On Jordan Triple High Derivations on Prime Γ-Semirings

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Abstract

In this study, we offer an emerging notion for triple high derivation, the third of which is triple high derivation on a semiring of Γ- According to our result, if the criteria on M are met, then any Jordan triple high derivation on M is a triple high derivation on M.
Keywords: derivation, high derivation, Jordan triple high derivation.

حول جوردان للمشتقات الثلاثية العليا على شبة الحلقات الاولية من نمط Γ−

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الملخص

في هذه الدراسة، نقدم مفهوماً جوردان للمشتقات الثلاثية العليا على شبة الحلقات الاولية من نمط Γ− ، هو اشتقاق ثلاثي مرتفع في نصف Γ− وفقاً لنتائجنا ، إذا تم استيفاء المعايير علىM ، فإن أي اشتقاق مرتفع لجوردان على M هو اشتقاق ثلاثي على M.

الكلمات المفتاحية : المشتقة، المشتقة العليا، جوردان للمشتقة الثلاثية العليا.

1. Introduction:

Through the present paper M will denote an associative Γ-semiring, the set of natural numbers including 0 will be denoted by N. [, ,] denotes the usual commutator operator such that $[a,b,c]_{\alpha,\beta}= a\alpha b\beta c-c\alpha b\beta a$, for all a,b,c∈M and α,β∈Γ.

The definition of prime Γ-semiring and semi-prime Γ-semiring was introduced in [2]. The definition of 2-torsion free Γ-semiring was introduced in [2].The definitions with multiplication of commutative, identity element with invertible are introduce in [7].The definitions with additive of abelian , identity element and invertible are introduce in [6].

In an attempt to generalize Hersteins result for high derivations, C.Haetinger [3] proved that on a prime semiring with 2-torsion free every Jordan high derivation is a high derivation.

Now the main purpose of this paper is to extend this result for triple high derivations in Γ-semirings.

We need the following lemma.

Lemma 1.1: [8]

Let M be a two-torsion free semi prime Γ-semiring with additive identity and inverse and supposing that a,b∈M, if aΓmΓb + bΓmΓa =0 for any m∈M, then aΓmΓb = bΓmΓa = 0.

2. Triple High Derivations on Prime Γ-Semirings:

Definition 2.1:

Let M be a Γ-semiring and $D= (d_i)_{i\in N}$ is an additive mapping family of M such that $d_0=Id_M$. Then D is a triple high derivations of M if for each n∈N , we get :

$$d_n(a\alpha b\beta c) = \sum_{i+j+k=n} d_i(a)\alpha\, d_j(b)\beta d_k(c) \quad ... (i)$$

For any a,bi,c ∈M and α,β ∈Γ.

D is a Jordan triple-high derivations of M if for each $n \in N$, we get :

$d_n(a\alpha b\beta a) = \sum_{i+j+k=n} d_i(a)\alpha\, d_j(b)\beta d_k(a)$ ... (ii)

For any $a,b \in M$ and $\alpha,\beta \in \Gamma$.

Lemma1:

Let M be a Γ-semiring and $D = (d_i)_{i\in N}$ is Jordan triple high derivations of M. Then for any $a,b,c \in M$; $\alpha,\beta \in \Gamma$ and $n \in N$:

$d_n(a\alpha b\beta c + c\alpha b\beta a) = \sum_{i+j+k=n} d_i(a)\alpha\, d_j(b)\beta d_k(c) + d_i(c)\alpha d_j(b)\beta d_k(a)$

Proof: Substitute a+c for a in definition (2.1)(ii)

$d_n((a+c)\alpha b\beta(a+c)) = \sum_{i+j+k=n} d_i(a+c)\alpha d_j(b)\beta d_k(a+c)$

$= \sum_{i+j+k=n}(d_i(a) + d_i(c))\alpha d_j(b)\beta(d_k(a) + d_k(c)) = \sum_{i+j+k=n} d_i(a)\alpha d_j(b)\beta d_k(a) +$
$d_i(a)\alpha d_j(b)\beta d_k(c) + d_i(c)\alpha d_j(b)\beta d_k(a) + d_i(c)\alpha d_j(b)\beta d_k(c)$ ... (1)

On the other hand

$d_n((a+c)\alpha b\beta(a+c)) = d_n(a\alpha b\beta a + a\alpha b\beta c + c\alpha b\beta a + c\alpha b\beta c)$

$= \sum_{i+j+k=n} d_i(a)\alpha d_j(b)\beta d_k(a) + d_i(ic)\alpha(b)\beta d_k(c) + d_n(a\alpha b\beta c + c\alpha b\beta a)$ ...(2)

Comparing (1) and (2) ,we get:

$d_n(a\alpha b\beta c + c\alpha b\beta a) = \sum_{i+j+k=n} d_i(a)\alpha d_j(b)\beta d_k(c) + d_i(c)\alpha d_j(b)\beta d_k(a)$.

Ramerk2.2:

Let $D = (d_i)_{i\in N}$ be a Jordan triple high derivations of Γ-semiring M with additive identity and inverse .for eveiry $n \in N$ and for each $a,b,c \in M$; $\alpha,\beta \in \Gamma$ we define

$\Psi_n(a,b,c)_{\alpha,\beta} = d_n(a\alpha b\beta c) - \Sigma_{i+j+k=n} d_i(a)\alpha d_j(b)\beta d_k(c)$

Lemma 2:

Let $D = (d_i)_{i\in N}$ be a Jordan triple-high derivations of Γ-semiring M with additive commutative, identity and inverse. Then for any $a,b,c,x \in M$; $\alpha,\beta,\gamma \in \Gamma$ and $n \in N$ then:

i) $\Psi_n(a,b,c)_{\alpha,\beta} = -\Psi_n(c,b,a)_{\alpha,\beta}$

ii) $\Psi_n(a+x,b,c)_{\alpha,\beta} = \Psi_n(a,bb,cc)_{\alpha,\beta} + \Psi_n(x,b,c)_{\alpha,\beta}$

iii) $\Psi_n(ai,i\, b+x,c)_{\alpha,\beta} = \Psi_n(a,bb,cc)_{\alpha,\beta} + \Psi_n(a,x,c)_{\alpha,\beta}$

iv) $\Psi_n(ai,i\, b,c+x)_{\alpha,\beta} = \Psi_n(a,bb,cc)_{\alpha,\beta} + \Psi_n(a,b,x)_{\alpha,\beta}$

v) $\Psi_n(a,bb,cc)_{\alpha,\beta+\gamma,\sigma} = \Psi_n(a,bb,cc)_{\alpha,\beta} + \Psi_n(a,b,c)_{\gamma,\sigma}$

Proof: i) By lemma1 and since d_n is additive mapping for each $n \in N$ then:

$d_n(a\alpha b\beta c + c\alpha b\beta a) = \sum_{i+j+k=n} d_i(a)\alpha d_j(b)\beta d_k(c) + d_i(c)\alpha d_j(b)\beta d_k(a)$

$d_n(a\alpha b\beta c) + d_n(c\alpha b\beta a) = \sum_{i+j+k=n} d_i(a)\alpha d_j(b)\beta d_k(c) + \sum_{i+j+k=n} d_i(c)\alpha d_j(b)\beta d_k(a)$
$d_n(a\alpha b\beta c) - \sum_{i+j+k=n} d_i(a)\alpha d_j(b)\beta d_k(c) = -d_n(c\alpha b\beta a) + \sum_{i+j+k=n} d_i(c)\alpha d_j(b)\beta d_k(a)$
$\Psi_n(a,b,c)_{\alpha,\beta} = -\Psi_n(c,b,a)_{\alpha,\beta}$
ii) $\Psi(a+x,b,c)_{\alpha,\beta} = d_n((a+x)\alpha b\beta c) - \sum_{i+j+k=n} d_i(a+x)\alpha d_j(b)\beta d_k(c)$
$= di_n(a\alpha b\beta c + x\alpha b\beta c) - \sum_{i+j+k} d_i(a)\alpha d_j(b)\beta d_k(c) + d_i(x)\alpha d_j(b)\beta d_k(c)$
$= d_n(a\alpha b\beta c) + d_n(x\alpha b\beta c) - \sum_{i+j+k=n} d_i(a)\alpha d_j(b)\beta d_k(c) - \sum_{i+j+k=n} d_i(x)\alpha d_j(b)\beta d_k(c)$
$= d_n(a\alpha b\beta c) - \sum_{i+j+k=n} d_i(a)\alpha d_j(b)\beta d_k(c) + d_n(x\alpha b\beta c) - \sum_{i+j+k=n} d_i(x)\alpha d_j(b)\beta d_k(c)$
$= \Psi(a,b,c)_{\alpha,\beta} + \Psi_n(x,b,c)_{\alpha,\beta}$.
iii),(iv) As the same way of (ii).

v) As the same way of (i) by interchanging $\alpha,\beta+\gamma,\tau$ by interchanging α & β in (i).

Remark 2.3:

Note that $D=(d_i)_{i\in N}$ is a triple high derivations of a Γ-semiring M with additive identity and inverse iff $\Psi_n(a,b,c)_{\alpha,\beta}=0$ for any $a,b,c\in M$; $\alpha,\beta\in\Gamma$ and $n\in N$.

Now, we prove some lemmas which make us able to give the next results.

3. The Main Results

Lemma3:

Let $D=(d_i)_{i\in N}$ be a Jordan triple high derivations of a Γ-semiring M with additive identity and inverse , assume that $n\in N$, $a,b,c,m\in M$; $\alpha,\beta,\gamma,\tau\in\Gamma$ if $\Psi_t(a,b,c)_{\alpha,\beta}=0$ for each $t<n$ then:

i) $\Psi_n(a,b,c)_{\alpha,\beta}\tau m\tau[a,b,c]_{\alpha,\beta}+[a,b,c]_{\alpha,\beta}\tau m\tau\Psi_n(a,b,c)_{\alpha,\beta}=0$

ii) $\Psi_n(a,b,c)_{\alpha,\beta}\beta m\beta[a,b,c]_{\alpha,\beta}+[a,b,c]_{\alpha,\beta}\beta m\beta\Psi_n(a,b,c)_{\alpha,\beta}=0$

iii) $\Psi_n(a,b,c)_{\alpha,\beta}\alpha m\alpha[a,b,c]_{\alpha,\beta}+[a,b,c]_{\alpha,\beta}\alpha m\alpha\Psi_n(a,b,c)_{\alpha,\beta}=0$

iv) $\Psi_n(a,b,c)_{\alpha,\alpha}\alpha m\alpha[a,b,c]_{\alpha,\alpha}+[a,b,c]_{\alpha,\alpha}\alpha m\alpha\Psi_n(a,b,c)_{\alpha,\alpha}=0$

v) $\Psi_n(a,b,c)_{\alpha,\beta+\gamma,\tau}\alpha m\alpha[a,b,c]_{\alpha,\beta+\gamma,\tau}+[a,b,c]_{\alpha,\beta+\gamma,\tau}\alpha m\alpha\Psi_n(a,b,c)_{\alpha,\beta+\gamma,\tau}=0$

Proof:i) By using Definition(3.1.1)(ii) we can commute

$W=a\alpha b\beta c\tau m\tau cba+c\alpha b\beta a\tau m\tau a\alpha b\beta c$

$d_n(w)=d_n(a\alpha b\beta c\tau m\tau cba+c\alpha b\beta a\tau m\tau a\alpha b\beta c)=$

$\sum_{i+j+k+p+q+t+m=n}d_i(a)\alpha d_j(b)\beta d_k(c)\tau d_p(m)\tau d_q(c)\alpha d_t(b)\beta d_m(a)+$
$d_i(c)\alpha d_j(b)\beta d_k(a)\tau d_p(m)\tau d_q(a)\alpha d_t(b)\beta d_m(c)$

$=\sum_{i+j+k=n}d_i(a)\alpha d_j(b)\beta d_k(c)\tau m\tau c\alpha b\beta a+$
$\sum_{i+j+k+p+q+t+m=n}^{i+j+k<n}d_i(a)\alpha d_j(b)\beta d_k(c)\tau d_p(m)\tau d_q(c)\alpha d_t(b)\beta d_m(a)+a\alpha b\beta c\tau m\tau$
$\sum_{q+t+m=n}d_q(c)\alpha d_t(b)\beta d_m(a)+$
$\sum_{i+j+k+p+q+t+m=n}^{q+t+m<n}d_i(a)\alpha d_j(b)\beta d_k(c)\tau d_p(m)\tau d_q(c)\alpha d_t(b)\beta d_m(a)+$
$\sum_{i+j+k=n}d_i(c)\alpha d_j(b)\beta d_k(a)\tau m\tau a\alpha b\beta c+$
$\sum_{i+j+k+p+q+t+m=n}^{i+j+k<n}d_i(c)\alpha d_j(b)\beta d_k(a)\tau d_p(m)\tau d_q(a)\alpha d_t(b)\beta d_m(c)+$
$c\alpha b\beta a\tau s\tau\sum_{q+t+m=n}d_q(a)\alpha d_t(b)\beta d_m(c)+$
$\sum_{i+j+k+p+q+t+m=n}^{q+t+m<n}d_i(c)\alpha d_j(b)\beta d_k(a)\tau d_p(m)\tau d_q(a)\alpha d_t(b)\beta d_m(c)$ ... (1)

On the other hand

$d_n(w)=d_n(a\alpha b\beta c\tau m\tau cba+c\alpha b\beta a\tau m\tau a\alpha b\beta c)=\sum_{i+j+k=n}d_i(a\alpha b\beta c)\tau d_j(m)\tau d_k(c\alpha b\beta a)+$
$d_i(c\alpha b\beta a)\tau d_j(m)\tau d_k(a\alpha b\beta c)$

$=d_n(a\alpha b\beta c)\tau m\tau c\alpha b\beta a+\sum_{i+j+k+p+q+t+m=n}^{i+j+k<n}d_i(a)\alpha d_j(b)\beta d_k(c)\tau d_p(m)\tau d_q(c)\alpha d_t(b)\beta d_m(a)+$
$a\alpha b\beta c\tau m\tau d_n(c\alpha b\beta a)+$
$\sum_{i+j+k+p+q+t+m=n}^{q+t+m<n}d_i(a)\alpha d_j(b)\beta d_k(c)\tau d_p(m)\tau d_q(c)\alpha d_t(b)\beta d_m(a)+d_n(c\alpha b\beta a)\tau m\tau a\alpha b\beta c+$
$\sum_{i+j+k+p+q+t+m=n}^{i+j+k<n}d_i(c)\alpha d_j(b)\beta d_k(a)\tau d_p(m)\tau d_q(a)\alpha d_t(b)\beta d_m(c)+c\alpha b\beta a\tau m\tau d_n(a\alpha b\beta c)+$
$\sum_{i+j+k+p+q+t+m=n}^{q+t+m<n}d_i(c)\alpha d_j(b)\beta d_k(a)\tau d_p(m)\tau d_q(a)\alpha d_t(b)\beta d_m(c)$ ... (2)

Compare(1),(2) and by assumption we get

$0=(d_n(a\alpha b\beta c)-\sum_{i+j+k=n}d_i(a)\alpha d_j(b)\beta d_k(c))\tau m\tau c\alpha b\beta a+(d_n(c\alpha b\beta a)-$
$\sum_{i+j+k=n}d_i(c)\alpha d_j(b)\beta d_k(a))\tau m\tau a\alpha b\beta c+a\alpha b\beta c\tau m\tau(d_n(c\alpha b\beta a)-$
$\sum_{i+j+k=n}d_i(c)\alpha d_j(b)\beta d_k(a))+c\alpha b\beta a\,\tau m\tau(d_n(a\alpha b\beta c)-\sum_{i+j+k=n}d_i(a)\alpha d_j(b)\beta d_k(c))$

$0=\Psi_n(a,b,c)_{\alpha,\beta}\tau m\tau c\alpha b\beta a+\Psi_n(c,b,a)_{\alpha,\beta}\tau m\tau a\alpha b\beta c+a\alpha b\beta c\tau m\tau\Psi_n(c,b,a)_{\alpha,\beta}+c\alpha b\beta a\,\tau m\tau\Psi_n(a,b,c)_{\alpha,\beta}$

$0=\Psi_n(a,b,c)_{\alpha,\beta}\tau m\tau c\alpha b\beta a-\Psi_n(a,b,c)_{\alpha,\beta}\tau m\tau a\alpha b\beta c-a\alpha b\beta c\tau m\tau\Psi_n(a,b,c)_{\alpha,\beta}+c\alpha b\beta a\,\tau m\tau\Psi_n(a,b,c)_{\alpha,\beta}$

$0=-(-\Psi_n(a,b,c)_{\alpha,\beta}\tau m\tau c\alpha b\beta a+\Psi_n(a,b,c)_{\alpha,\beta}\tau m\tau a\alpha b\beta c)-(a\alpha b\beta c\tau m\tau\Psi_n(a,b,c)_{\alpha,\beta}-c\alpha b\beta a\,\tau m\tau$
$\Psi_n(a,b,c)_{\alpha,\beta})$

$0=-\Psi_n(a,b,c)_{\alpha,\beta}\tau m\tau[a,b,c]_{\alpha,\beta}-[a,b,c]_{\alpha,\beta}\tau m\tau\Psi_n(a,b,c)_{\alpha,\beta}$

Hence

$\Psi_n(a,b,c)_{\alpha,\beta}\,\tau m\tau\,[a,b,c]_{\alpha,\beta} + [a,b,c]_{\alpha,\beta}\,\tau m\tau\,\Psi_n(a,b,c)_{\alpha,\beta} = 0.$

ii) Replacing β for τ in (i) we get the require result.

iii) Replacing α for τ in (i) we get the require result.

iv) Interchanging α and τ and α for β in (i),we obtain(iv).

v) As the same way of (i) by interchanging $\alpha,\beta+\gamma,\tau$ by interchanging α and τ in (i),we obtain(v).

Lemma 4:

Let $D=(d_i)_{i\in N}$ be a Jordan triple high derivations of a Γ-semiring M with addative identity and inverse. Then for any $a,b,c,m\in M$; $\alpha,\beta,\gamma,\tau \in\Gamma$and $n\in N$:

i)$\Psi_n(a,b,c)_{\alpha,\beta}\,\tau m\tau\,[a,b,c]_{\alpha,\beta} = [a,b,c]_{\alpha,\beta}\,\tau m\tau\,\Psi_n(a,b,c)_{\alpha,\beta} = 0$

ii) $\Psi_n(a,b,c)_{\alpha,\beta}\,\beta m\beta\,[a,b,c]_{\alpha,\beta} = [a,b,c]_{\alpha,\beta}\,\beta m\beta\,\Psi_n(a,b,c)_{\alpha,\beta} = 0$

iii) $\Psi_n(a,b,c)_{\alpha,\beta}\,\alpha m\alpha\,[a,b,c]_{\alpha,\beta} = [a,b,c]_{\alpha,\beta}\,\alpha m\alpha\,\Psi_n(a,b,c)_{\alpha,\beta} = 0$

iv) $\Psi_n(a,b,c)_{\alpha,\alpha}\,\alpha m\alpha\,[a,b,c]_{\alpha,\alpha} = [a,b,c]_{\alpha,\alpha}\,\alpha m\alpha\,\Psi_n(a,b,c)_{\alpha,\alpha} = 0$

v)) $\Psi_n(a,b,c)_{\alpha,\beta+\gamma,\tau}\,\alpha m\alpha\,[a,b,c]_{\alpha,\beta+\gamma,\tau} = [a,b,c]_{\alpha,\beta+\gamma,\tau}\,\alpha m\alpha\,\Psi_n(a,b,c)_{\alpha,\beta+\gamma,\tau} = 0$

Proof:i) By Lemma 3(i), we get:

$\Psi_n(a,b,c)_{\alpha,\beta}\,\tau m\tau\,[a,b,c]_{\alpha,\beta} + [a,b,c]_{\alpha,\beta}\,\tau m\tau\,\Psi_n(a,b,c)_{\alpha,\beta} = 0$

by Lemma1.1 we get:

$\Psi_n(a,b,c)_{\alpha,\beta}\,\tau m\tau\,[a,b,c]_{\alpha,\beta} = [a,b,c]_{\alpha,\beta}\,\tau m\tau\,\Psi_n(a,b,c)_{\alpha,\beta} = 0.$

(ii),(iii),(iv) and (v) As the same technique of (i).

Lemma 5:

Let $D=(d_i)_{i\in N}$ be a Jordan triple high derivations of a two-torsion free prime Γ-semiring M with additive identity and inverse .Then for any $a,b,c,m,x,y,z \in M$; $\alpha,\beta,\gamma,\tau \in\Gamma$ and $n\in N$:

i)$\Psi_n(a,b,c)_{\alpha,\beta}\,\tau m\tau\,[x,y,z]_{\alpha,\beta} = 0$

ii) $\Psi_n(a,b,c)_{\alpha,\beta}\,\beta m\beta\,[x,y,z]_{\alpha,\beta} = 0$

iii) $\Psi_n(a,b,c)_{\alpha,\beta}\,\alpha m\alpha\,[x,y,z]_{\alpha,\beta} = 0$

iv) $\Psi_n(a,b,c)_{\alpha,\alpha}\,\alpha m\alpha\,[x,y,z]_{\alpha,\alpha} = 0$

v) $\Psi_n(a,b,c)_{\alpha,\beta+\gamma,\tau}\,\alpha m\alpha\,[x,y,z]_{\alpha,\beta+\gamma,\tau} = 0$

Proof:i) Substitute a by a+x in Lemma 4(i) ,we get:

$\Psi_n(a+x,b,c)_{\alpha,\beta}\,\tau m\tau\,[a+x,b,c]_{\alpha,\beta} = 0$

$\Psi_n(a,b,c)_{\alpha,\beta}\,\tau m\tau\,[a,b,c]_{\alpha,\beta} + \Psi_n(x,b,c)_{\alpha,\beta}\,\tau m\tau\,[a,b,c]_{\alpha,\beta} + \Psi_n(a,b,c)_{\alpha,\beta}\,\tau m\tau\,[x,b,c]_{\alpha,\beta} + \Psi_n(x,b,c)_{\alpha,\beta}\,\tau m\tau\,[x,b,c]_{\alpha,\beta} = 0$

By Lemma 4(i), we get:

$\Psi_n(a,b,c)_{\alpha,\beta}\,\tau m\tau\,[a,b,c]_{\alpha,\beta} = \Psi_n(x,b,c)_{\alpha,\beta}\,\tau m\tau\,[x,b,c]_{\alpha,\beta} = 0$

$\Psi_n(x,b,c)_{\alpha,\beta}\,\tau m\tau\,[a,b,c]_{\alpha,\beta} + \Psi_n(a,b,c)_{\alpha,\beta}\,\tau m\tau\,[x,b,c]_{\alpha,\beta} = 0$

Therefore we get:

$\Psi_n(x,b,c)_{\alpha,\beta}\,\tau m\tau\,[a,b,c]_{\alpha,\beta}\,\tau m\tau\,\Psi_n(x,b,c)_{\alpha,\beta}\,\tau m\tau\,[a,b,c]_{\alpha,\beta} = 0$

$-\,\Psi_n(x,b,c)_{\alpha,\beta}\,\tau m\tau\,[a,b,c]_{\alpha,\beta}\,\tau m\tau\,\Psi_n(a,b,c)_{\alpha,\beta}\,\tau m\tau\,[x,b,c]_{\alpha,\beta} = 0$

Hence, by the primness of M:

$\Psi_n(a,b,c)_{\alpha,\beta}\,\tau m\tau\,[x,b,c]_{\alpha,\beta} = 0$ ….(1)

Substitute b by b+y in Lemma 4(i), we get:

$\Psi_n(a,b+y,c)_{\alpha,\beta}\,\tau m\tau\,[a,b+y,c]_{\alpha,\beta} = 0$

$\Psi_n(a,b,c)_{\alpha,\beta}\,\tau m\tau\,[a,b,c]_{\alpha,\beta} + \Psi_n(a,y,c)_{\alpha,\beta}\,\tau m\tau\,[a,b,c]_{\alpha,\beta} + \Psi_n(a,b,c)_{\alpha,\beta}\,\tau m\tau\,[a,y,c]_{\alpha,\beta} + \Psi_n(a,y,c)_{\alpha,\beta}\,\tau m\tau\,[a,y,c]_{\alpha,\beta} = 0$

By Lemma 4(i),we get:

$\Psi_n(a,b,c)_{\alpha,\beta}\,\tau m\tau\,[a,b,c]_{\alpha,\beta} = \Psi_n(a,y,c)_{\alpha,\beta}\,\tau m\tau\,[a,y,c]_{\alpha,\beta} = 0$

$\Psi_n(a,y,c)_{\alpha,\beta}\,\tau m\tau\,[a,b,c]_{\alpha,\beta} + \Psi_n(a,b,c)_{\alpha,\beta}\,\tau m\tau\,[a,y,c]_{\alpha,\beta} = 0$

Therefore, we get:

$\Psi_n(a,y,c)_{\alpha,\beta}\, \tau m\tau\, [a,b,c]_{\alpha,\beta}\, \tau m\tau\, \Psi_n(a,y,c)_{\alpha,\beta}\, \tau m\tau\, [a,b,c]_{\alpha,\beta} = 0$

$-\Psi_n(a,x,c)_{\alpha,\beta}\, \tau m\tau\, [a,b,c]_{\alpha,\beta}\, \tau m\tau\, \Psi_n(a,b,c)_{\alpha,\beta}\, \tau m\tau\, [a,y,c]_{\alpha,\beta} = 0$

Hence, by the primness of M:

$\Psi_n(a,b,c)_{\alpha,\beta}\, \tau m\tau\, [a,y,c]_{\alpha,\beta} = 0$ ….(2)

Now, Substitute c by c+z in Lemma 4(i), we get:

$\Psi_n(a,b,c+z)_{\alpha,\beta}\, \tau m\tau\, [a,b,c+z]_{\alpha,\beta} = 0$

$\Psi_n(a,b,c)_{\alpha,\beta}\, \tau m\tau\, [a,b,c]_{\alpha,\beta} + \Psi_n(a,b,z)_{\alpha,\beta}\, \tau m\tau\, [a,b,c]_{\alpha,\beta} + \Psi_n(a,b,c)_{\alpha,\beta}\, \tau m\tau\, [a,b,z]_{\alpha,\beta} + \Psi_n(a,b,z)_{\alpha,\beta}\, \tau m\tau\, [a,b,z]_{\alpha,\beta} = 0$

By Lemma 4(i), we get:

$\Psi_n(a,b,c)_{\alpha,\beta}\, \tau m\tau\, [a,b,c]_{\alpha,\beta} = \Psi_n(a,b,z)_{\alpha,\beta}\, \tau m\tau\, [a,b,z]_{\alpha,\beta} = 0$

$\Psi_n(a,b,z)_{\alpha,\beta}\, \tau m\tau\, [a,b,c]_{\alpha,\beta} + \Psi_n(a,b,c)_{\alpha,\beta}\, \tau m\tau\, [a,b,z]_{\alpha,\beta} = 0$

Therefore, we get:

$\Psi_n(a,b,z)_{\alpha,\beta}\, \tau m\tau\, [a,b,c]_{\alpha,\beta}\, \tau m\tau\, \Psi_n(a,b,z)_{\alpha,\beta}\, \tau m\tau\, [a,b,c]_{\alpha,\beta} = 0$

$-\Psi_n(a,b,z)_{\alpha,\beta}\, \tau m\tau\, [a,b,c]_{\alpha,\beta}\, \tau m\tau\, \Psi_n(a,b,c)_{\alpha,\beta}\, \tau m\tau\, [a,b,z]_{\alpha,\beta} = 0$

Hence, by the primness of M:

$\Psi_n(a,b,c)_{\alpha,\beta}\, \tau m\tau\, [a,b,z]_{\alpha,\beta} = 0$ …(3)

Thus: $\Psi_n(a,b,c)_{\alpha,\beta}\, \tau m\tau\, [a+x,b+y,c+z]_{\alpha,\beta} = 0$

$\Psi_n(a,b,c)_{\alpha,\beta}\, \tau m\tau\, [a,b,c]_{\alpha,\beta} + \Psi_n(a,b,c)_{\alpha,\beta}\, \tau m\tau\, [a,b,z]_{\alpha,\beta} + \Psi_n(a,b,c)_{\alpha,\beta}\, \tau m\tau\, [a,y,c]_{\alpha,\beta} + \Psi_n(a,b,c)_{\alpha,\beta}\, \tau m\tau\, [a,y,z]_{\alpha,\beta} + \Psi_n(a,b,c)_{\alpha,\beta}\, \tau m\tau\, [x,b,c]_{\alpha,\beta} + \Psi_n(a,b,c)_{\alpha,\beta}\, \tau m\tau\, [x,b,z]_{\alpha,\beta} + \Psi_n(a,b,c)_{\alpha,\beta}\, \tau m\tau\, [x,y,c]_{\alpha,\beta} + \Psi_n(a,b,c)_{\alpha,\beta}\, \tau m\tau\, [x,y,z]_{\alpha,\beta} = 0$

by (1),(2) and (3) and lemma4(i), we get:

$\Psi_n(a,b,c)_{\alpha,\beta}\tau m\tau[x,y,z]_{\alpha,\beta} = 0.$

(ii),(iii),(iv) and (v) As the same technique of (i).

Theorem 6:

Every Jordan triple high derivations of a two-torsion free prime Γ-semiring M with additive identity and inverse is triple high derivations of M.

Proof: Let $D=(d_i)_{i\in N}$ be a Jordan triple high derivations of a two-torsion free prime Γ-semiring M. Since M is prime we get from Lemma5(i) either $\Psi_n(a,b,c)_{\alpha,\beta} = 0$ or $[x,y,z]_{\alpha,\beta} = 0$ for any a,b,c,x,y,z ∈M α,β ∈Γand n∈N.

If $[x,y,z]_{\alpha,\beta} \neq 0$ for any x,y,z∈M, α,β ∈Γ . Then $\Psi_n(a,b,c)_{\alpha,\beta} = 0$ for any a,b,c ∈M α,β ∈Γ and n∈N then by Remark 2.3 we get, D is triple high derivations on M.

If $[x,y,z]_{\alpha,\beta} = 0$ for any x,y,z ∈M , α,β ∈Γ and n∈N, then M is a commutative Γ-semiring and by Lemma1 we get

$d_n(2a\alpha b\beta c) = 2\sum_{i+j+k=n} d_i(a)\alpha\, d_j(b)\beta d_k(c)$

Since M is two-torsion free we get D is a triple high derivations of M.

Proposition7:

Every Jordan high derivations of two-torsion free Γ-semiring M with additive identity and inverse such that aαbβc=aβbαc for any a,b,c∈M and α,β∈Γ is Jordan triple high derivations of M.

Proof: Let $D=(d_i)_{i\in N}$ be a Jordan high derivations of M.

Substitute b by aβb+bβa in definition 2.1(i), we get:

$d_n(a\alpha(a\beta b+b\beta a)+(a\beta b+b\beta a)\alpha a) =$
$\sum_{i+j=n} d_i(a)\,\alpha d_j(a\beta b + b\beta a) + d_i(a\beta b + b\beta a)\alpha d_j(a) = \sum_{i+j=n} d_i(a)\alpha(\sum_{h+w=j} d_h(a)\beta d_w(b) + d_h(b)\beta d_w(a)) + (\sum_{r+e=i} d_r(a)\beta d_e(b) + d_r(b)\beta d_e(a))\alpha d_j(a)$

$= \sum_{i+j=n}\sum_{h+w=j} d_i(a)\,\alpha d_h(a)\beta d_w(b) + d_i(a)\alpha d_h(b)\beta d_w(a) + \sum_{i+j=n}\sum_{r+e=i} d_r(a)\beta d_e(b)\,\alpha d_j(a) + d_r(b)\beta d_e(a)\alpha d_j(a)$

$=\sum_{i+h+w=n} d_i(a)\alpha d_h(a)\beta d_w(b) + d_i(a)\alpha d_h(b)\beta d_w(a) + d_i(a)\beta d_h(b)\alpha d_w(a) + d_i(b)\beta d_h(a)\alpha d_w(a)$ .... (1)

On the other hand:

$d_n(a\alpha(a\beta b+b\beta a)+ (a\beta b+b\beta a)\alpha a)$

$= d_n(a\alpha a\beta b+a\alpha b\beta a+a\beta b\alpha a+b\beta a\alpha a)$

$=\sum_{i+h+w=n} d_i(a)\alpha d_h(a)\beta d_w(b) + d_i(b)\beta d_h(a)\alpha d_w(a)+d_n(a\alpha b\beta a+a\beta b\alpha a)$ ..... (2)

Compare (1) and (2) and since $a\alpha b\beta c=a\beta b\alpha c$ for any $a,b,c\in M$ and $\alpha,\beta\in\Gamma$ we get:

$2d_n(a\alpha b\beta a) =2\sum_{i+h+w=n} d_i(a)\alpha d_h(b)\beta d_w(a)$

Since M is a two-torsion free we get :

$d_n(a\alpha b\beta a) =\sum_{i+h+w=n} d_i(a)\alpha d_h(b)\beta d_w(a)$.

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