1,ρ_2) = {M, ϕ, $\{p\}$, $\{q\},\{q,r\},\{p,q\}\}$. Define λ :(H,$\mathcal{T}_1,\mathcal{T}_2$) ⟶(M, ρ_1,ρ_2) by λ (p)= q , λ (q)=b and λ (r)=r , clearly λ is contra_(1,2)*_open function , but λ is not contra_(1,2)*_sg*_open , since for (1,2)*_sg*_open set S=$\{r\}$ in H , λ (S)= λ ($\{r\}$)=$\{r\}$ is not (1,2)*_sg*_closed set in M .

To make the converse true we give the following proposition :

Proposition (3,19):

If λ :(H,$\mathcal{T}_1,\mathcal{T}_2$) ⟶ (M, ρ_1,ρ_2) is a contra_(1,2)* _sg**_open function and M is RM-space , then λ is contra _(1,2)* _open .

Proof: Let S be $\mathcal{T}_{1,2}$_open in H, by (2,7)step-i- we get , S is (1,2)*_ sg*_open in H , also since λ is contra_ (1,2)* _sg**_open function . Thus , λ (S) is a (1,2)*_sg*_ closed set in M, by hypothesis M is RM-space , then by Theorem (2,10) we get , λ (S) is a $\rho_{1,2}$_open set in M. Therefore , λ is contra _(1,2)*_open function

In the same way, we prove the next proposition .

Proposition (3,20): If λ :(H,$\mathcal{T}_1,\mathcal{T}_2$) ⟶ (M, ρ_1,ρ_2) is a contra_(1,2)*_open function and H is a RM-space , then λ is contra_(1,2)*_sg**_open .

Next, we Give some propositions about the composition of contra _(1,2)*_sg**_open function with other (1,2)* _open and (1,2)*_closed function types :

Proposition (3,21): Let λ:(H,$\mathcal{T}_1,\mathcal{T}_2$) ⟶ (M, ρ_1,ρ_2) be any function and γ: (M, ρ_1,ρ_2) ⟶(N , ξ_1,ξ_2) be contra_(1,2)*_sg**_open function ,then $\gamma\circ\lambda$:(H,$\mathcal{T}_1,\mathcal{T}_2$) ⟶(M , ξ_1,ξ_2) is contra_(1,2)* _sg*_open . If λ is

(i) (1,2)* _sg*_open nction.

(ii)(1,2)* _open nction .

Proof

(i) Let S be $\mathcal{T}_{1,2}$_open in H ,by hypotheses λ is (1,2)* _sg*_open function . Thus , λ (S) is (1,2)* _sg*_open set in M . Also , since γ is a contra _(1,2)* _sg**_open function , then γ (λ (S))= ($\gamma\circ\lambda$)(S) is (1,2)* _sg*_ closedin N . Therefore , $\gamma\circ\lambda$:(H,$\mathcal{T}_1,\mathcal{T}_2$) ⟶(N , ξ_1,ξ_2) is contra_(1,2)*_sg*_open.

The prove of part-ii- similar to part -i- .

Proposition (3,22): Suppose λ :(H,$\mathcal{T}_1,\mathcal{T}_2$) ⟶ (M, ρ_1,ρ_2**)** be a pre_(1,2)* _sg**_open function and γ :(M, ρ_1,ρ_2) ⟶(N , ξ_1,ξ_2) be a contra _(1,2)*_sg**_open function , then $\gamma\circ\lambda$:(H,$\mathcal{T}_1,\mathcal{T}_2$) ⟶(N , ξ_1,ξ_2) is contra_(1,2)* _sg**_open function .

Poof :- Suppose S is (1,2)* _sg*_open set in H , since λ is a pre_(1,2)*_sg_open function . Thus , λ (S) is (1,2)*_–sg*_open set in M . Also , since γ is a contra _(1,2)* _sg**_open function , then γ (λ (S))= $\gamma\circ\lambda$ (S) is(1,2)*_sg*_ closed in N . Therefore , $\gamma\circ\lambda$:(H,$\mathcal{T}_1,\mathcal{T}_2$) ⟶(N , ξ_1,ξ_2) is contra_(1,2)* _sg**_open .

Proposition (3,23):

Let λ:(H ,$\mathcal{T}_1,\mathcal{T}_2$) ⟶(M,ρ_1,ρ_2) be a contra_(1,2)* _sg**_open function and γ :(M, ρ_1,ρ_2) ⟶(N , ξ_1,ξ_2) be a pre _(1,2)*_sg*_closed function , then $\gamma\circ\lambda$:(H,$\mathcal{T}_1,\mathcal{T}_2$) ⟶(N , ξ_1,ξ_2) is contra(1,2)* _sg**_open .

Proof:- Suppose S is (1,2)*_sg*_open in H, since λ is a contra_(1,2)*_sg**_open function .Thus, λ (S) is (1,2)*_sg*_closed in M. Also , since γ is a pre _(1,2) _sg*_closed , then γ (λ (S))= $\gamma\circ\lambda$ (S) is a (1,2)* _sg*_ closedin N . Therefore , $\gamma\circ\lambda$:(H,$\mathcal{T}_1,\mathcal{T}_2$) ⟶(N , ξ_1,ξ_2) is contra_(1,2)*_sg**_open function .

Remark(3,24):
If λ :(H ,$\mathcal{T}_1$,$\mathcal{T}_2$) ⟶(M, ρ_1 , ρ_2)is contra_(1,2)*_sg**_open and γ :(M,ρ_1 , ρ_2) ⟶(N , ξ_1, ξ_2) is (1,2)* _closed ((1,2)*_sg*_closed)function, then $\gamma \circ \lambda$:(H,$\mathcal{T}_1$,$\mathcal{T}_2$)⟶(N, ξ_1, ξ_2) is not necessary contra_(1,2)*_sg**_open function . As shows in (3,25)

Example(3,25): Let H =M=N={p,q , r} and let $\mathcal{T}_1$ ={H , ϕ ,{p},{p ,r}} , $\mathcal{T}_2$ ={H , ϕ,{q ,r}}, ρ_1 ={m , ϕ} , ρ_2 ={M,ϕ,{p},{q ,r}}, ξ_1={N,ϕ,{p}}and ξ_2={N,ϕ,{q},{p,q}},then $\mathcal{T}_{1,2}$_open in(H,$\mathcal{T}_1$,$\mathcal{T}_2$)={H,ϕ ,{p},{p,r},{q,r}},$\rho_{1,2}$ _open in (M, ρ_1 , ρ_2)={M,ϕ,{p}, {q ,r}} , $\rho_{1,2}$ _closed set s in (M,ρ_1 , ρ_2)={M,ϕ,{p},{q,r}},then $\xi_{1,2}$_open set s in (N,ξ_1, ξ_2)= {N,ϕ, {p},{q},{p ,q}} , define a function λ:(H,$\mathcal{T}_1$,$\mathcal{T}_2$) ⟶(M,ρ_1 , ρ_2)by λ (p)= p,λ (q)=q and λ (r)=r and define γ:(M,ρ_1 , ρ_2) ⟶(N , ξ_1, ξ_2)by γ (p)=γ (q)=r and γ (r)=q. It is observe that λ is contra_(1,2)* _sg**_open function and γ is [(1,2)*_sg*_closed and (1,2)*_closed] function. But $\gamma \circ \lambda$:(H,$\mathcal{T}_1$,$\mathcal{T}_2$) ⟶(N, ξ_1, ξ_2) is not contra_(1,2)*_sg**_open function, since for(1,2)*_sg*_open set S={r} in H , $\gamma \circ \lambda$ (S)= $\gamma \circ \lambda$ ({r})= γ (λ ({r})=γ (=γ (r)=q is not (1,2)*_sg*_closed set s in N .

To make (3,24) true we must add another condition as we will notice in (3,26):

Proposition(3,26):
Let λ:(H,$\mathcal{T}_1$,$\mathcal{T}_2$) ⟶ (M, ρ_1 , ρ_2)be a contra_(1,2)*_sg**_open function and γ :(M,ρ_1 , ρ_2) ⟶(N, ξ_1, ξ_2) be any open function, then $\gamma \circ \lambda$:(H,$\mathcal{T}_1$,$\mathcal{T}_2$) ⟶(N, ξ_1, ξ_2)is contra_(1,2)*_sg**_open .If M is RM_ space and

(i) γ is (1,2)* _sg*_closed function.

(ii) γ is (1,2)* –closed function .

Proof:-
(i) Suppose S is (1, 2)* _sg*_open s in H,. Thus , λ (S) is (1,2)*_sg*_closed in M , by hypothesis M is RM_ space and by Theorem(2,10) we get λ (S) is a $\rho_{1,2}$_closed in M, and also since γ is (1,2)* _sg*_ closed function , then γ (λ (S)) = $\gamma \circ \lambda$ (S). is a (1,2)* _sg*_ closedin N . Therefore , $\gamma \circ \lambda$:(H,$\mathcal{T}_1$,$\mathcal{T}_2$)⟶(N, ξ_1 ξ_2) is contra _(1, 2)*_g**_open function .

The prove of part-ii- similar to part -i- .

In the following , we will Give another type of contra_(1,2)*_sg*_open functions which is called contra_(1,2)*_ g***_open :

Definition (3,27): A function :(H,$\mathcal{T}_1$,$\mathcal{T}_2$) ⟶ (M, ρ_1 , ρ_2) is contra _(1,2)* _sg***_open function if for Every (1,2)* _sg*_open set S in (H,$\mathcal{T}_1$,$\mathcal{T}_2$), λ (S) is $\rho_{1,2}$ _closed set in (M, ρ_1 , ρ_2) .

Proposition (3,28): Every contra_(1,2)* _sg***_open function is

(i) contra_(1,2)*_open .

(ii) contra_(1,2)* _sg**_open .

Proof:
(i) Let λ :(H,$\mathcal{T}_1$,$\mathcal{T}_2$) ⟶ (M, ρ_1 , ρ_2) be a contra_(1,2)*_sg***_open function and let S is $\mathcal{T}_{1,2}$ –open set in H , by Remark(2,7)step-i- [Every $\mathcal{T}_{1,2}$_open set is (1, 2)* _sg*_open] so we get S is (1,2)*_sg*_open in H . Also , since λ is a contra_(1,2)*_sg***_ open function . Thus, (S) is $\rho_{1,2}$ _ closed in M . Hence , λ is contra _(1,2)*_open function .

(ii): Let λ :(H,$\mathcal{T}_1$,$\mathcal{T}_2$) ⟶(M,ρ_1 , ρ_2) is contra_(1,2)* _sg***_open function and let S be (1, 2)* _sg*_open in H ,since λ _is contra_(1,2)* _sg***_ open function . Then , λ (S) is $\rho_{1,2}$ _

closed in M and by (2,7) step-i- So , we get λ (S) is (1,2)*_sg*_closed in M. thus , λ is contra _(1,2)*)* _sg**_open function .

Corollary (3,29): Every contra_(1, 2)*_sg***_open function is contra_(1,2)*_sg*_open .

Proof:- This follows proposition (3,28) part (i) and proposition (3,3) .

To demonstrate that the inverse of the proposition (3,28) and Corollary (3,29) not always correct, we have the next example:

Example(3,30):-

(i) Let H =M={p,q,r} , $\mathcal{T}_1$ ={H , ϕ,{p, r}}, $\mathcal{T}_2$ = {H , ϕ},{M, ϕ ,{p} ,{p, r} } , and ρ_2 ={M, ϕ ,{q ,r}}, then $\mathcal{T}_{1,2}$ −open set s in(H ,$\mathcal{T}_1$,$\mathcal{T}_2$)={H ,ϕ ,{p ,r}} , $\rho_{1,2}$ _open set s in (M, ρ_1 , ρ_2) ={M,ϕ ,{p},{p ,r} ,{q ,r}}, $\rho_{1,2}$ _closed in (M,ρ_1 , ρ_2)={M, ϕ ,{p}, {q},{q ,r}}. Define λ :(H,$\mathcal{T}_1$, $\mathcal{T}_2$)⟶(M, ρ_1 , ρ_2) by λ (p)= q , λ (q)=b and λ (r)=r. It is observe that λ is [contra_(1,2)*_open and contra_(1,2)* _sg*_open] function , but λ is not contra _(1,2)*_sg***_open , since for (1,2)*_sg*_open set S={r} in H , λ (S)= λ ({r})={r} is not $\rho_{1,2}$ _closed in M.

(ii) Let H =M={p , q , r} , $\mathcal{T}_1$ = {H , ϕ} , $\mathcal{T}_2$ = {H , ϕ,{p} , {p ,r}}, ρ_1 ={M, ϕ} , and ρ_2 ={M, ϕ , {p, r}}, then $\mathcal{T}_{1,2}$_open set s in(H ,$\mathcal{T}_1$,$\mathcal{T}_2$)= {H ,ϕ,{p} ,{p ,r}} , $\rho_{1,2}$ _open set s in (M, ρ_1 , ρ_2) ={M,ϕ ,{p ,r}}, $\rho_{1,2}$ _closed set s in (M,ρ_1 , ρ_2)={M, ϕ , {q}}. Define λ :(H,$\mathcal{T}_1$, $\mathcal{T}_2$)⟶(M, ρ_1 , ρ_2) by λ (p)= q , λ (q)=b and λ (r)=r. It is observe that λ is contra_(1,2)* _sg**_open , but λ is not contra_(1,2)*_sg***_open , since for (1,2)*_sg*_open set S={p ,r} in H , λ (S)= λ ({p ,r})={q ,r} is not $\rho_{1,2}$ _closed set in M.

To make (3,28) and(3,29) are true we must add another condition as we will notice in (3,31):

Proposition (3,31): If λ :(H,$\mathcal{T}_1$, $\mathcal{T}_2$) ⟶ (M, ρ_1 , ρ_2) is a contra_(1,2)*_open function and H is a RM_ space then λ is contra _(1,2)*_sg***_open .

Proof:- Suppose S be (1,2)* _sg*_open in H , since H is a RM _space .Hence, S is $\mathcal{T}_{1,2}$_open in H . Also , since λ is contra(1,2)* _open function .Thus, λ (S) is $\rho_{1,2}$ _closed in M. thus λ is contra_(1,2)* _sg***_open

Proposition (3,32): If λ :(H,$\mathcal{T}_1$, $\mathcal{T}_2$) ⟶ (M, ρ_1 , ρ_2) is contra_(1,2)* _sg*_open function and H, M are two RM-spaces , then λ is contra _(1,2)* _sg***_open .

Proof :- Let S be (1,2)* _sg*_open in H , since H is a RM _space .Hence, S is $\mathcal{T}_{1,2}$_open in H .Since, λ is contra_(1,2)* _sg*_open , this lead λ (S) is (1,2)* _sg*_closed in M and by assumpotion M is RM _space . Hence , λ (S) is $\rho_{1,2}$ _closed in M . Therefo , λ is contra _(1,2)* _sg***_open .

In the same way we will prove (3,33) :

Proposition (3,33) : If λ :(H,$\mathcal{T}_1$, $\mathcal{T}_2$) ⟶ (M, ρ_1 , ρ_2) is a contra_(1,2)* _sg**_open function and M is RM−spaces , then λ is contra _(1,2)*_sg***_open .

The composition of contra _(1,2)*_sg***_(1,2)*_open functions with other (1,2)*_open and (1,2)*_closed functions types will be given in the following propositions :

Proposition(3,34): Let λ :(H , $\mathcal{T}_1$, $\mathcal{T}_2$)⟶(M ,ρ_1 , ρ_2) be contra_(1,2)* _sg***_open function and γ :(M, ρ_1 , ρ_2) ⟶ (N , ξ_1, ξ_2) be any function ,then $\gamma \circ \lambda$:(H , $\mathcal{T}_1$, $\mathcal{T}_2$) ⟶ (N, ξ_1, ξ_2) is contra_(1,2)* _ sg***_open , if γ is

- **(i)** (1,2)*_closed function
- **(ii)** (1,2)*–sg*–closed function

Proof:

(i) Suppose S is (1,2)*_sg*_open in H ,. Thus , λ (S) is $\rho_{1,2}$ _closed set in M. Also , since γ is (1,2)*_closed function, then $\gamma(\lambda$ (S))= $\gamma \circ \lambda$(S).is a $\xi_{1,2}$_ closedin N. Hence, $\gamma \circ \lambda$:(H,$\mathcal{T}_1$,$\mathcal{T}_2$)⟶(N, ξ_1, ξ_2)is contra_(1,2)*_sg***_open function . And in the same way ,part(ii) can be proved

Proposition(3,35): Let λ:(H ,$\mathcal{T}_1$,$\mathcal{T}_2$) ⟶(M, ρ_1 , ρ_2) be contra_(1,2)* _sg***_open function and γ:(M, ρ_1 , ρ_2) ⟶ (N, ξ_1, ξ_2) be pre_(1,2)*_sg*_closed function , then $\gamma \circ \lambda$:(H , $\mathcal{T}_1$, $\mathcal{T}_2$) ⟶ (N, ξ_1, ξ_2) is contra_(1,2)*_sg**_open .

Proof:- Let S be (1, 2)*_sg*_open in H,. Thus, λ (S) is $\rho_{1,2}$ _closed in M, by (2,7) step-i- we get ,λ (S) is (1, 2)* _sg*_closed in M and since γ is a pre_ (1, 2)* _sg*_closed function, then γ (λ (S))= $\gamma \circ \lambda$(S) is (1, 2)* _sg*_closed in N. Hence, $\gamma \circ \lambda$:(H,$\mathcal{T}_1$,$\mathcal{T}_2$) ⟶(N, ξ_1, ξ_2)is contra_(1,2)*_sg**_open function .

In the same way we will prove (3,36) :

Proposition (3,36): Let λ :(H,$\mathcal{T}_1$,$\mathcal{T}_2$) ⟶ (M, ρ_1 , ρ_2) be (1,2)* _open (resp . (1,2)* _sg*_open) function and γ:(M,ρ_1 , ρ_2) ⟶(N, ξ_1, ξ_2) be contra_(1,2)*_sg***_open function, then $\gamma \circ \lambda$:(H,$\mathcal{T}_1$,$\mathcal{T}_2$) ⟶(N, ξ_1, ξ_2) is contra_(1,2)*_sg*_open .

Proposition(3,37) :

Let λ:(H, $\mathcal{T}_1$,$\mathcal{T}_2$) ⟶ (M, ρ_1 , ρ_2)be a pre_(1,2)* _sg*_open function and γ : (M, ρ_1 , ρ_2) ⟶(N , ξ_1, ξ_2) be a contra _(1,2)*_sg***_open function , then $\gamma \circ \lambda$:(H,$\mathcal{T}_1$,$\mathcal{T}_2$) ⟶(N , ξ_1, ξ_2) is contra_(1,2)*_sg***_open

Proof:- Let S be (1,2)* _sg*_open set in H, since λ is pre_ (1,2)*_sg*_open function .Thus , λ (S) is(1,2)* _sg*_open in M . Also , since γ is a contra_(1,2)*_sg***–open function , then γ (λ (S))= $\gamma \circ \lambda$ (S) is $\xi_{1,2}$_ closed in N . Therefore , $\gamma \circ \lambda$:(H,$\mathcal{T}_1$,$\mathcal{T}_2$) ⟶(N , ξ_1, ξ_2) is contra_(1,2)*_sg***_open function

4.Contra_(sg*, g)_open functions and Contra_(g , sg*)_open functions :

In this section, we will Give and study new types of contra_(1, 2)*_sg*_open functions namely [contra_(sg*, g) _open functions and contra_(g, sg*) _open functions] in bitopological spaces.

Definition (4,1): A function λ :(H,$\mathcal{T}_1$,$\mathcal{T}_2$) ⟶ (M, ρ_1 , ρ_2) is contra _(1,2)* _(sg*,g)_open function if for every (1,2)*_sg*_open set , S in (H,$\mathcal{T}_1$, $\mathcal{T}_2$), λ (S) is (1,2)*_g_ closed set in (M, ρ_1 , ρ_2) .

Proposition (4,2): Every contra_(1, 2)*_sg**_open function is contra _(1,2)* _(sg*,g)_open .

Proof : Suppose S be (1,2)* _sg*_open in H, since λ is a contra_(1, 2)* _sg**_open function .Thus , λ (S) is(1,2)* _sg*_closed in M , by[Every (1,2)*_sg*_closedis (1,2)*_g_ closed]. Then, λ (S) is (1,2)*_g_ closed in M. Therefore , λ is contra _(1,2)*_(sg*,g)_open function.

Corollary (4,3): Every contra_(1, 2)*_sg***_open function is contra_(1,2)* _(sg*,g)_open .

proof : It can be proven using proposition (3,28)part-ii- and proposition(4,2) .

The converse of above proposition and Corollary need not be true as seen from the following Example:

Example(4,4): Let H =M={p , q , r} and let $\mathcal{T}_1$ = {H , ϕ , {p} } and $\mathcal{T}_2$ = {H , ϕ,{q},{p ,q} }, ρ_1 ={M, ϕ} , and ρ_2 ={M,ϕ,{p}}.Then, the $\mathcal{T}_{1,2}$_open in (H,$\mathcal{T}_1$,$\mathcal{T}_2$)=Sg*O(H,$\mathcal{T}_1$,$\mathcal{T}_2$)={H,ϕ,{p},{q},{p ,q}}, $\rho_{1,2}$_open set s in (M, ρ_1 , ρ_2)={M, ϕ,{p}}, $\rho_{1,2}$_closed set s in (M, ρ_1 , ρ_2)= Sg*C(M, ρ_1 , ρ_2)= {M, ϕ,{q ,r}} and gC(M, ρ_1 , ρ_2)={M, ϕ ,{q} ,{r},{q ,r},{p ,q},{p ,r}} .Define λ :(M,$\mathcal{T}_1$,$\mathcal{T}_2$) ⟶ (M, ρ_1 , ρ_2) by λ (p)=p , λ (q)=r and λ (r)=q . Clearly λ is a contra _(1, 2)* _(sg*, g) _open nction. But λ is not [contra-(1, 2)* _sg***_open and is not contra_(1, 2)* _sg**_open] function , since for (1,2)* _sg*_open S={p ,q} in H , λ (S)= λ ({p ,q})={p ,r} is not $\rho_{1,2}$_closed(resp. (1,2)* _sg*_closed) in M.

Proposition (4,5) Give the condition to make proposition(4,2) and Corollary(4,3) are true :

proposition(4,5): If λ :(H,$\mathcal{T}_1$,$\mathcal{T}_2$) $\longrightarrow$ (M, ρ_1 ,ρ_2) is a contra_(1,2)*_(sg*,g)_open function and M is a (1,2)*_$T_{1/2}$ space , then λ is

- **(i)** Contra _(1, 2)* _sg***_open function.
- **(ii)** Contra _(1, 2)*_sg**_open function .

Proof

(i): Let S be(1,2)*_sg*_open set in H, since λ is contra_(1, 2)*_(sg*,g)_open function .Thus λ (S) is(1,2)*_g_ closed in M ,since M is (1,2)*_$T_{1/2}$ space , then λ (S) is a $\rho_{1,2}$_closed set in M. Therefore , λ is contra _(1,2)*_sg***_open function. And in the same way, part (ii) can be proved

Remark(4,6): contra_(1,2)*_open functions and contra _(1,2)* _sg*_open functions are independent with contra_(1, 2)* _(sg*,g)_open functions :

Example(4,7):

(i) Suppose H =M={p,q ,r}, $\mathcal{T}_1$ = {H , ϕ} ,$\mathcal{T}_2$ ={H , ϕ,{p ,r} }, ρ_1={M , ϕ,{p}} , and ρ_2={M,ϕ,{p ,r}} Then , the $\mathcal{T}_{1,2}$_open set s in (H,$\mathcal{T}_1$,$\mathcal{T}_2$)={H,ϕ,{p ,r} }, $\rho_{1,2}$ –open set s in (M, ρ_1 ,ρ_2)={M,ϕ,{p},{p ,r}}, and $\rho_{1,2}$_closedin (M, ρ_1 ,ρ_2) ={M, ϕ,{q},{q ,r}.Define λ :(H,$\mathcal{T}_1$,$\mathcal{T}_2$)$\longrightarrow$ (M,ρ_1 ,ρ_2) by λ (p)=q, λ (q)=p and λ (r)=r. Clearly λ is [contra _(1, 2)*_open and contra_(1, 2)*_sg*_open] function. But λ is not contra_(1, 2)*_(sg*,g)_open function , since for (1,2)*_sg*_open set S={r} in H , λ (S)= λ ({r})={r} is not (1,2)*_g_ closed in M.

(ii) Suppose H =M={p,q, r}, $\mathcal{T}_1$ ={H, ϕ}and $\mathcal{T}_2$ ={H, ϕ,{p}}, then the $\mathcal{T}_{1,2}$_open set s in (H,$\mathcal{T}_1$,$\mathcal{T}_2$) ={H , ϕ,{p}}and $\mathcal{T}_{1,2}$_closed in (H,$\mathcal{T}_1$,$\mathcal{T}_2$)={M,ϕ,{q,r}} .Define λ :(H,$\mathcal{T}_1$,$\mathcal{T}_2$) $\longrightarrow$ (H,$\mathcal{T}_1$,$\mathcal{T}_2$)by λ (p)=r,λ (q)=q and λ (r)=p . Clearly λ is a contra_(1, 2)*_(sg*,)_open function . But λ is not [contra _(1, 2)*_open and contra_(1, 2)*_sg*_open] function, since for $\mathcal{T}_{1,2}$_open set S={p} in H , λ (S)= λ ({p})={r} is not [$\mathcal{T}_{1,2}$_closed and is not (1,2)* _sg*_closed]set in M.

To make (4,6) true we must add another condition as we will notice in (4,8):

Proposition(4,8): If λ :(H,$\mathcal{T}_1$,$\mathcal{T}_2$) $\longrightarrow$ (M, ρ_1 ,ρ_2) is a contra_(1,2)*_(sg*,g)_open function and M is (1,2)*_$T_{1/2}$ space , then λ is

- **(i)** Contra _(1, 2)*_open function.
- **(ii)** Contra _(1, 2)*_sg*_open function .

Proof

(i): Let S be $\mathcal{T}_{1,2}$ – open set in H , since [all $\mathcal{T}_{1,2}$ – open set is (1,2)*_sg*_open] . Thu , S is(1,2)*_sg*_open in H . Also ,since λ is contra_(1, 2)*_(sg*,g)_open function .Thus λ (S) is(1,2)*_g_ closed in M ,since M is (1,2)*_$T_{1/2}$ space , then λ (S) is a $\rho_{1,2}$_closed set in M. Therefore , λ is contra _(1,2)*_open function. And in the same way, part (ii) can be proved

Proposition(4,9): If λ :(H,$\mathcal{T}_1$,$\mathcal{T}_2$) $\longrightarrow$ (M, ρ_1 ,ρ_2) is a contra_(1,2)*_open function and H is RM_ space , then λ is contra_(1,2)* _(sg*,g)_open function .

Proof : Suppose S is (1,2)* _sg*_open in H , since H is a RM _ space , then S is $\mathcal{T}_{1,2}$_open in H .Also , since λ is contra_(1,2)*_open function .Thus, λ (S) is $\rho_{1,2}$_closed in M and by using (2,7)step-ii- we obtain , λ (S) is (1,2)*_g_ closed in M. Therefore , λ is contra_(1,2)* _(sg*,g)_open function .

Proposition(4,10): If λ :(H,$\mathcal{T}_1$,$\mathcal{T}_2$) ⟶ (M, ρ_1 ,ρ_2) is a contra_(1,2)*_sg*_open function and H is RM _space , then λ is contra_(1,2)*_(sg*,g)_open function .

Proof : Suppose S is (1,2)* _sg*_open in H , since H is RM _ space , then S is $\mathcal{T}_{1,2}$_open in H .Also , since λ is contra_(1,2)* $sg*$_open function . Thus , λ (S) is a (1,2)* _sg*_ closed in M and by using Remark(2,7)step-iii-we obtain, λ (S) is (1,2)*_g_ closedin M. Therefore , λ is contra_(1,2)*_(sg*,g)_open .

The following anther type of contra_(1,2)*_(sg*,g)_open functions, which is called contra(1,2)*_(g ,sg*)_open function .

Definition(4,11): A function λ :(H,$\mathcal{T}_1$,$\mathcal{T}_2$) ⟶ (M, ρ_1 ,ρ_2) is said to be contra _(1,2)*_(g ,sg*)_open function if for every (1,2)*_g_ open set S in (H,$\mathcal{T}_1$,$\mathcal{T}_2$), λ (S) is (1,2)* _sg*_ closed set in (M, ρ_1 , ρ_2) .

Proposition(4,12): Every contra_(1,2)*_(g, sg*)_open function is contra_(1,2)* _sg**_open function .

Proof: Let S be (1,2)*_ sg*_open set in H , by Remark(2,7)step-iii-,we get S is (1,2)*_g_ open in H, since λ is a contra_(1, 2)* _(g ,sg*)_open function .Thus , λ (S) is(1,2)* _sg*_closed set in M . Therefore λ is contra _(1,2)* _sg**_open function.

Corollary (4,13): Every contra _(1, 2)*_(g, sg*)_open is

(i) Contra_(1,2)* _(sg*,g)_open function .

(ii) Contra_(1,2)*_sg*_open function .

proof:(i) It can be proven using proposition (4,12) and proposition(4,2).

(ii) It can be proven using proposition (4,12) and proposition(3,13).

The next Example show that the inverse of proposition(4,12) and Corollary(4,13) need not be true:

Example(4,14):

Suppose H =M={$\mathcal{p}$,$\mathcal{q}$,$\mathcal{r}$}, $\mathcal{T}_1$ ={H, ϕ,{$\mathcal{p}$}} , $\mathcal{T}_2$={H , ϕ}, ρ_1 ={M, ϕ,{$\mathcal{p}$}}, and ρ_2 ={M, ϕ, {$\mathcal{q}$},{$\mathcal{p}$,$\mathcal{q}$}}.Then, the $\mathcal{T}_{1,2}$_open in (H,$\mathcal{T}_1$,$\mathcal{T}_2$) ={H,ϕ,{$\mathcal{p}$}} , $\rho_{1,2}$_open set s in (M, ρ_1 ,ρ_2)={M, ϕ,{$\mathcal{p}$} ,{$\mathcal{q}$},{$\mathcal{p}$,$\mathcal{q}$}} and $\rho_{1,2}$_closed in (M, ρ_1 ,ρ_2)={ M , ϕ ,{$\mathcal{r}$} , {$\mathcal{p}$,$\mathcal{r}$} , {$\mathcal{q}$,$\mathcal{r}$} } . Define λ :(H , $\mathcal{T}_1$,$\mathcal{T}_2$) ⟶ (M, ρ_1 ,ρ_2) by λ ($\mathcal{p}$)=$\mathcal{r}$, λ ($\mathcal{q}$)=$\mathcal{q}$ and λ ($\mathcal{r}$)=$\mathcal{p}$. It is observe that λ is [contra -(1,2)*_sg**_open and contra_(1, 2)* _sg*_open and contra_(1,2)* _(sg*,g)_open function . But λ is not contra_(1, 2)* _(g ,sg*)_open function since for (1,2)*_g_ open set S={$\mathcal{q}$} in H , λ (S)= λ ({$\mathcal{q}$})={$\mathcal{q}$} is not (1,2)* _sg*_closed set in M.

To make (4,12) and(4,13) are true we must add another condition as we will notice in (4,15):

Proposition (4,15): A function λ :(H,$\mathcal{T}_1$,$\mathcal{T}_2$) ⟶ (M, ρ_1 ,ρ_2) is a contra_(1,2)*_(g ,sg*)_open function if H is (1,2)*_$T_{1/2}$ space and λ is

(i) Contra _(1, 2)* _sg*_open function.

(ii) Contra _(1, 2)*_sg**_open function .

Proof

(i): Suppose S is (1,2)*_g_ open in H, since H is (1,2)*_$T_{1/2}$ space , then S is $\mathcal{T}_{1,2}$_open set in H. Also, since λ is contra_(1, 2)*_sg*_open function .Thus λ (S) is(1,2)*_s$g*$_closed in M . Therefore , λ is contra _(1,2)* _sg*_open function. And in the same way ,part(ii)can be proved.

Proposition (4,16): If λ :(H,$\mathcal{T}_1$,$\mathcal{T}_2$) ⟶ (M, ρ_1 ,ρ_2) is a contra_(1,2)*_(sg*,g)_open function and H ,M are (1,2)*_$T_{1/2}$ spaces , then λ is contra_(1,2)*_(g, sg*)_open function.

Proof: Let S be (1,2)*_g_ open in H, since H is a(1,2)*_$T_{1/2}$ space , then S is $\mathcal{T}_{1,2}$_open in H and by (2,7) step-i- , we obtain S is (1,2)* _sg*_open in H . Also, since λ is contra_(1, 2)* _(sg*,g)_open function .Thus λ (S) is(1,2)*_g_ closed set in M, by hypotheses M is (1,2)*_$T_{1/2}$ space . Then , λ (S) is a $\rho_{1,2}$_closedin M and by (2,6)step-i- , we get λ (S) is (1,2)* _s$g*$_closed in M . Therefore , λ is contra _(1,2)* _sg*_open .

Remark(4,17): contra _(1,2)*_open functions and contra _(1,2)* _sg***_open functions are independent to contra_(1, 2)* _(g, sg*)_open functions . see the next Examples :

Example(4,18):

(i)Suppose H=M={p, q ,r}, $\mathcal{T}_1$ ={H ,ϕ,{p}}and $\mathcal{T}_2$ = {H ,ϕ,{p,r}} .Then , the $\mathcal{T}_{1,2}$_open in(H,$\mathcal{T}_1$,$\mathcal{T}_2$)={H , ϕ,{p},{p,r}}, and $\mathcal{T}_{1,2}$ _closed set s in (H,$\mathcal{T}_1$,$\mathcal{T}_2$) ={ H , ϕ,{q} ,{q ,r}}.Define :(H,$\mathcal{T}_1$,$\mathcal{T}_2$) ⟶(H,$\mathcal{T}_1$,$\mathcal{T}_2$)by λ (p)=q, λ (q)=p and λ (r)=r.Clearly λ is [contra_(1, 2)*_open and contra_(1, 2)* _sg***_open] function. But λ is not contra_(1, 2)*_(g ,sg*)_open function ,since for (1,2)*_g_open S={r} in H, λ (S)= λ ({r})={r} is not (1,2)* _sg*_closed set in M.

(ii) Let H =M={p, q , r} and let $\mathcal{T}_1$ = {H , ϕ} , $\mathcal{T}_2$ = {H , ϕ,{p ,r} }, then the $\mathcal{T}_{1,2}$_open set s in (H,$\mathcal{T}_1$,$\mathcal{T}_2$) = {H, ϕ,{p}} , ρ_1 ={M,ϕ,{p},{q, r}}, ρ_2 ={M,ϕ,{p ,r}}, then $\rho_{1,2}$ _open in (M , ρ_1 , ρ_2)={ M,ϕ,{p},{p ,r},{q,r}}, $\rho_{1,2}$ _closed in(M, ρ_1 , ρ_2)={M,ϕ,{p},{q},{q,r}},.Define λ :(H,$\mathcal{T}_1$,$\mathcal{T}_2$)⟶(M, ρ_1 ,ρ_2) by λ (p)=p , λ (q)=r and λ (r)=q . Clearly λ is contra __(1, 2)* _(sg*, g) _open nction. But λ is not [contra_(1,2)*_sg***_open and is not contra_(1,2)*_open]function , since for (1,2)* _sg*_open set S={p ,r} in H , λ (S)= λ ({p ,r})={p ,q} is not $\rho_{1,2}$_closed set in M.

To make (4,17) true we must add another condition as we will notice in (4,19):

Proposition (4,19):A function λ :(H,$\mathcal{T}_1$,$\mathcal{T}_2$) ⟶ (M, ρ_1 ,ρ_2) is a contra_(1,2)*_(g ,sg*)_open function if H is (1,2)*_$T_{1/2}$ space , and λ is

(i) Contra _(1, 2)*_open function. ,

(ii)Contra _(1, 2)* _sg***_open function .

Proof

(**i**):Let S is (1,2)*_g_ open in H, by hypotheses H is (1,2)*_$T_{1/2}$ space .Thus,S is $\mathcal{T}_{1,2}$_open in H. Also, since λ is contra_(1, 2)* _open .This lead λ(S) is $\rho_{1,2}$ _closed in M (since all $\rho_{1,2}$ _closed is(1,2)*_sg*_closed).Hence, λ (S) is (1,2)*_sg*_closedin M. Therefore , λ is contra_(1,2)*_(g ,sg*)_open function .

And in the same way , part(ii)can be proved.

Proposition (4,20): If λ :(H,$\mathcal{T}_1$,$\mathcal{T}_2$) ⟶ (M, ρ_1 ,ρ_2) is a contra_(1,2)* _(g ,sg*)_open function and M is a (1,2)*_ RM_ space , then λ is

(i) Contra_(1,2)*_open function ,

(ii) Contra (1,2)* _sg***_open function .

Proof

(**i**):Let S be $\mathcal{T}_{1,2}$_open in H , by (2,7) step-ii- we get , S be (1,2)*_g_ open in H ,and since λ is contra_(1,2)* _(g ,sg*)_open function .Thus , λ (S) is (1,2)* _sg*_closed in M . BY hypotheses M is RM _ space , then , λ (S) is $\rho_{1,2}$_closed in M Therefore , λ is contra_(1,2)*_open function. And in the same way ,part(ii)can be proved.

Some properties and results about the composition of contra_(1,2)*_(sg*, g)_open functions and contra_(1,2)*_(g ,sg*)_open functions will be Given in the following .

Proposition(4,21):

Let λ :(H , $\mathcal{T}_1$, $\mathcal{T}_2$) ⟶ (M, ρ_1 , ρ_2)**is** pre_(1,2)* _sg*_open function and γ : (M, ρ_1 , ρ_2) ⟶(N , ξ_1, ξ_2) be contra_(1,2)*_(sg*,g)_open function, then $\gamma\circ\lambda$:(H,$\mathcal{T}_1$,$\mathcal{T}_2$)⟶(N , ξ_1, ξ_2) is contra _(1,2)*_(sg*,g)_open .

Proof:-suppose S is (1,2)* _sg*_open in H, .Thus , λ (S) is(1,2)* _sg*_open in M . Also , since γ is contra _(1,2)*_(sg*,g)_open function , then γ (λ (S)) is (1,2)* _g_closed set in N. But, $\gamma(\lambda$(S))= $\gamma\circ\lambda$ (S).. Therefore , $\gamma\circ\lambda$:(H,$\mathcal{T}_1$,$\mathcal{T}_2$)⟶(N , ξ_1, ξ_2) is contra_(1,2)*_(sg*,g)_open .

Proposition(4,22):

Let λ :(H , $\mathcal{T}_1$, $\mathcal{T}_2$) ⟶ (M, ρ_1 , ρ_2) be a pre-(1,2)* _sg*_open function and γ : (M, ρ_1 , ρ_2) ⟶(N , ξ_1, ξ_2) be contra_(1,2)*_(g ,sg*)_open , then $\gamma\circ\lambda$:(H,$\mathcal{T}_1$,$\mathcal{T}_2$)⟶(N , ξ_1, ξ_2) is contra_(1,2)*_sg**_open function

Proof:- Let S be a (1,2)* _sg*_open in H, since λ is a pre_ (1,2)* _sg*_open function .Thus , λ (S) is(1,2)*_–sg*_open in M and by Remark(2,7) step-iii- we get , λ (S) is (1,2)*_g_ open in M. Also , since γ is contra _(1,2)*_(g ,sg*)_open function , then γ (λ (S)) is (1,2)* _sg*_closed in N. But, $\gamma(\lambda$(S))= $\gamma\circ\lambda$ (S). Therefore , $\gamma\circ\lambda$:(H,$\mathcal{T}_1$,$\mathcal{T}_2$) ⟶(N , ξ_1, ξ_2) is contra _(1,2)*_sg**_open function.

Corollary(4,23): Let λ :(H, $\mathcal{T}_1$,$\mathcal{T}_2$)⟶(M,ρ_1 , ρ_2) be a pre_(1,2)*_sg*_open function and γ :(M,ρ_1 , ρ_2) ⟶(N , ξ_1, ξ_2) be contra_(1,2)*_(g ,sg*)_open function, then $\gamma\circ\lambda$:(H,$\mathcal{T}_1$,$\mathcal{T}_2$)⟶(N, ξ_1,ξ_2) is

(i) Contra_(1,2)*_sg*_open function . , (ii) Contra_(1,2)*_(sg*,g)_open .

proof(i):It is follows from (4,22) and (3,13).

proof(ii):It is follows from (4,22) and (4,2).

Proposition(4,24): suppose λ:(H,$\mathcal{T}_1$,$\mathcal{T}_2$) ⟶(M, ρ_1 , ρ_2)be any function and γ: (M, ρ_1 , ρ_2)⟶(N , ξ_1, ξ_2) be contra_(1,2)*_(g ,sg*)_open function, then $\gamma\circ\lambda$:(H,$\mathcal{T}_1$,$\mathcal{T}_2$)⟶(N,ξ_1, ξ_2)is contra_(1,2)*_sg*_open . If λ is

(i) (1,2)* _open nction.

(ii) (1,2)* _sg*_open nction.

(iii) (1,2)*_g_open nction.

Proof

(i): Suppose S is $\mathcal{T}_{1,2}$_open in H .Thus,λ (S) is $\rho_{1,2}$_open in M, by (2,7)step-ii- we get, λ (S) is (1,2)*_g_open in M Also , since γ is a contra_(1,2)_(g ,sg*)_open , then γ (λ (S))=$\gamma\circ\lambda$ (S) is a (1,2)*_sg*_ closed in N . Therefore , $\gamma\circ\lambda$:(H,$\mathcal{T}_1$,$\mathcal{T}_2$) ⟶(N , ξ_1, ξ_2) is contra(1,2)*_sg*_open function . And in the same way ,part(ii)can be proved.

Proposition (4 ,25): Let λ : (H , $\mathcal{T}_1$, $\mathcal{T}_2$) ⟶ (M, ρ_1 , ρ_2) be a contra_(1,2)*_(g ,sg*) _open function and γ :(M, ρ_1 , ρ_2) ⟶(N , ξ_1, ξ_2) be a pre_(1,2)*_sg*_closed function, then $\gamma\circ\lambda$:(H,$\mathcal{T}_1$,$\mathcal{T}_2$)⟶(N , ξ_1, ξ_2) is contra_(1,2)*_(g ,sg*)_open function.

Proof:- Suppose S is (1,2)*_g_ open in H, since λ is contra_ (1,2)* _(g , sg*)_open function. Thus , λ (S) is(1,2)*_sg*_closed in M Also , since γ is pre (1,2)*_sg*_closed ,then γ (λ (S)) is (1,2)*_sg*_closed set in N. But, (λ (S))=$\gamma\circ\lambda$ (S).Therefore , $\gamma\circ\lambda$:(H,$\mathcal{T}_1$,$\mathcal{T}_2$) ⟶(N , ξ_1, ξ_2) is contra _(1,2)*_(g ,sg*)_open.

Corollary(4,26): Let λ:(H, $\mathcal{T}_1$,$\mathcal{T}_2$)⟶(M, ρ_1 , ρ_2) be a contra _(1,2)*_(g ,sg*) _open function and γ :(M, ρ_1 , ρ_2) ⟶(N , ξ_1, ξ_2) be a pre_(1,2)* _sg*_closed function, then $\gamma\circ\lambda$:(H,$\mathcal{T}_1$,$\mathcal{T}_2$)⟶(N , ξ_1, ξ_2) is

(i) Contra_(1,2)*_sg*_open function . ,

(ii) Contra_(1,2)*_sg**_open .

Proof

(i)**:** Let S is $\mathcal{T}_{1,2}$_open in H ., by (2,7)step-ii- we get, λ (S) is (1,2)*_g_open in H, since λ is contra_ (1,2)* _(g , sg*)_open function. Thus , λ (S) is(1,2)*_sg*_closed in M. Also , since γ is pre (1,2)*_sg*_closed ,then γ (λ (S)) is (1,2)*_sg*_closed set in N. But, (λ (S))=$\gamma \circ \lambda$ (S).Therefore , $\gamma \circ \lambda$:(H,$\mathcal{T}_1$,$\mathcal{T}_2$) ⟶(N , ξ_1, ξ_2) is contra _(1,2)*_sg*_open function.

And in the same way ,part(ii)can be proved.

Remark(4,27):

(i) If λ :(H, $\mathcal{T}_1$,$\mathcal{T}_2$)⟶(M,ρ_1 , ρ_2) is (1,2)*_open [resp.(1,2)*_sg*_open ,(1,2)*_g_ open] function and γ :(M,ρ_1 , ρ_2)⟶(N, ξ_1, ξ_2) is contra_(1,2)*_(sg* ,g) _open function, then $\gamma \circ \lambda$:(H,$\mathcal{T}_1$, $\mathcal{T}_2$)⟶(N , ξ_1, ξ_2) is not necessarily contra_(1,2)*_(sg* ,g)_open function .

(ii) If λ :(H, $\mathcal{T}_1$,$\mathcal{T}_2$)⟶(M,ρ_1 , ρ_2) is (1,2)*_open [(1,2)*_sg*_open , (1,2)*_g_ open , pre-(1,2)*_sg*_open]function and γ :(M, ρ_1 , ρ_2) ⟶ (N , ξ_1 , ξ_2) is a contra_(1,2)*_ (g , sg*) _open function , then $\gamma \circ \lambda$:(H,$\mathcal{T}_1$, $\mathcal{T}_2$)⟶(N , ξ_1, ξ_2) is not necessarily contra_(1,2)*_(g ,sg*)_open function .

To make (4,27) true we must add another condition as we will notice in (4,28):

Proposition (4,28): Let λ :(H,$\mathcal{T}_1$, $\mathcal{T}_2$) ⟶ (M, ρ_1 , ρ_2) be any function and γ : (M, ρ_1 , ρ_2) ⟶(N , ξ_1, ξ_2) be contra_(1,2)*_(sg*, g)_open function, then $\gamma \circ \lambda$:(H,$\mathcal{T}_1$, $\mathcal{T}_2$)⟶(N,ξ_1, ξ_2)is contra_(1,2)*_(sg*,g)_open . If H is RM_ space and λ is

(i) (1,2)* _open nction. , (ii) (1,2)* _sg*_open function.

Proof

(i): suppose S is (1,2)* _sg*_open in H .Since H is RM_ space , then by S is a $\mathcal{T}_{1,2}$_open in H . Also since λ is (1,2)*_open function. Thus, λ (S) is $\rho_{1,2}$_open s in M, by Remark(2,7) step-ii-we get, λ (S) is (1,2)* _sg*_open in M. Also , since γ is a contra_(1,2)*_(sg* ,g)_open , then γ(λ (S)) is (1,2)*_g_ closed in N. But , γ (λ (S))=$\gamma \circ \lambda$ (S). Therefore , $\gamma \circ \lambda$:(H,$\mathcal{T}_1$, $\mathcal{T}_2$) ⟶(N , ξ_1, ξ_2) is contra(1,2)* _(sg*,g)_open function . And in the same way ,part(ii)can be proved.

In the same way we prove proposition (4,29):

Proposition(4,29): If λ:(H,$\mathcal{T}_1$, $\mathcal{T}_2$) ⟶(M,ρ_1 , ρ_2) is(1,2)*_g_ open function and γ :(M,ρ_1, ρ_2) ⟶(N ,ξ_1,ξ_2) is contra_(1,2)*_(sg*, g)_open function, then $\gamma \circ \lambda$:(H,$\mathcal{T}_1$, $\mathcal{T}_2$)⟶(N,ξ_1, ξ_2)is contra_(1,2)*_(sg* ,g)_open if H is RM_ space and M is (1,2)*_$T_{1/2}$_space .

Proposition (4,30): Let λ :(H,$\mathcal{T}_1$, $\mathcal{T}_2$) ⟶ (M, ρ_1 , ρ_2) be any function, γ : (M, ρ_1 , ρ_2) ⟶(N , ξ_1, ξ_2) be contra_(1,2)*_(g ,sg*)_open function, and H is (1,2)*_$T_{1/2}$ _space , then $\gamma \circ \lambda$:(H,$\mathcal{T}_1$, $\mathcal{T}_2$)⟶(N,ξ_1, ξ_2)is contra_(1,2)*_(g ,sg*)_open . If H is RM_ space and λ is

(i) (1,2)*_open nction.

(ii) (1,2)* _sg*_open nction.

(iii) (1,2)*_g_ open nction.

(iv) pre_(1,2)*_sg*_open function.

Proof:

(i) Let S be a (1,2)*_g_ open set in H .Since H is (1,2)*_$T_{1/2}$ _space, then S is a $\mathcal{T}_{1,2}$_open in H . Also since λ is (1,2)*_open . Thus, λ (S) is $\rho_{1,2}$_open in M, by (2,7)step-ii-we get, λ (S) is (1,2)*_g_open in M Also, since γ is contra_(1,2)*_(g ,sg*)_open , then γ (λ (S)) is a (1,2)*_sg*_ closed in N. But, γ(λ (S))=$\gamma \circ \lambda$(S).Therefore , $\gamma \circ \lambda$:(H,$\mathcal{T}_1$, $\mathcal{T}_2$)⟶(N, ξ_1, ξ_2) is contra(1,2)*_(g, sg*)_open function .

The proof of part-ii- ,-iii- ,and-iv- are similar to part-i- .

Remark (4,31):

(i) If λ :(H, $\mathcal{T}_1,\mathcal{T}_2$) ⟶(M, ρ_1,ρ_2) is contra_(1,2)*_(sg* ,g) _open function and γ :(M,ρ_1,ρ_2)⟶(N, ξ_1,ξ_2) is (1,2)*_closed [(1,2)*_sg*_closed ,(1,2)*_g_ closed and pre_(1,2)*_sg*_closed] function ,then $\gamma\circ\lambda$:(H,$\mathcal{T}_1,\mathcal{T}_2$)⟶(N , ξ_1,ξ_2) is not necessarily contra_(1,2)*_(sg* ,g)_open function.

(ii) If λ :(H,$\mathcal{T}_1,\mathcal{T}_2$)⟶(M, ρ_1,ρ_2) is contra_(1,2)*_(g, sg*) _open function and γ :(M,ρ_1,ρ_2)⟶(N, ξ_1,ξ_2) is (1,2)*_closed [(1,2)*_sg*_closed , (1,2)*_g_ closed] function and , then $\gamma\circ\lambda$:(H,$\mathcal{T}_1,\mathcal{T}_2$)⟶(N , ξ_1,ξ_2) is not necessarily contra_(1,2)*_(g, sg*)_open function .

The following Examples to show that :

Example(4,32):

(i) Let H=M=N={p, q, r} and let $\mathcal{T}_1$={H,ϕ,{p}}, $\mathcal{T}_2$={H,ϕ,{p ,r}}, ρ_1 ={M,ϕ}, and ρ_2 ={M,ϕ ,{p}}, ξ_1={N,ϕ,{p},{p, r}}, ξ_2={N,ϕ,{q, r}}, then $\mathcal{T}_{1,2}$_open set s in (H,$\mathcal{T}_1,\mathcal{T}_2$)={H, ϕ,{$p$},{$p$,r}}, $\rho_{1,2}$ _open set s in (M, ρ_1,ρ_2)={M,ϕ,{p}}, $\rho_{1,2}$_closed in M={M,ϕ,{q ,r}}, $\xi_{1,2}$_open set s in (N,ξ_1,ξ_2) ={N,ϕ ,{p},{p ,r},{q ,r}}, $\xi_{1,2}$_closed set s in (N, ξ_1,ξ_2)={N,ϕ,{p},{q},{q,r}}and define λ :(H, τ_1,τ_2) ⟶(M, ρ_1,ρ_2) by λ (p)= r , λ (q)=q and λ (r)=p and define γ :(M,ρ_1,ρ_2)⟶(N,ξ_1,ξ_2)by γ (p)=p, γ (q)=q and γ (r)=r. It is observe that λ is contra_(1,2)*_(sg* ,g) _open function and γ is(1,2)*_closed [(1,2)*_sg*_closed , (1,2)*_g_ closed, pre_(1,2)*_sg*_closed] function, but $\gamma\circ\lambda$:(H,$\mathcal{T}_1,\mathcal{T}_2$)⟶(N , ξ_1,ξ_2) is not contra_(1,2)*_(sg* ,g)_open function , since for(1,2)*_sg*_open set S={p} in H , $\gamma\circ\lambda$ (S)= $\gamma\circ\lambda$ ({p})= γ (λ ({p})= γ (r)=r is not (1,2)*_g_ closed set s in N

(ii)Let H=M=N={p,q,r},$\mathcal{T}_1$={H,ϕ,{p}}, $\mathcal{T}_2$={H,ϕ,{p ,r}}, ρ_1 ={M,ϕ,{q}}, ρ_2 ={M,ϕ,{p, r}}, ξ_1={N,ϕ,{q, r}} and ξ_2={N,ϕ,{p},{p, r}}, then $\mathcal{T}_{1,2}$_open set s in(H,$\mathcal{T}_1,\mathcal{T}_2$)={H,ϕ,{$p$},{$p$,r}},$\rho_{1,2}$ _open set s in (M, ρ_1,ρ_2)=$\rho_{1,2}$_closed in M={M,ϕ,{p},{q ,r}}, $\xi_{1,2}$_open set s in (N, ξ_1,ξ_2)={N,ϕ,{p},{p, r},{q, r}}, $\xi_{1,2}$_closed in (N,ξ_1,ξ_2)={N,ϕ,{p},{q},{q, r}},. Define λ :(H,τ_1,τ_2) ⟶(M, ρ_1,ρ_2) by λ (p)= r, λ (q)=q λ(r)=p and γ:(M,ρ_1,ρ_2)⟶(N,ξ_1,ξ_2) by γ(p)=p, γ(q)=q and γ(r)=r.Clearly λ is contra_(1,2)*_(g, sg*) _open function and γ is (1,2)*_closed [(1,2)*_sg*_closed , (1,2)*_g_ closed] function and , then sg*)_open . But $\gamma\circ\lambda$:(H,$\mathcal{T}_1,\mathcal{T}_2$)⟶(N, ξ_1,ξ_2) is not contra_(1,2)*_(g, sg*)_open , since for(1,2)*_g_open set S={p} in H , $\gamma\circ\lambda$ (S)=$\gamma\circ\lambda$ ({p})= γ (λ ({p})= γ (r)=r is not (1,2)*_sg*_closed set s in N .

To make (4,31) true we must add another condition as we will notice in (4,33):

Proposition(4,33): Let λ :(H, $\mathcal{T}_1,\mathcal{T}_2$) ⟶(M, ρ_1,ρ_2) is contra_(1,2)*_(sg*,g)_open function and γ:(M,ρ_1,ρ_2)⟶(N,ξ_1,ξ_2)be(1,2)*_closedand M is (1,2)*_$T_{1/2}$space, then $\gamma\circ\lambda$:(M,$\mathcal{T}_1,\mathcal{T}_2$)⟶(N, ξ_1,ξ_2) is

(i)contra_(1,2)*_sg***_open function .

(ii)contra_(1,2)*_(sg* ,g)_open function .

Proof:

(i) Suppose S is (1,2)* _sg*_open in H . Thus, λ (S) is (1,2)*_g_ closed in M, by hypotheses M is (1,2)*_$T_{1/2}$ _space, then λ (S) is $\rho_{1,2}$_closed in M. Also , since γ is (1,2)*_closed , then γ (λ (S))=$\gamma\circ\lambda$ (S) is a $\xi_{1,2}$_closed in N . Therefore , $\gamma\circ\lambda$:(H,$\mathcal{T}_1,\mathcal{T}_2$) ⟶(N , ξ_1,ξ_2) is contra(1,2)* _sg***_open .And in the same way ,part(ii)can be proved.

In the same way ,we will prove (4,34) :

Corollary(4,34):

If λ:(H,$\mathcal{T}_1$,$\mathcal{T}_2$) ⟶(M, ρ_1 , ρ_2) is contra_(1,2)*_(sg*,g) _open function ,and γ:(M,ρ_1 , ρ_2)⟶(N, ξ_1, ξ_2) be any function and M is (1,2)*_$T_{1/2}$ space ,then $\gamma \circ \lambda$:(H,$\mathcal{T}_1$,$\mathcal{T}_2$)⟶(N , ξ_1, ξ_2) is contra_(1,2)*_sg**_open function if

(i) γ is (1,2)*_sg*_closed function. ,

(ii) γ is pre_(1,2)*_sg*_closedfunction.

Corollary(4,35):

If λ :(H, $\mathcal{T}_1$, $\mathcal{T}_2$)⟶(M,ρ_1 , ρ_2) is contra_(1,2)*_(sg*,g) _open function, γ :(M,ρ_1 , ρ_2)⟶(N, ξ_1, ξ_2) be (1,2)*_ g_ closed function and M is (1,2)*_$T_{1/2}$ space , then $\gamma \circ \lambda$:(H,$\mathcal{T}_1$,$\mathcal{T}_2$)⟶(N , ξ_1, ξ_2) is contra_(1,2)*_(sg* ,g)_open function.

Proof : Suppose S is (1,2)* _sg*_open in H . Thus, λ (S) is (1,2)*_g_ closed in M, by hypotheses M is (1,2)*_$T_{1/2}$ _space, then λ (S) is $\rho_{1,2}$_closed in M. Also , since γ is (1,2)*_ g _closed , then γ (λ (S))=$\gamma \circ \lambda$ (S) is a $\xi_{1,2}$_closed in N [since all (1,2)*_closed is (1,2)*_ g _closed]set , so we get γ (λ (S))=$\gamma \circ \lambda$ (S) is (1,2)*_ g _closed in N . Therefore , $\gamma \circ \lambda$:(H,$\mathcal{T}_1$,$\mathcal{T}_2$) ⟶(N , ξ_1, ξ_2) is contra_(1,2)*_(sg* ,g)_open function.

Proposition(4,36):

If λ :(H,$\mathcal{T}_1$,$\mathcal{T}_2$)⟶(M, ρ_1 , ρ_2) is contra_(1,2)*_(g ,sg*)_open , γ:(M,ρ_1 , ρ_2)⟶(N, ξ_1, ξ_2) is (1,2)*_closedfunction and M is RM_space ,then $\gamma \circ \lambda$:(H,$\mathcal{T}_1$,$\mathcal{T}_2$)⟶(N , ξ_1, ξ_2)is

(i) contra_(1,2)*_(g, sg*)_open function .

(ii) contra_(1,2)*_sg**_open function .

Proof

(i):suppose S is (1,2)*_g_ open in H . Thus, λ (S) is (1,2)*_sg*_closed in M, by hypotheses M is RM _space, then λ (S) is $\rho_{1,2}$_ closed in M. Also , since γ is (1,2)*_closed , then γ (λ (S))= $\gamma \circ \lambda$ (S) is a $\xi_{1,2}$_ closed in N and by using Remark(2,6) step-i- we get $\gamma \circ \lambda$ (S) is (1,2)* _sg*_closed set in N. Therefore , $\gamma \circ \lambda$:(H,$\mathcal{T}_1$,$\mathcal{T}_2$) ⟶(N , ξ_1, ξ_2) is contra(1,2)*_(g ,sg*)_open function .

And in the same way ,part(ii)can be proved.

In the same way we prove (4,37) :

Corollary(4,37):

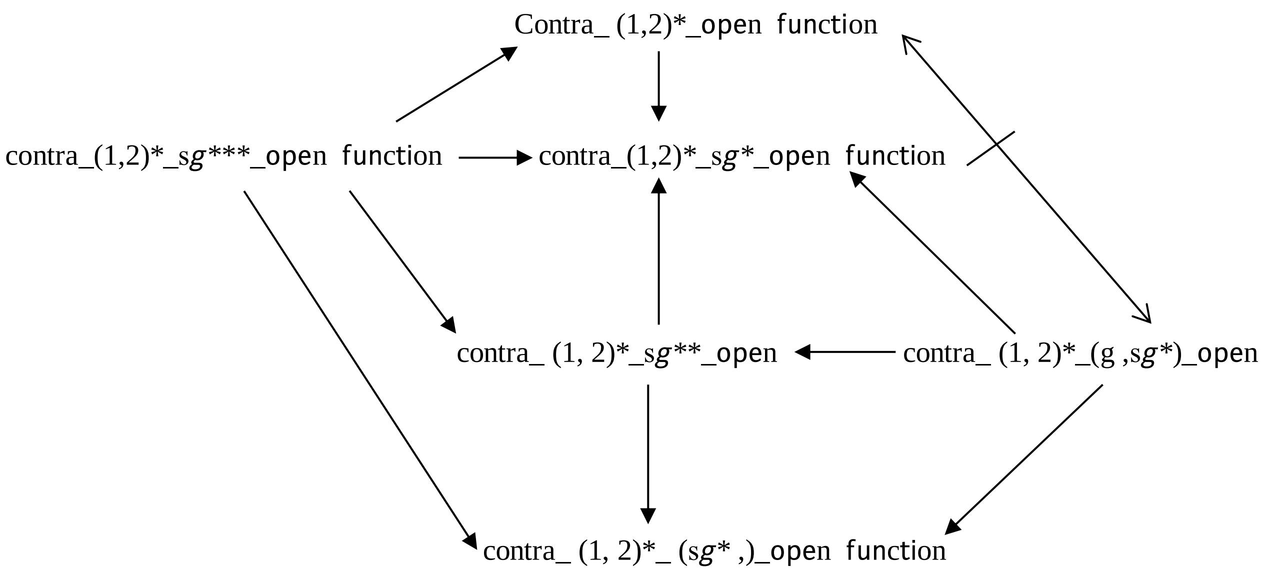
If λ :(H,$\mathcal{T}_1\mathcal{T}_2$) ⟶(M, ρ_1 , ρ_2) is contra_(1,2)*_(g ,sg*)_open function, γ :(M,ρ_1 , ρ_2)⟶(N, ξ_1, ξ_2) is any function and M is RM_space, then $\gamma \circ \lambda$:(H,$\mathcal{T}_1$,$\mathcal{T}_2$)⟶(N , ξ_1, ξ_2)is contra_(1,2)*_(g ,sg*)_open if γ is a

i- (1,2)*_sg*_closed function.

ii- (1,2)*_g_ closed function .

Remark (4,38):

Here in the following diagram illustrates the relation between the contra_(1,2)*_sg*_open functions types (without using condition), where the converse is not necessarily true.



Conclusion :

This work has led to find a new types of contra_(1,2)*_open functions in bitopological spaces , it also compare and investigated the relationships between these types of functions , and also several Definitions and results were presented to study the characteristics of those functions .

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