On Jordan Generalized High Homorphics on Prime Γ-Semirings

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Abstract
In this research, we will introduce generalized high Homorphics, and explain certain features of generalized high Homorphics, as well as discuss certain significant connections and distinctions..
Key word: generalized high Homorphics, Jordan generalized high Homorphics, Γ-semiring.

حول تعميمات جوردان التشاكلات العليا على شبة الحلقات الاولية من نمط–Γ

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الملخص
في هذا البحث ، سوف نقدم التشاكلات العليا على شبة الحلقات الاولية من نمط–Γ ، و شرح سمات معينة من التشاكلات العاليا المعممة ، بالاضافة الى مناقشة بعض الروابط و الاختلافات المهمة .
الكلمات المفتاحية: تعميمات التشاكلات العليا ، تعميمات جوردان التشاكلات العليا، شبة حلقة من نمط كاما.

1.Introduction

The notions of high Homorphics are built upon in [3]. More details on the semiring named Γ may be found in [10, 11]. In this work, we investigate the connection between upper Homorphics, and the related notions of Jordan high Homorphics and Jordan triple high Homorphics, which are all connected to the study of prime semiring and Γsemi-ring.

Lemma 1.1: [9]

Assume that M is a two-torsion free semiprime Γ-semiring with additive identity and inverse. a,b∈M, if $a\Gamma m\Gamma b + b\Gamma m\Gamma a = 0$ for any m∈M, then $a\Gamma m\Gamma b = b\Gamma m\Gamma a = o$.

2. Generalized High Homorphics on Prime Γ-Semiring

Definition 2.1:

Letss F be the family of mappings of an arbitrary semiring into another semiring that add (in addition) the specified members of the family. F is known as a generalized upper Homorphics from M to M'. This means that a high Homorphics exist that does the same things as the one described. $\theta=(\varphi_i)_{i\in N}$ from M into M', for any n∈N we have:

$f_n(a\alpha b) = \sum_{i=1}^{n} f_i(a)\alpha\varphi_i(b)$

for any a,b∈M and α∈Γ, where θ is the named high Homorphics.

Definition 2.2:

Lets, the mapping F = (fi)i∈N is a family of additive mappings from a nonnegative integer semiring to a nonnegative integer semiring. Therefore, F is named a Jordan generalized upper Homorphics if a Jordan high Homorphics occurs. $\theta=(\varphi_i)_{i\in N}$ from M into M' for any n∈N we have:

$$f_n(a\alpha a) = \sum_{i=1}^{n} f_i(a)\ \alpha\ \varphi_i\ (a)$$

for any a∈M and α∈Γ.

θ is named the relating Jordan high Homorphics.

Definition2.3:

Lets $F=(f_i)_{i\in N}$ resulting in a set of additive mappings of Γ-semiring M into Γ-semiring M', then F is supposed to be a Jordan generalized triple high Homorphics if there exist a Jordan triple high Homorphics $\theta=(\phi_i)_{i\in N}$ of Γ-semiring M into Γ-semiring M' for any n∈N we have:

$$f_n(a\alpha b\beta a) = \sum_{i=1}^{n} f_i(a)\ \alpha\ \phi_i(b)\ \beta\ \phi_i(a)$$

for any a,b∈M and α,β∈Γ. θ is named the involving Jordan triple high Homorphics.

Lemma 1:

Lets $F=(f_i)_{i\in N}$ be a Jordan generalized high Homorphics of Γ-semiring M into Γ-semiring M' additive commutative, as for any a,b,c∈M, α,β∈Γ and n∈N:

i) $f_n\ (a\alpha b+ b\alpha a) =\sum_{i=1}^{n} fi(a)\ \alpha\ \varphi i(b)\ +\ fi(b)\ \alpha\varphi i(a)$

ii) $f_n(a\alpha b\beta a+a\beta b\alpha a) =\sum_{i=1}^{n} fi(a)\alpha\varphi i(b)\beta\varphi i(a)\ +\ fi(a)\beta\varphi i(b)\alpha\varphi i(a)$

iii) $f_n(a\alpha b\alpha a) =\sum_{i=1}^{n} fi(a)\ \alpha\ \varphi i(b)\ \alpha\ \varphi i(a)$

iv) $f_n(a\alpha b\beta c+c\beta b\alpha a)= \sum_{i=1}^{n} fi(a)\alpha\ \varphi i(b)\beta\ \varphi i(c) +\ fi(c)\ \beta\ \varphi i(b)\ \alpha\ \varphi i(a)$

v) $f_n(a\alpha b\alpha c+c\alpha b\alpha a)= \sum_{i=1}^{n} fi(a)\ \alpha\ \varphi i(b)\alpha\varphi i(c)\ +\ fi(c)\alpha\varphi i(b)\alpha\ \varphi i(a)$

Proof:

i) $f_n\ ((a+b)\alpha\ (a+b)) = \sum_{i=1}^{n} fi(a + b)\ \alpha\ \varphi i(a + b)$

$=\sum_{i=1}^{n}\ fi(a)\alpha\varphi i(a) +\ fi(a)\alpha\varphi i(b) +\ fi(b)\alpha\ \varphi i(a) +\ fi(b)\alpha\varphi i(b)$ ....(1)

Alternatively,:

$f_n\ ((a+b)\alpha\ (a+b)) = f_n\ (a\alpha a+a\alpha b+b\alpha a+b\alpha b)$

$= \sum_{i=1}^{n} fi(a)\ \alpha\ \varphi i(a)\ +\ fi(b)\ \alpha\ \varphi i(b)\ +\ fn(a\alpha b + b\alpha a)$ … (2)

Compare (1) and (2), we found :

$$f_n(a\alpha b+b\alpha a) = \sum_{i=1}^{n} fi(a)\ \alpha\ \varphi i(b)\ +\ fi(b)\ \alpha\ \varphi i(a)$$

ii) Replace aβb+bβa for b in (i), we found :

$f_n(a\alpha(a\beta b+b\beta a)+(a\beta b+b\beta a)\alpha a) =\sum_{i=1}^{n} fi(a)\alpha\varphi i(a\beta b + b\beta a)\ +\ fi(a\beta b + b\beta a)\alpha\ \varphi i(a)$

$=\sum_{i=1}^{n} fi(a)\alpha(\ \varphi i(a)\beta\varphi i(b)\ +\ \varphi i(b)\beta\varphi i(a)\ ) + (\ fi(a)\beta\varphi i(b)\ +\ fi(b)\beta\varphi i(a)\ )\alpha\ \varphi i(a)$

$=\sum_{i=1}^{n} fi(a)\alpha\varphi i(a)\beta\varphi i(b) + fi(a)\alpha\varphi i(b)\beta\varphi i(a) + fi(a)\beta\varphi i(b)\alpha\varphi i(a) +\ fi(b)\beta\varphi i(a)\alpha\varphi i(a)$ … (1)

Alternatively,:

$f_n(a\alpha(a\beta b+b\beta a)+(a\beta b+b\beta a)\alpha a) = f_n(a\alpha a\beta b+a\alpha b\beta a+a\beta b\alpha a+b\beta a\alpha a)$

$=\sum_{i=1}^{n} fi(a)\alpha\varphi i(a)\beta\varphi i(b) + fi(b)\beta\varphi i(a)\alpha\varphi i(a) + fn(a\alpha b\beta a + a\beta b\alpha a)$ …(2)

Compare (1) and (2), we found :

$f_n(a\alpha b\beta a+a\beta b\alpha a) = \sum_{i=1}^{n} fi(a)\alpha\varphi i(b)\beta\varphi i(a) + fi(a)\beta\varphi i(b)\alpha\varphi i(a)$

iii) Substituting α for β in definition 2.3 we found the require result.

iv) Substituting a+c for a in definition 3.1.6, we found :

$f_n((a+c)\ \alpha b\beta(a+c))=\sum_{i=1}^{n} fi(a + c)\ \alpha\ \varphi i(b)\ \beta\ \varphi i(a + c)$

$=\sum_{i=1}^{n} fi(a)\ \alpha\ \varphi i(b)\ \beta\ \varphi i(a) + fi(a)\ \alpha\ \varphi i(b)\ \beta\ \varphi i(c) + fi(c)\ \alpha\ \varphi i(b)\ \beta\ \varphi i(a)\ +$
$fi(c)\ \alpha\ \varphi i(b)\ \beta\ \varphi i(c)$ ... (1)

Alternatively,:

$f_n((a+c)\ \alpha b\beta(a+c)) = f_n(a\alpha b\beta a+a\alpha b\beta c+c\alpha b\beta a+c\alpha b\beta c)$

$=\sum_{i=1}^{n} fi(a)\alpha\varphi i(b)\beta\varphi i(a) + fi(c)\alpha\varphi i(b)\beta\varphi i(c)$ …(2)

Compare (1) and (2), we found :

$f_n(a\alpha b\beta c+c\alpha b\beta a)=\sum_{i=1}^{n} fi(a)\ \alpha\ \varphi i(b)\ \beta\ \varphi i(c) + fi(c)\ \alpha\ \varphi i(b)\ \beta\ \varphi i(a)$.

v) Substituting α for β in (iv) we found the require result.

Definition 2.4:

Lets $F=(f_i)_{i\in N}$ be a Jordan generalized high Homorphics of Γ-semiring M into Γ-semiring M' with additive identity and inverse. As for any a,b∈M and α∈Γ, we define $\delta_n\ (a, b)_{\alpha}$: M×M⟶ M' by:

$$\delta_n(a, b)_{\alpha} = f_n(a\alpha b)-\sum_{i=1}^{n} f_i(a)\ \alpha\ \varphi_i(b)$$

Lemma 2:

If $F=(f_i)_{i\in N}$ is a Jordan generalized high Homorphics from Γ-semiring M into Γ-semiring M' with additive commutative, identity and inverse as for any a,b,c∈M and α,β∈Γ.

i) $\delta_n(a+b, c)_\alpha = \delta_n(a,c)_\alpha + \delta_n(b,c)_\alpha$

ii) $\delta_n(a, b+c)_\alpha = \delta_n(a,b)_\alpha + \delta_n(a,c)_\alpha$

iii) $\delta_n(a, b)_{\alpha+\beta} = \delta_n(a, b)_\alpha + \delta_n(a, b)_\beta$

iv) $\delta_n(a, b)_\alpha = - \delta_n(b,a)_\alpha$

Proof:

i) $\delta_n(a+b, c)_\alpha = f_n((a+b)\alpha c)-\sum_{i=1}^{n} fi(a+b)\,\alpha\,\varphi i(c)$

$= f_n(a\alpha c + b\alpha c)-\sum_{i=1}^{n} fi(a)\alpha\,\varphi i(c) - \sum_{i=1}^{n} fi(b)\alpha\,\varphi i(c)$

$= f_n(a\alpha c) -\sum_{i=1}^{n} fi(a)\alpha\,\varphi i(c) + f_n(b\alpha c)- \sum_{i=1}^{n} fi(b)\alpha\varphi i(c)$

$= \delta_n(a,c)_\alpha + \delta_n(b,c)_\alpha$

ii) by the same way of (i).

iii) $\delta_n(a, b)_{\alpha+\beta} = f_n(a(\alpha+\beta)b) - \sum_{i=1}^{n} fi(a)\,(\alpha+\beta)\,\varphi i(b)$

Since f_n is additive mappings for any n.

$=f_n(a\alpha b)-\sum_{i=1}^{n} fi(a)\,\alpha\,\varphi i(b) + fn(a\beta b) -\sum_{i=1}^{n} fi(a)\,\beta\,\varphi i(b)$

$= \delta_n(ai,ib)_\alpha + \delta_n(ai,ib)_\beta$.

iv) T.P. $\delta_n(ai,ib)_\alpha = - \delta_n(b,a)_\alpha$

by lemma 1(i) and since f_n is additive for each $n\in N$ then:

$$f_n(a\alpha b + b\alpha a) = \sum_{i=1}^{n} f_i(a)\alpha\,\varphi_i(b) + f_i(b)\alpha\varphi i(a)$$

$f_n(a\alpha b)+ f_n(b\alpha a) = \sum_{i=1}^{n} f_i(a)\alpha\varphi_i(b) + \sum_{i=1}^{n} f_i(b)\alpha\varphi_i(a)$

$\varphi_n(a\alpha b) - \sum_{i=1}^{n} f_i(a)\alpha\varphi_i(b) = -f_n(b\alpha a) + \sum_{i=1}^{n} fi(b)\alpha\varphi i(a)$

$\delta_n(ai,ib)_\alpha = - \delta_n(b,a)_\alpha$

3. The Main Results

Lemma 3:

Lets $F=(f_i)_{i\in N}$ be a Jordan generalized high Homorphics of Γ-semiring M into Γ-semiring M' with additive identity and inverse. As for any a,b,m∈M , α,β∈Γ and n∈N.

i) $\delta_n(a,b)_\alpha\,\beta\,\varphi_n(m)\,\beta\,G_n(b,a)_\alpha + \delta_n(b,a)_\alpha\,\beta\,\varphi_n(m)\,\beta\,G_n(a,b)_\alpha=0$

ii) $\delta_n(a,b)_\alpha\,\alpha\,\varphi_n(m)\,\alpha\,G_n(b,a)_\alpha+ \delta_n(b,a)_\alpha\,\alpha\,\varphi_n(m)\,\alpha\,G_n(a,b)_\alpha=0$

iii) $\delta_n(ai,ib)_\beta\,\alpha\,\varphi_n(m)\,\alpha\,G_n(b,a)_\beta + \delta_n(b,a)_\beta\,\alpha\,\varphi_n(m)\,\alpha\,G_n(a,b)_\beta=0$

Proof:

We proceed by induction on n∈N. If n=1,

Lets $w=a\alpha b\beta m\beta b\alpha a + b\alpha a\beta m\beta a\alpha b$

since F is a Jordan high Homorphics, then:

$f_1(w) = f_1(a\alpha(b\beta m\beta b)\alpha a + b\alpha(a\beta m\beta a)\alpha b)$

$= f_1(a)\alpha\varphi_1(b\beta m\beta b)\alpha\varphi_1(a) + f_1(b)\alpha\varphi_1(a\beta m\beta a)\alpha\varphi_1(b)$

$= f_1(a)\alpha\varphi_1(b)\beta\varphi_1(m)\beta\varphi_1(b)\alpha\varphi_1(a)$

$+ f_1(b)\alpha\varphi_1(a)\beta\varphi_1(m)\beta\varphi_1(a)\alpha\varphi_1(b)$ … (1)

Alternatively,:

$f_1(w) = f_1((a\alpha b)\beta m\beta(b\alpha a) + (b\alpha a)\beta m\beta(a\alpha b))$

$= f_1(a\alpha b)\beta\varphi_1(m)\beta\varphi_1(b\alpha a) + f_1(b\alpha a)\beta\varphi_1(m)\,\beta\varphi_1(a\alpha b)$

$=f_1(a\alpha b)\beta\varphi_1(m)\beta(\varphi_1(a)\alpha\varphi_1(b)+f_1(b)\alpha\varphi_1(a) - f_1(a\alpha b))$

$+(-f_1(a\alpha b)+f_1(a)\alpha\varphi_1(b)+f_1(b)\alpha\varphi_1(a))\beta\varphi_1(m)\,\beta\varphi_1(a\alpha b)$

$= -f_1(a\alpha b)\beta\varphi_1(m)\beta(\varphi_1(a\alpha b)-f_1(a)\alpha\varphi_1(b))-f_1(a\alpha b)\,\beta$

$\varphi_1(m)\beta(\varphi_1(a\alpha b)-f_1(b)\alpha\varphi_1(a))+f_1(a)\alpha\varphi_1(b)\beta\varphi_1(m)$

$\beta\varphi_1(a\alpha b) + f_1(b)\alpha\varphi_1(a)\,\beta\varphi_1(m)\,\beta\varphi_1(a\alpha b)$ … (2)

By comparing (1) and (2), we found :

$0 = -f_1(a\alpha b)\beta\varphi_1(m)\beta G_1(ai,ib)_\alpha - f_1(a\alpha b)\beta\varphi_1(m)\,\beta G_1(b,a)_\alpha$
$+f_1(a)\alpha\varphi_1(b)\beta\varphi_1(m)\beta\varphi_1(a\alpha b) + f_1(b)\alpha\varphi_1(a)\beta\varphi_1(m)$
$\beta\varphi_1(a\alpha b) - f_1(a)\alpha\varphi_1(b)\beta\varphi_1(m)\beta\varphi_1(b)\alpha\varphi_1(a)$
$-f_1(b)\alpha\varphi_1(a)\,\beta\varphi_1(m)\,\beta\varphi_1(a)\alpha\varphi_1(b)$
$=-f_1(a\alpha b)\beta\varphi_1(m)\beta G_1(ai,ib)_\alpha - f_1(a\alpha b)\beta\varphi_1(m)\,\beta G_1(b,a)_\alpha$
$+f_1(a)\alpha\varphi_1(b)\beta\varphi_1(m)\beta G_1(b,a)_\alpha + f_1(b)\alpha\varphi_1(a)\beta)\varphi_1(m)\beta G_1(ai,ib)_\alpha$
$=-(f_1(a\alpha b)-f_1(b)\alpha\varphi_1(a))\beta\varphi_1(m)\beta G_1(ai,ib)_\alpha$
$-(f_1(a\alpha b)-f_1(a)\alpha\varphi_1(b))\beta\varphi_1(m)\beta G_1(b,a)_\alpha$

Thus we found:

$G_1(a,b)_\alpha\beta\varphi_1(m)\,\beta G_1(b,a)_\alpha + G_1(b,a)_\alpha\beta\varphi_1(m)\,\beta G_1(ai,ib)_\alpha=0$

for any a,b,m∈M and $\alpha,\beta\in\Gamma$.

Then, therefore, we assume:

$G_s(a,b)_\alpha\,\beta\varphi_s(m)\,\beta G_s(b,a)_\alpha + G_s(b,a)_\alpha\beta\varphi_s(m)\,\beta G_s(a,b)_\alpha=0$

for any a,b,m∈M, and n,s∈N, s<n.

Now, Lets $w=a\alpha b\beta m\beta b\alpha a + b\alpha a\beta m\beta a\alpha b$, then:

$f_n(w) = f_n\,(a\alpha(b\beta m\beta b)\alpha a + b\alpha(a\beta m\beta a)\alpha b)$
$=\sum_{i=1}^{n} fi(a)\alpha\varphi i(b\beta m\beta b)\alpha\varphi i(a) + \sum_{i=1}^{n} fi(b)\alpha\varphi i(a\beta m\beta a)\alpha\varphi i(b)$
$=\sum_{i=1}^{n} fi(a)\alpha\,(\sum_{j=1}^{n} fj(b)\beta\varphi j(m)\beta\varphi j(b))\,\alpha\varphi i(a)$
$+\sum_{i=1}^{n} fi(b)\alpha(\sum_{j=1}^{n} fj(a)\beta\varphi j(m)\beta\varphi j(a))\,\alpha\varphi i(b)$
$=\sum_{i=1}^{n} fi(a)\alpha\varphi i(b)\beta\varphi i(m)\beta\varphi i(b)\,\alpha\varphi i(a)$
$+\sum_{i=1}^{n} fi(b)\alpha\varphi i(a)\beta\varphi i(m)\beta\varphi i(a)\,\alpha\varphi i(b)$
$=\sum_{i=1}^{n} fi(a)\alpha\varphi i(b)\beta\varphi i(m)\beta \sum_{j=1}^{n} fj(b)\,\alpha\varphi j(a)$
$+\sum_{i=1}^{n} fi(b)\alpha\varphi i(a)\beta\varphi i(m)\beta \sum_{j=1}^{n} fj(a)\,\alpha\varphi j(b)$
$= f_n(a)\alpha\varphi_n(b)\beta\varphi_n(m)\beta\sum_{j=1}^{n} fj(b)\,\alpha\varphi j(a)$
$+\sum_{i=1}^{n} \varphi i(a)\alpha\varphi i(b)\beta\varphi i(m)\beta \sum_{j=1}^{n} \varphi j(b)\,\alpha\varphi j(a)$
$+\varphi_n(b)\alpha\varphi_n(a)\beta\varphi_n(m)\beta\sum_{j=1}^{n} \varphi j(a)\,\alpha\varphi j(b)$
$+\sum_{i=1}^{n-1} \varphi i(b)\alpha\varphi i(a)\beta\varphi i(m)\beta \sum_{j=1}^{i} \varphi j(a)\,\alpha\varphi j(b)$

Alternatively,:

$f_n(w) = f_n(\,(a\alpha b)\beta m\beta(b\alpha a) + (b\alpha a)\beta m\beta(a\alpha b)\,)$

$f_n(w) = \sum_{i=1}^{n} fi(a\alpha b)\beta\varphi i(m)\beta\;(\sum_{j=1}^{i} fj(a)\,\alpha\varphi j(b) + fj(b)\alpha\varphi j(a)$
$- f_i(a\alpha b)) + \sum_{i=1}^{n}\sum_{j=1}^{i}(fj(a)\,\alpha\varphi j(b) + fj(b)\,\alpha\varphi j(a)$
$- f_i(a\alpha b))\,\beta\varphi_i(m)\beta\varphi_i(a\alpha b)$
$= \sum_{i=1}^{n} fi(a\alpha b)\beta\varphi i(m)\beta \sum_{j=1}^{n} \varphi j(a)\,\alpha\varphi j(b) + \sum_{j=1}^{n} \varphi i(a\alpha b)\beta\varphi i(m)$
$\beta\sum_{j=1}^{n} fj(b)\,\alpha\varphi j(a) - \sum_{i=1}^{n} fi(a\alpha b)\,\beta\varphi i(m)\beta\varphi i(a\alpha b)$
$+\sum_{i=1}^{n} fi(a)\alpha\varphi i(b)\beta\varphi i(m)\beta\varphi i(a\alpha b)$
$+\sum_{i=1}^{n} fi(b)\alpha\varphi i(a)\beta\varphi i(m)\beta\varphi i(a\alpha b)$
$-\sum_{i=1}^{n} fi(a\alpha b)\,\beta\varphi i(m)\beta\varphi i(a\alpha b)$
$= -\sum_{i=1}^{n} fi(a\alpha b)\beta\varphi i(m)\beta\,(f_i(a\alpha b) - \sum_{j=1}^{i} fj(a)\alpha\varphi j(b)\,)$
$-\sum_{i=1}^{n} fj(a\alpha b)\,\beta\varphi i(m)\beta\,(f_i(a\alpha b) - \sum_{j=1}^{i} fj(b)\alpha\varphi j(a)\,)$
$+\sum_{i=1}^{n} fi(a)\alpha\varphi i(b)\beta\varphi i(m)\beta\varphi i(a\alpha b)$
$+\sum_{i=1}^{n} fi(b)\alpha\varphi i(a)\beta\varphi i(m)\beta\varphi i(a\alpha b)$

$= -\sum_{i=1}^{n} fi(a\alpha b)\beta\varphi i(m)\beta Gi(ai, ib)\alpha - \sum_{i=1}^{n} fi(a\alpha b)\beta\varphi i(m)\beta$

$G_i(b,a)_\alpha - \sum_{i=1}^{n} fi(a)ki(\alpha)\varphi i(b)\beta\varphi i(m)\beta\varphi i(a\alpha b)$

$+ \sum_{i=1}^{n} fi(b)\alpha\varphi i(a)\beta\varphi i(m)\beta\varphi i(a\alpha b)$

$=-f_n(a\alpha b)\beta\varphi_n(m)\beta G_n(ai,ib)_\alpha - \sum_{i=1}^{n-1} fi(a\alpha b)\beta\varphi i(m)\beta$

$G_i(ai,ib)_\alpha - f_n(a\alpha b)\beta\varphi_n(m)\beta G_n(b,a)_\alpha$

$- \sum_{i=1}^{n-1} fi(a\alpha b)\beta\varphi i(m)\beta\, Gi(b\, ,a)\alpha + fn(a)\alpha\varphi n(b)\beta\varphi n(m)\beta\varphi n(a\alpha b)$

$+\sum_{i=1}^{n-1} fi(a)\alpha\varphi i(b)\beta\varphi i(m)\beta\varphi i(a\alpha b) + fn(b)\alpha\varphi n(a)\beta\varphi n(m)\beta\varphi n(a\alpha b)$

$+ \sum_{i=1}^{n-1} fi(b)\alpha\varphi i(a)\beta\varphi i(m)\beta\varphi i(a\alpha b)$

Compare the right hand sides of $f_n(w)$, we found :

$0 = - f_n(a\alpha b)\beta\varphi_n(m)\beta G_n(ai,ib)_\alpha - f_n(a\alpha b)\beta\varphi_n(m)\beta G_n(b,a)_\alpha$

$+ f_n(a)\alpha\varphi_n(b)\beta\varphi_n(m)\beta(\varphi_n(a\alpha b) - \sum_{j=1}^{n} fi(b)\alpha\varphi i(a))$

$+ f_n(b)\alpha\varphi_n(a)\beta\varphi_n(m)\beta(\varphi_n(a\alpha b) - \sum_{i=1}^{n} fi(a)\alpha\varphi i(b))$

$- \sum_{i=1}^{n-1} fi(a\alpha b)\beta\varphi i(m)\beta Gi(ai, ib)\alpha - \sum_{i=1}^{n-1} fi(a\alpha b)\beta\varphi i(m)\beta Gi(b, a)\alpha$

$+ \sum_{i=1}^{n-1} fi(a)\alpha\varphi i(b)\beta\varphi i(m)\beta\ (\ \varphi_i(a\alpha b)\ - \sum_{j=1}^{i} fj(b)\alpha\varphi j(a))$

$+ \sum_{i=1}^{n-1} fi(b)\alpha\varphi i(a)\beta\varphi i(m)\beta\ \ (\varphi_i(a\alpha b) - \sum_{j=1}^{i} fj(a)\alpha\varphi j(b))$

$= - f_n(a\alpha b)\beta\varphi_n(m)\beta G_n(ai,ib)_\alpha - f_n(a\alpha b)\beta\varphi_n(m)\beta G_n(b,a)_\alpha$

$+ f_n(a)\alpha\varphi_n(b)\beta\varphi_n(m)\beta G_n(b,a)_\alpha$

$+ f_n(b)\alpha\varphi_n(a)\beta\varphi_n(m)\beta G_n\ (ai,ib)_\alpha$

$- \sum_{i=1}^{n-1} fi(a\alpha b)\beta\varphi i(m)\beta Gi(ai, ib)\alpha - \sum_{i=1}^{n-1} fi(a\alpha b)\beta\varphi i(m)\beta Gi(b, a)\alpha$

$+\sum_{i=1}^{n-1} fi(a)\alpha\varphi i(b)\beta\varphi i(m)\beta Gi(b, a)\alpha + \sum_{i=1}^{n-1} fi(b)\alpha\varphi i(a)\beta\varphi i(m)\beta Gi(ai, ib)\alpha$

$= - G_n(b, a)_\alpha\beta\varphi_n(m)\beta G_n(ai,ib)_\alpha - G_n(a, b)_\alpha\ \beta\varphi_n(m)\beta G_n(b,a)_\alpha$

$- \sum_{i=1}^{n-1} Gi(b, a)\alpha\ \beta\varphi i(m)\beta Gi(ai, ib)\alpha - \sum_{i=1}^{n-1} Gi(ai, ib)\alpha\ \beta\varphi i(m)\beta Gi(b, a)\alpha$

$=-(G_n(b,a)_\alpha\beta\varphi_n(m)\beta G_n(ai,ib)_\alpha + G_n(ai,ib)_\alpha\ \beta\varphi_n(m)\beta G_n(b,a)_\alpha)$

$-(\sum_{i=1}^{n-1} Gi(ai, ib)\alpha\beta\varphi i(m)\beta Gi(b, a)\alpha + \sum_{i=1}^{n-1} Gi(b, a)\alpha\ \beta\varphi i(m)\beta Gi(ai, ib)\alpha\ )$

Our assumption is that we have::

$G_n(ai,ib)_\alpha\ \beta\varphi_n(m)\beta G_n(b,a)_\alpha + G_n(b,a)_\alpha\ \beta\varphi_n(m)\beta G_n(ai,ib)_\alpha=0$,

for any $a,b,m\in M$, $\alpha,\beta\in\Gamma$ and $n\in N$.

Replacing β by α in I and continuing in the same manner as we did in the proof of I we discovered (ii).

When we interchange α and β in I we discovered that (iii).

Lemma 4:

Lets $F=(f_i)_{i\in N}$ be a Jordan generalized high Homorphics of Γ-semiing M into Γ-semiring M' with additive identity and inverse. As for any $a,b,m\in M$, $\alpha,\beta\in\Gamma$ and $n\in N$.

i) $\delta_n(a,b)_\alpha\ \beta\varphi_n(m)\beta\ G_n\ (b,a)_\alpha= \delta_n(b,a)_\alpha\beta\varphi_n(m)\beta\ G_n\ (a,b)_\alpha=0$

ii) $\delta_n(a,b)_\alpha\ \alpha\varphi_n(m)\alpha\ G_n\ (b,a)_\alpha= \delta_n(b,a)_\alpha\ \alpha\varphi_n(m)\alpha\ G_n(a,b)_\alpha=0$

iii) $\delta_n(a,b)_\beta\ \alpha\varphi_n(m)\alpha\ G_n\ (b,a)_\beta= \delta_n(b,a)_\beta\ \alpha\varphi_n(m)\alpha\ G_n\ (a,b)_\beta=0$

Theorem 5:

Lets $F=(f_i)_{i\in N}$ be a Jordan generalized high Homorphics of Γ-semiring M into prime Γ-semiring M' wth additive identity and inverse. As for any $a,b,c,d,m\in M$, $\alpha,\beta\in\Gamma$ and $n\in N$:

i) $\delta_n(a,b)_\alpha\ \beta\varphi_n(m)\beta G_n(d,c)_\alpha= 0$

ii) $\delta_n(a,b)_\alpha\alpha\varphi_n(m)\alpha G_n(d,c)_\alpha=0$

iii) $\delta_n(a,b)_\alpha\ \alpha\varphi_n(m)\alpha G_n(d,c)_\beta=0$

Proof:

i) Substituting a+c for a in lemma 4(i) , we found ::

$\delta_n(a+c,b)_\alpha\ \beta\varphi_n(m)\beta G_n(b,a+c)_\alpha= 0$

$\delta_n(a,b)_\alpha\ \beta\varphi_n(m)\beta G_n(b,a)_\alpha+ \delta_n(a,b)_\alpha\ \beta\varphi_n(m)\beta G_n(b,c)_\alpha$

$+ \delta_n(c,b)_\alpha\ \beta\varphi_n(m)\beta G_n(b,a)_\alpha+ \delta_n(c,b)_\alpha\ \beta\varphi_n(m)\beta G_n(b,c)_\alpha=0$

By lemma4 (i), we found :

$\delta_n(a,b)_\alpha\ \beta\varphi_n(m)\beta G_n(b,c)_\alpha+ \delta_n(c,b)_\alpha\ \beta\varphi_n(m)\beta G_n(b,a)_\alpha =0$

Therefore, we found :

$\delta_n(a,b)_\alpha\ \beta\varphi_n(m)\beta G_n(b,c)_\alpha\ \beta\varphi_n(m)\beta G_n(ai,ib)_\alpha\ \beta\varphi_n(m)\beta G_n(b\ ,c)_\alpha=0$

$=- \delta_n(a,b)_\alpha\beta\varphi_n(m)\beta G_n(b,c)_\alpha\beta\varphi_n(m)\beta G_n(c,b)_\alpha\ \beta\varphi_n(m)\beta G_n(b,a)_\alpha=0$

Hence, by the primness of M'

$$\delta_n(a,b)_\alpha\ \beta\varphi_n(m)\beta G_n(b,c)_\alpha=0 \qquad ...\ (1)$$

Now, substituting b+d for b in lemma 4(i), we found :

$\delta_n(a,\ b+d)_\alpha\ \beta\varphi_n(m)\beta G_n\ (b+d,a)_\alpha= 0$

$\delta_n(a,b)_\alpha\ \beta\varphi_n(m)\beta G_n(b,a)_\alpha+ \delta_n(a,\ b)_\alpha\ \beta\varphi_n(m)\beta G_n(d,a)_\alpha$

$+ \delta_n(a,d)_\alpha\ \beta\varphi_n(m)\beta G_n(b,a)_\alpha+ \delta_n(a,d)_\alpha\ \beta\varphi_n(m)\beta G_n(d,a)_\alpha=0$

By lemma 4(i), we found :

$\delta_n(a,b)_\alpha\ \beta\varphi_n(m)\beta G_n(d,a)_\alpha+ \delta_n(a,d)_\alpha\ \beta\varphi_n(m)\beta G_n\ (b,a)_\alpha =0$

Then we found :

$0=\delta_n(a,b)_\alpha\beta\varphi_n(m)\beta G_n(d,a)_\alpha\beta\varphi_n(m)\beta G_n(a,b)_\alpha\beta\varphi_n(m)\beta G_n(d,a)_\alpha$

$0=- \delta_n(a,b)_\alpha\beta\varphi_n(m)\beta G_n(d,a)_\alpha\beta\varphi_n(m)\beta G_n(a,d)_\alpha\ \beta\varphi_n(m)\beta G_n(b,a)_\alpha$

Since M' is prime Γ-semiring, then:

$$\delta_n(a,b)_\alpha\ \beta\varphi_n(m)\ \beta\ G_n(d,a)_\alpha=0 \qquad ...\ (2)$$

Thus:

$\delta_n(ai,ib)_\alpha\ \beta\varphi_n(m)\ \beta G_n(b+d,a+c)_\alpha=0$

$\delta_n(ai,ib)_\alpha\ \beta\varphi_n(m)\beta G_n\ (b,a)_\alpha+ \delta_n(ai,ib)_\alpha\ \beta\varphi_n(m)\beta G_n(b,c)_\alpha$

$+ \delta_n(ai,ib)_\alpha\ \beta\varphi_n(m)\beta G_n\ (d,a)_\alpha + \delta_n(ai,ib)_\alpha\ \beta\varphi_n(m)\beta G_n(di,ci)_\alpha =0$

Since by (1) and (2) and lemma 4(i), we found :

$$\delta_n(ai,ib)_\alpha\ \beta\varphi_n(m)\beta G_n\ (di,ci)_\alpha =0$$

ii) Replace α for β in (i), we found (ii).

iii) By the same way of (i).

Theorem 6:

Every Jordan generalized high Homorphics of Γ-semiring M into prime Γ-semiring M' with additive identity and inverse is either a generalized high Homorphics of M into M' or high anti-Homorphics from M into M'.

Proof:

Lets $F=(f_i)_{i\in N}$ be Jordan generalized high Homorphics of Γ-semiring M into prime Γ-semiring M'.

Since M' is prime, we found from theorem5 (i):

$\delta_n(a,b)_\alpha=0$ or $G_n(d,c)_\alpha=0$ for any $a,b,c,d\in M$ and $\alpha\in\Gamma$.

If $G_n(d,c)_\alpha\neq 0$ for any $c,d\in M$, $\alpha\in\Gamma$ and $n\in N$ then $\delta_n(a,b)_\alpha=0$ for any $a,b\in M$, $\alpha\in\Gamma$ and $n\in N$ and hence we found F is high Homorphics from Γ-semiring M into prime Γ-semiring M'.

But if $G_n(d,c)_\alpha=0$ for any $c,d\in M$, $\alpha\in\Gamma$ and $n\in N$ then we found F is high anti-Homorphics from M into M'.

Proposition 7:

Lets $F=(f_i)_{i\in N}$ be a Jordan generalized high Homorphics from Γ-semiring M into 2-torsion free Γ-semiring M' with additive commutative
, identity and invers such that aαbβc=aβbαc, for any a,b,c∈M, α,β∈Γ, and a'αb'βc'=a'βb'αc', for any a',b',c'∈M' and α,β∈Γ then F is Jordan triple high Homorphics.

Proof:

Replace b by aβb+bβa in the definition 2.1, we found :

f_n (aα(aβb+bβa) + (aβb+bβa)αa)

$=\sum_{i=1}^{n} f_i(a)\alpha\varphi_i(a\beta b + b\beta a) + \sum_{i=1}^{n} f_i(a\beta b + b\beta a)\alpha\varphi_i(a)$

$=\sum_{i=1}^{n} f_i(a)\alpha\,(\sum_{j=1}^{i} f_j(a)\beta\varphi_j(b) + fj(b)\beta\varphi_j(a))$

$+\sum_{i=1}^{n}(\sum_{j=1}^{i} f_j(a)\beta\varphi_j(b) + f_j(b)\beta\varphi_j(a))\,\alpha\varphi_i(a)$

$=\sum_{i=1}^{n}\sum_{j=1}^{i} f_i(a)\alpha\varphi_j(a)\beta\varphi_j(b) + \sum_{i=1}^{n}\sum_{j=1}^{i} f_i(a)\alpha\varphi_j(b)\beta\varphi_j(a)$

$+\sum_{i=1}^{n}\sum_{j=1}^{i} f_j(a)\beta\varphi_j(b)\alpha\varphi_i(a) + \sum_{i=1}^{n}\sum_{j=1}^{i} f_j(b)\beta\varphi_j(a)\alpha\varphi_i(a)$

Since a'αb'βc'=a'βb'αc', for any a',b',c'∈M' and α,β∈ Γ, we found :

$=\sum_{i=1}^{n} f_i(a)\alpha\varphi_i(a)\beta\varphi_i(b) + 2\sum_{i=1}^{n} f_i(a)\alpha\varphi_i(b)\beta\varphi_i(a)$

$+\sum_{i=1}^{n} f_i(b)\beta\varphi_i(a)\alpha\varphi_i(a)$ … (1)

Alternatively,:

f_n (aα(aβb+bβa) + (aβb+bβa)αa)

= f_n (aαaβb+ aαbβa + aβbαa +bβaαa)

$$=\sum_{i=1}^{n} f_i(a)\alpha\phi_i(a)\beta\phi_i(b) + f_i(b)\beta\phi_i(a)\alpha\phi_i(a) + f_n(a\alpha b\beta a+a\beta b\alpha a) \quad \ldots(2)$$

Compare (1) and (2) and since aαbβa=aβbαa, for any a,b∈M and α,β∈Γ, and a'αb'βa'=a'βb'αa', for any a',b'∈M' and α,β∈Γ, we found :

$$f_n(2a\alpha b\beta a) = 2\sum_{i=1}^{n} f_i(a)\alpha\phi_i(b)\beta\phi_i(a)$$

Since M and M' are 2-torsion free we found :

$$f_n(a\alpha b\beta a) = \sum_{i=1}^{n} f_i(a)\alpha\phi_i(b)\beta\phi_i(a)$$

Hence, F is Jordan generalized triple high k-Homorphics.

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